2-D FIR WIENER FILTER REALIZATIONS USING ORTHOGONAL LATTICE STRUCTURES

by

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Abstract: The authors propose a new realization algorithm for the 2-D FIR Wiener filters. The reduced-order 2-D orthogonal lattice filter structure is used as its principal component as in the 1-D case. A numerical example is included to verify the formulation.

Introduction: One-dimensional (1-D) lattice structures have been wiedly used, as a subsystem, to solve FIR Wiener filtering problem. The lattice realization for the 1-D FIR Wiener is shown in Fig. 8.25 in [1]. The resulting implementation exhibits excellent numerical behavior, and has a modular structure.

Recently, an efficient solution was given in [2] for the determination of 2-D orthogonal lattice filters for autoregressive modeling of random fields. In this letter, we shall introduce a simple and computationally efficient algorithm to obtain for 2-D Wiener filtering using the orthogonal backward prediction error fields of the 2-D lattice configuration in [2]. Moreover, it will be shown that the set of normalized versions of the transfer functions of the backward prediction error filters are the 2-D analogues of the single-variable Szegö polynomials discussed in [3].

2-D FIR Wiener Filter: We suppose that we are given arrays $\boldsymbol{x} = \{\, x(k_1, k_2)\}$ and $\boldsymbol{y} = \{y(k_1, k_2)\}$, $k_1 \geq 0$, $k_2 \geq 0$ and we are required to find on $(n+1) - by - (m+1)$ filter array, $\boldsymbol{a} = \{a(k_1, k_2)\}$ for which it is true that the convolution $\boldsymbol{x} * \boldsymbol{a}$ is the best least-square approximation to the output array $\boldsymbol{y}$ [3]. It is well-known that the solution satisfies the following normal equations,

$$\boldsymbol{\Phi a} = \boldsymbol{g} \tag{1}$$

The block-Toeplitz matrix consists of blocks, $\mathbf{\Phi} = \{\boldsymbol{\varphi}_{jk}\},\ 0 \le j, k \le n,$ where $\boldsymbol{\varphi}_{jk}$ is an $(m+1)-by-(m+1)$ matrix and its elements are the autocorrelation of the input array. $\boldsymbol{g}$ is the cross-correlation of the output array $\boldsymbol{y}$ with the input array $\boldsymbol{x}$.

Several algorithms for solving systems in eqn.1 are reported in the literature [3-5]. Justice [3] has extend the single-variable orthogonal Szegö polynomials into the 2-D case to develop an efficient algorithm for solving the system in eqn.1. Wax and Kailath [4] generalized Trench's [6] algorithm to the Toeplit-block Toeplitz matrix. The use of this algorithm for an iterative solution of a Toeplitz system of linear equations was also presented. Recently, Tummala [5] has presented a new iterative algorithm to solve both Toeplitz and non-Toeplitz block matrix equations.

2-D Lattice Representation: The 2-D FIR Wiener filter can be expressed as

$$\hat{y}(k_1, k_2) = \boldsymbol{a}^T \boldsymbol{x}_M(k_1, k_2) \tag{2}$$

where $\boldsymbol{x}_M(k_1, k_2)$ is defined as the vector observations

$$\boldsymbol{x}_M(k_1, k_2) = \left[x(k_1, k_2)\ x((k_1, k_2)-1) \cdots x((k_1, k_2)-M)\right]^T \tag{3}$$

and $\boldsymbol{a} = [a_0 a_1 ... a_M]^T$ is the filter coefficient vector and satisfies the normal equations in eqn.1. M denotes the number of points in the support region of the prediction error filter and the notation $((i, j) - p)$ denotes the pth element behind (i, j) in the prediction region.

Our approach here is different in that for the vector of observations in eqn.2. We can use the set of orthogonal backward prediction errors,

$$\boldsymbol{b}_M(k_1, k_2) = \left[b_0^{(0)}(k_1, k_2)\ b_1^{(1)}((k_1, k_2) \cdots b_M^{(M)}((k_1, k_2)-M)\right]^T \tag{4}$$

Where $b_i^{(i)}(k_1, k_2)$'s for $i = 0,1,...,M$ are obtained by feeding the input of the orthogonal lattice with the input array $\boldsymbol{x}$ as shown in [1]. The 2-D lattice predictor transforms the sequence of correlated input samples $x(k_1,k_2),\ x((k_1,k_2)-1),\ldots,x((k_1,k_2)-M)$ into corresponding sequence of uncorrelated backward prediction errors given in eqn.4. The transformation between the input vector $\boldsymbol{x}_M(k_1,k_2)$ and the backward prediction error vector $\boldsymbol{b}_M(k_1,k_2)$ can be written as

$$\boldsymbol{b}_M(k_1,k_2) = \boldsymbol{L}_{b,M}\boldsymbol{x}_M(k_1,k_2) \tag{5}$$

where $\boldsymbol{L}_{b,M}$ is a nonsingular lower triangular matrix. Using eqns. 1-5 the output of the 2-D Wiener filter can be written as

