Analysis of Brushless Permanent Magnet Synchronous Motors

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Abstract—A brief design review of the Permanent Magnet Synchronous Motors has been presented. A procedure has been developed to predict the steady state and dynamic performances of a brushless permanent magnet synchronous motor. Finite element analysis has been combined with a lumped parameter circuit model in order to provide satisfactory engineering information. To this end, two coordinated steps are involved. One is to develop a unified lumped parameter circuit model for both steady state and dynamic analysis. The second step is to extract the individual lumped parameters from finite element solutions based on corresponding equivalent circuits, each with a pre-determined topology. The proposed techniques have been experimentally verified in a laboratory permanent magnet synchronous motor.

I. INTRODUCTION

There are many types of synchronous motors. Prominent among these are wound-field and permanent magnet types. Synchronous motors are widely used in constant speed drives at line frequency as well as in variable speed drives with inverter-fed variable frequency supplies [1]–[3]. The main advantages, as compared with induction motors, are the absence of rotor slip power loss and the natural ability to supply reactive current. Since the magnetic excitation may be provided from the rotor side instead of the stator, the machine can be built with a larger airgap without degraded performance. The ability to supply reactive current also permits the use of natural-commutated dc link converters. These motors also have lower weight, volume, and inertia compared to dc motors for the same ratings.

Wound-field synchronous motors are well-known in industrial drives requiring constant speed irrespective of load. Separately controllable dc excitation in the form of dc field current is a unique feature in the power factor control of such a motor from the lagging to leading mode. The leading power factor synchronous motor is also known as synchronous condenser used in power factor correction and reactive power compensation in transmission systems [4]. The operation at unity power factor mode reduces the stator and system copper losses and permits inverter size reduction with simplicity of commutation circuits, switching logic and control. These are used in high-power mill-drives, compressors, conveyers, extruders, etc. The dc excitation current to the motor is usually fed from a static power semiconductor-based exciter system through brushes and slip rings. When the brushes and slip rings are not acceptable because of environmental constraints and frequent maintenance requirements, they are replaced by a brushless excitation system having shaft-mounted rectifiers in the ac excitation system, and rotating with the rotor. These brushless synchronous motors are used in medium- and low-power industrial applications.

In certain applications, the field excitation can be provided using permanent magnets, thus dispensing with brushes, slip rings and the dc field winding losses [5], [6]. Applications of these permanently excited synchronous motors are found in various industrial drive systems, such as aerospace, machine tools, robotics, precision textiles, etc. The brushless permanent magnet synchronous motors are simply known as permanent magnet ac motors. These motors are either line-start or inverter-fed types whose polyphase stator windings are simultaneously switched on via balanced polyphase supply voltages [7].

There is another type of modern small brushless permanent magnet dc motors, commonly called brushless dc motors. Incidentally, these brushless dc motors are polyphase permanent magnet synchronous motors which are electronically commutated and sequentially switched on. These are usually operated in the self-control mode [8], [9]. These are mostly used for control purposes such as computer disk drives, robotics, automation, high-quality turntables and tape capstans, etc.

Modern brushless permanent magnet synchronous motors may operate near unity power factor and have a large pull-out torque for a given frame size [10]. Depending on the location of the magnets on the rotor, typical brushless permanent magnet synchronous motors can be broadly classified into three kinds, i.e., interior PM synchronous motors which have the permanent magnets buried inside the rotor; surface PM synchronous motors which have their permanent magnets mounted on the surface of the rotor; and inset PM synchronous motors in which the permanent magnets are inset or partially inset into the rotor [11]. Depending on the magnetization orientation for the magnets, the rotor can also be classified into radially or circumferentially oriented type configurations. For surface type configuration, the only direction of magnetization is radial [8]. In a brushless PM synchronous motor, there is no provision for rotor side excitation control. The control is done entirely through the stator terminals [1], [12]. For variable speed drive applications, both the constant torque and constant power operating modes are required. As the magnet flux of
a PM motor is usually fixed, the airgap flux weakening is accomplished by controlling the direct-axis current at high speed [13]. This results in low efficiency at high speed; and the permanent magnet may be subjected to irreversible demagnetization states [14].

A synchronous motor, regardless of wound-field or permanent magnet type, is unable to self-start when fed from a constant frequency source [4]. Some sort of starting by either a rotor squirrel cage conduction winding or even a hysteresis/ferromagnetic ring [15]-[16] is required to develop the starting torque. In brushless PM synchronous motors, the squirrel cage winding also protects the magnets from demagnetization during the start-up transient and sudden increase in the stator current, and also acts as a damper to the machine oscillation. This helps to keep the rotor in synchronism, whenever the rotor speed deviates from the synchronous speed.

Due to increased competition in world drive markets and the pressure of increased cost of electrical energy, it is essential to predict synchronous and dynamic performances of a PM motor in order to avoid the design misjudgment that can prove costly once the motor is manufactured. Equivalent circuits and lumped parameter models have been the traditional tools to calculate the performance of electrical machines and to conduct the design optimization with idealized and simplified physical representation [1]. However, it is difficult to take into account the effects of magnetic saturation, complex configuration, eddy currents and external system impedances at the same time. Numerical techniques have been recognized as practical and accurate method of field computation to aid in the machine design. Finite elements, amongst numerical methods, have emerged as a suitable technique for electrical machine design and performance evaluation in low frequency applications, because its elements can be easily adapted to any shape of boundary and interface geometry [4], [17]. But this is a time-consuming process. In order to be computationally simple and at the same time functionally accurate, field and circuit combined analysis is a desirable solution. Due to the fact that the PM synchronous motor is receiving increasing attention with the advancement of permanent magnet materials for high efficiency and high performance drive applications, only interior permanent magnet type synchronous motor is employed as an example to detail the procedure of field and circuit combined approach in the following discussion.

II. DESIGN CONSIDERATION

Recent advances in permanent magnet materials, power semiconductor switching devices and control strategies have given rise to a great variety of PM synchronous motor designs and applications [6], [7]. The motor designer has the control over several variables such as magnet material and configuration, the number of poles and the placement of conduction cage winding.

Proper selection of magnetic materials is important from both economic as well as performance considerations. Alnico has high remanence but the coercivity is low, with a nonlinear demagnetization curve. Therefore, it is not suitable for the use in ac motor design except for very small machines. Ferrite material is low in cost and has excellent linearity in demagnetization, but the remanence is low and more material is required to achieve a given flux level in the airgap. Hence ferrite magnets tend to be used in drives where high dynamic performance is less important than motor cost. Samarium-Cobalt has substantially increased residual flux density and coercivity, but cost is high and likely to remain so. As a result, it is used in high performance servo drives, where a high torque to inertia ratio is desirable. A further increase in residual flux density and coercivity has been achieved with the introduction of Neodymium-Boron-Iron magnet material at a lower cost than the Samarium-Cobalt material [18]. Factors that can limit the applications include the low Curie temperature and the susceptibility of the iron alloy to corrosion.

The surface mounting design tends to yield a small rotor diameter with low inertia, which is conducive to good dynamic performance. For this reason, many servo drives employ the surface magnet motor design. The inset magnet design can lead to establish that the q-axis inductance is larger than the d-axis inductance. The saliency will produce significant reluctance torque as well as the magnet torque. The interior magnet design offers the advantages of mechanical robustness and a smaller airgap, which allows for a degree of flux weakening when the motors are expected to operate in constant power mode at high speed. In the interior magnet design it is possible to produce greater flux density than the flux density at the magnet surface by circumferential magnetization with the poles in opposition to focus the flux at the pole faces [19].

As the number of the pole pairs increases, the ratio between the pole pitch and the radius of the rotor decreases and some constructions become better than others. Higher number of poles allows the production of higher airgap flux density for a given magnet material. Iron loss considerations encourage the use of fewer poles in order to achieve high speed operation at low frequencies.