$$\hat{y}(k_1,k_2) = \boldsymbol{w}^T\boldsymbol{b}_M(k_1,k_2) \tag{6a}$$

with

$$\boldsymbol{w} = \boldsymbol{E}_{b,M}^{-1}\boldsymbol{L}_{b,M}\boldsymbol{g} \tag{6b}$$

where $\boldsymbol{E}_{b,M} = diag.matrix\left[E_{b_0}^{(0)}\ E_{b_1}^{(1)}\cdots E_{b_M}^{(M)}\right]$ and $E_{b_i}^{(i)}$'s $i = 0,1,\ldots,M$ are the backward prediction error powers [2]. Eqn. 6a shows that the output of estimate can be formed as a linear combination of the backward errors. It is interesting to note that the weights in the weight vector $\boldsymbol{w}$ in eqn. 6b are the normalized cross-correlations between the backward errors and the desired output array **y**. From eqns. 5 and 6b, we observe that $\boldsymbol{w}$ is just normalized version of the 2-D lattice structure response that would result if the sequence corresponding to $\boldsymbol{g}$ were applied as input. As in the 1-D case [1], this eliminates the need to determine elements of the matrix $\boldsymbol{L}_{b,M}$. On the other hand, it is possible to show that the transfer functions of the normalized backward prediction errors form a set 2-D of orthogonal Szegö polynomials [3] on the unit bidisc $P_i(z_1,z_2) = \left(E_{b_i}^{(i)}\right)^{-1/2} B_i^{(i)}(z_1,z_2),\ i = 0,1,2,\ldots,M$.

Example: The following 2-D wiener filtering example is described by Justice [3]. The input and output arrays are infinite, but only nonzero portion of each array which actually enters into calculations. The desired input mask of the 2-D FIR filter is a 3-by-2 coefficient array. The nonzero portions of the input autocorrelation and the input-output crosscorelation values are given in Fig.1.

In order to generate a 3-by-2 input array, we can consider a reduced order 2-D lattice structure [2] as shown in Fig.2a. The ordering arrangements of the prediction region for the first-quadrant model is depicted in Fig.2b. It is possible to obtain the forward and backward lattice parameters in each lattice section from scant knowledge of autocorrelation values of the input array, $R_{xx}(m,n)$, given in Fig.1a by using the 2-D Schur algorithm in [7]. Then the six orthogonal polynomials are obtained from the structure in Fig.2b as follows.

$$
\begin{aligned}
&B_0^{(0)}(z_1,z_2) = 1.0 \\
&B_1^{(1)}(z_1,z_2) = -0.6374 + 1.0z_1^{-1} \\
&B_2^{(2)}(z_1,z_2) = -0.3209 - 0.0482z_1^{-1} + 1.0z_2^{-1} \\
&B_3^{(3)}(z_1,z_2) = 0.2684 - 0.3608z_1^{-1} - 0.6406z_2^{-1} + 1.0z_1^{-1}z_2^{-1} \\
&B_4^{(4)}(z_1,z_2) = 0.1146 + 0.0386z_1^{-1} - 0.3677z_2^{-1} - 0.0534z_1^{-1}z_2^{-1} + 1.0z_2^{-1} \\
&B_5^{(5)}(z_1,z_2) = -0.1259 + 0.1480z_1^{-1} + 0.3109z_2^{-1} - 0.4162z_1^{-1}z_2^{-1} - 0.6515z_2^{-1} + 1.0z_1^{-1}z_2^{-2}
\end{aligned}
\tag{7}
$$

and the corresponding backward prediction error powers are calculated as

$$
\boldsymbol{E}_{b,5} = diag.matrix\begin{bmatrix} 91.0 & 54.03 & 79.62 & 46.86 & 78.01 & 45.76 \end{bmatrix} \tag{8}
$$

From eqns. 6-8 and the corss-correlation values givenin Fig 1b, one can obtain the weights shown in Fig.2b,

$$
\boldsymbol{w} = \begin{bmatrix} 2.9341 & 3.7352 & 1.2984 & 2.5333 & 2.6415 & 0.9166 \end{bmatrix}^T \tag{9}
$$

In order to verify our algorithm, we can calculate the 2-D FIR Wiener filter coefficients. From eqns. 2 and 6a, the filter coefficient vector $\boldsymbol{a}$ is given by

$$
\boldsymbol{a} = \boldsymbol{L}_{b_1M}^T \boldsymbol{w} = \begin{bmatrix} 1.0 & 3.0 & -1.0 & 2.0 & 2.0 & 1.0 \end{bmatrix}^T \tag{10}
$$

If we compare this calculated coefficient vector with filter coefficients obtained by solving normal equations in eqn.1, is it appearent that the proposed lattice realization algorithm exactly determines the 2-D FIR Wiener filter coefficient values.

Conclusion : In this letter, a new method is developed for the realization of the 2-D Wiener filters from the given correlation values. The proposed algorithm is based on the 2-D orthogonal lattice structure. The complexity of our algorithm is less than all existing methods [3-5].

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