For a variable speed PM motor fed from the current source inverter, although there is no need to develop the starting conduction cage torque, as the motor drive can be started from standstill, the conduction cage windings are helpful in reducing the distortion and spikes of line-to-line voltage [3]. The damper windings can also reduce the commutating inductance, and thus help in reducing the commutation overlap. Therefore, damper windings are also employed in these drives. However, for the drive with voltage-source inverter, dampers in the rotor would provide a path for the flow of harmonic currents induced from the inverter nonsinusoidal voltage waveforms [20]. Therefore, it is advantageous not to have dampers in this kind of drives with voltage source inverters.

In a PM motor design, since the space in the rotor is very limited, proper utilization of magnetic material is a vital consideration. The stator punchings are made of low loss coated modern electrical steels with laser welded stackings. Good designs require the optimum use of material by operating
at the highest energy density possible, dimensioning of the magnets for proper aspect ratios at load conditions and at the same time to operate at optimum flux density level. This is a complex problem and best solved using finite element based field analysis.

The field and circuit combined approach can not only greatly simplify the analysis and evaluation of machine performance, but also aid to complete the two-dimensional (2-D) finite element analysis by introducing the possibility to take into account the three-dimensional (3-D) part of the machine. Another advantage is the ability to consider such a machine as a component of a much wider system. This approach will involve two coordinated stages. One is to develop an appropriate lumped parameter model. The other one is to extract individual lumped parameters from the associated finite element solutions based on corresponding equivalent circuits, each with a predetermined topology.

III. LUMPED PARAMETER MODEL

An analysis will be presented for a 3-phase motor, since this class of PM motor is the most likely candidate for wide industrial applications. The standard two-axis theory with fixed rotor reference frame will be used. Although a transformation that compact the three phase model to such an equivalent two-phase description is not difficult to find, it is desirable to utilize a particular transformation for the voltage and currents such that the representation of electrical power flow in the machine is preserved. As a result, the developed torques are invariant over the transformation. At the same time, it would be convenient that the transformation be the same for both the voltage and current. Assuming that a square transformation matrix \( [C] \) satisfies these two requirements and transforms the variables \( \{i\}_{abc} \) and \( \{v\}_{abc} \) in 3-phase abc system to a new set of variables \( \{i\}_{dq0} \) and \( \{v\}_{dq0} \) in dq0 system as

\[
\{v\}_{dq0} = [C]\{v\}_{abc}
\]

\[
\{i\}_{dq0} = [C]\{i\}_{abc}
\]

Power invariance is described by

\[
\{v\}_{abc} \cdot \{i\}_{abc} = \{v\}_{dq0} \cdot \{i\}_{dq0} = ([C]\{v\}_{abc})^t \cdot ([C]\{i\}_{abc})
\]

\[
= \{v\}_{abc}^t[C]^t[C]\{i\}_{abc}
\]

where superscript \( t \) denotes the transpose of a matrix. Therefore,

\[
[C]^t[C] = [I]
\]

where \([I]\) is an identity matrix. Thus, a necessary condition on the transformation matrix \([C]\) satisfying the requirements of invariant power and identical transformations for both current and voltage is

\[
[C]^{-1} = [C]^t.
\]

That is, the transformation matrix \([C]\) must be Hermitian or orthogonal. Equivalently, \([C]\) must be unitary.

For the model of a PM motor, the following specific matrix \([C]\) will be applied to define the transformation of variables from the stationary reference frame \( abc \) to the rotor reference frame \( dq0 \) [21]–[22]:

\[
[C] = \sqrt{\frac{2}{3}}
\]

\[
\begin{bmatrix}
\cos \theta_r & \cos \left(\theta_r - \frac{2\pi}{3}\right) & \cos \left(\theta_r + \frac{2\pi}{3}\right)
\sin \theta_r & \sin \left(\theta_r - \frac{2\pi}{3}\right) & \sin \left(\theta_r + \frac{2\pi}{3}\right)
\end{bmatrix}
\]

\[
\begin{bmatrix}
1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}
\end{bmatrix}
\]

where \( \theta_r \) is the angle between the \( d \)-axis and the axis of phase A. It can be easily verified that this transformation matrix \([C]\) satisfies the condition of (5). Since three-phase PM motors usually have balanced wye or delta connected windings with no neutral connection and thus effectively have only two independent phases, the transformation matrix \([C]\) can be further reduced by eliminating the third row in the absence of zero-sequence component. When such a transformation is applied to the variables of a three-phase PM motor, the three phase symmetrical windings A, B and C can be replaced by equivalent two-phase windings d and q as shown in Fig. 1, in which the magnets are identified by \( \psi_f \) and rotor winding is represented by equivalent shorted \( D, Q \) windings.

If the three-phase terminal voltages are given as

\[
\begin{bmatrix}
v_A \\
v_B \\
v_C
\end{bmatrix}
= \begin{bmatrix}
\sqrt{2}V \cos \omega t \\
\sqrt{2}V \cos(\omega t - 120^\circ) \\
\sqrt{2}V \cos(\omega t - 240^\circ)
\end{bmatrix}
\]

then the corresponding voltages in \( dq0 \) system will be

\[
\begin{bmatrix}
v_d \\
v_q
\end{bmatrix}
= \sqrt{3}V \begin{bmatrix}
\cos(\omega t - \theta_r) \\
-\sin(\omega t - \theta_r)
\end{bmatrix}
\]

For such an equivalent primitive commutator machine, with the conventional directions shown in Fig. 1 and subject to
motor conventions, the system governing equation can be written as [22].

$$\{v\} = [Z]\{i\} = ([R] + [L]p + [G]\omega)\{i\}$$ (9)

where $p$ is the derivative operator, $[R]$ is the diagonal resistance matrix, $[L]$ is the symmetric inductance matrix, and $[G]$ is the asymmetric torque matrix. These matrices are given by (10)–(12), respectively

$$[R] = \text{diag}(r_1, r_1, r_{2d}, r_{2q}, r_f)$$ (10)

$$[L] = \begin{bmatrix}
L_d & 0 & L_{md} & 0 & 0 \\
0 & L_q & 0 & L_{mq} & 0 \\
L_{md} & 0 & L_D & 0 & 0 \\
0 & L_{mq} & 0 & L_Q & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}$$ (11)

$$[G] = \begin{bmatrix}
0 & L_q & 0 & L_{mq} & 0 \\
0 & 0 & -L_{md} & 0 & -L_{md} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}$$ (12)

and the voltage and current vectors are given by (13) and (14), respectively

$$\{v\} = \{v_d \ v_q \ v_D \ v_Q \ E_0\}'$$

$$\{i\} = \{i_d \ i_q \ i_D \ i_Q \ i_f\}'$$ (14)

where suffixes $d$ and $q$ denote the $d$-axis and $q$-axis quantities of the stator, respectively. Suffixes $D$ and $Q$ denote the $d$-axis and $q$-axis quantities of the rotor, respectively. Letter $L$ denotes the inductances, and suffix $m$ denotes the mutual quantities between the $d$-axis and $q$-axis. The magnet is represented by equivalent current $i_f$, whose value is defined by

$$E_0 = r_f i_f = \frac{1}{\sqrt{3}} \omega L_{md} i_f$$ (15)

where $E_0$ is the induced phase-voltage in $abc$ system by the magnet only at the synchronous speed. The torque matrix is derived from the fact that the motion induced voltage terms are the ones that contribute to the electromechanical power conversion process and, therefore, are associated with the torque

$$T_e = P\{i\}'[G]\{i\} = P([L_q - L_d]i_d i_q - L_{md} i_q i_D + L_{mq} i_d i_Q - L_{md} i_q i_f)$$ (16)

where $P$ is the number of pole pairs.

It should be noted that (9) and (16) are quite general. They describe steady-state as well as transient conditions. The parameters in these equations will change with loads due to the effects of magnetic saturation and skin effect. These load dependent parameters will be determined from the finite element solutions.

Although the established generalized system model (9) has provided the capability for complete analysis of any operation mode of a brushless PM synchronous motor, it is still advisable to specialize the generalized system equations for a particular operation mode. It will not only simplify the calculation but also provide a convenient way to explore some distinguishing features of a PM motor.

In order to have a deep insight of starting characteristics of a brushless PM synchronous motor fed from a fixed frequency line voltage source, two approaches cooperate to deal with asynchronous performance during the run-up period. One is the quasi-dynamic analysis approach with pseudo-constant speed characteristic. The other is transient analysis approach. The former can lead to an identification of different current and torque components, presents a clear physical interpretation of the cause of each component, and allows convenient parameter sensitivity analysis on the overall system; while the latter will provide more detailed information such as possible demagnetizing effect on the magnets, maximum current requirement for the inverter and the time of run-up to stable synchronization, as well as synchronization process.

A. Quasi-Dynamic Model

The quasi-dynamic analysis model is based on the assumption that the motor operates under steady-state at each slip point during run-up assuming that the mechanical time constants are very large compared with the electrical time-constants. Therefore, using quasi-dynamic analysis model, no electromagnetic transient is taken into account.

If we assume magnetic materials are linear, starting process can be treated as the superposition of two operation modes. One mode is the operation of an unsymmetric asynchronous motor alone. The other one is the operation mode of a magnet-excited asynchronous generator with short-circuit stator windings. However, saturation phenomenon in fact is unavoidable. This difficulty is surmounted by adopting adjusted saturated parameters, where the interaction between the two modes has been appropriately considered. Consequently, the superposition scheme is assumed for the analysis [21].

1) Unsymmetric Asynchronous Motor Mode: It has shown in (8) that for a balanced three phase terminal voltages, the corresponding transformed two-phase voltages in the rotating rotor frame will be

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \sqrt{3} V \begin{bmatrix} \cos(\omega t - \theta) \\ -\sin(\omega t - \theta) \end{bmatrix}$$

$$= \sqrt{3} V \begin{bmatrix} \cos(\omega t + \delta) \\ -\sin(\omega t + \delta) \end{bmatrix}$$ (17)

where $s$ is the slip; $\delta$ is the angle at which the system would have operated had the machine synchronized. The concern at hand is only run-up speed that ends just under synchronism and not to synchronism, itself. In this case, the angle $\delta$ is irrelevant and is equated to zero. Since all the exciting functions are sinusoidal and of the same angular frequency $\omega$ in $d$-$q$ axis system, the use of complex phasor is preferred. With $V_d$ as reference phasor, the applied voltage source can be expressed as

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \sqrt{2} V \begin{bmatrix} 1 \\ j \end{bmatrix}$$. (18)
The operator of the impedance matrix can be replaced by \( jw_s \) in order to obtain the steady-state impedance matrix, and if \( \mu_s = \omega_s / \omega_s = 1 - s \), then \( \omega_s \) can be replaced by \( \mu_s \). Omitting the row of the impedance matrix corresponding to permanent magnet, because only asynchronous motor mode is considered at hand, we have

\[
[Z] = \begin{bmatrix}
    r_1 + jw_s X_d & \mu X_q & jw_s X_{md} & \mu X_{mq} \\
    -\mu X_d & r_1 + jw_s X_q & -\mu X_{md} & jw_s X_{mq} \\
    jw_s X_{md} & 0 & r_{2d} + jw_s X_D & 0 \\
    0 & jw_s X_{mq} & 0 & r_{2q} + jw_s X_Q \\
\end{bmatrix}
\]

(19)

For the sake of convenience, it is suitable to eliminate the shorted rotor part first, which leads to the reduced impedance matrix

\[
[Z'] = \begin{bmatrix}
    r_1 + jw_s X_d + \frac{s^2}{r_{2d} + jw_s X_D} X_{md} & \mu X_q - \frac{jw_s \mu X_{mq}}{r_{2q} + jw_s X_Q} \\
    -\mu X_d & r_1 + jw_s X_q + \frac{s^2}{r_{2d} + jw_s X_D} X_{md} & \mu X_q - \frac{jw_s \mu X_{mq}}{r_{2q} + jw_s X_Q} \\
\end{bmatrix}
\]

(20)

Hence, the stator currents can be figured out by the inverse of impedance matrix \([Z']\). Let the solution be represented by

\[
\{i^m\} = (T_d \quad T_q) \quad \text{as the state-variables. Thus, with necessary rearrangement the above equations can be arranged in state variable form as}
\]

\[
\begin{align*}
    \psi_d &= \omega - \omega_r \\
    \psi_q &= \frac{\partial}{\partial J} (T_e - T_L) - B \frac{\partial}{\partial \omega_r} \\
    p d &= \frac{1}{\Delta_d} \left( r_L L_d i_d + L_L i_d \omega_r \right) - L_m r_d \psi_d + L_m \psi_d \omega_r
\end{align*}
\]

(27)

2) Magnet-Excited Asynchronous Generator Mode: In this case the stator windings are shorted and thus \( V_d = V_q = 0 \). At the same time, it is obvious that no induced currents exist in rotor windings, that is, \( i_q = 0 \). It is also noted that the only excitation source \( \omega_r L_{md} i_f \) is constant and the steady-state impedance matrix is obtained by replacing \( p \) by zero. If the term \( \omega_r L_{md} i_f \) is moved to the left of the equality, (9) can be written as

\[
\begin{align*}
    \{i^m\} &= \begin{bmatrix}
        i_d^m \\
        i_q^m \\
    \end{bmatrix} = \begin{bmatrix}
        r_1 & \mu X_q \\
        -\mu X_d & r_1
    \end{bmatrix} \begin{bmatrix}
        \sqrt{2} (A \cos \omega_r t - B \sin \omega_r t) \\
        \sqrt{2} (E \cos \omega_r t - F \sin \omega_r t)
    \end{bmatrix}
\end{align*}
\]

(22)

The short-circuit currents in the stator can be readily obtained as

\[
\{i^q\} = \begin{bmatrix}
    i_d^q \\
    i_q^q \\
\end{bmatrix} = \frac{\sqrt{3}}{r_1 + \mu X_d X_q} \begin{bmatrix}
    -\mu^2 X_d X_q E_0 \\
    \mu r_1 E_0 \\
    0
\end{bmatrix}
\]

(24)

3) Current of Each Winding: Superimposing the results of the above two operating modes results in the \( d-q \) axis instantaneous current components for each winding as

\[
\{i\} = \{i^m\} + \{i^q\}.
\]

The results can be transformed back to \( abc \) system to get the instantaneous value of each phase current if necessary, for example for \( A \)-phase in the absence of zero-sequence component

\[
i_A = \sqrt{2} (\cos \omega_r t + \sin \omega_r t).
\]

The total instantaneous torque containing both the average and pulsating components can be determined by substituting (25) in the torque expression of (16).

B. Dynamic Model

If both the mechanical and electromagnetic transients are needed to be included in the analysis, a realistic dynamic model is necessary [27]. In addition to the system (9), the other necessary equations to deal with transient behavior are the mechanical equations as follows:

\[
\begin{align*}
    \psi_d &= \psi - \omega_r \\
    \psi_q &= \frac{\partial}{\partial J} (T_e - T_L) - B \frac{\partial}{\partial \omega_r} \\
    p d &= \frac{1}{\Delta_d} \left( r_L L_d i_d + L_L i_d \omega_r \right) - L_m r_d \psi_d + L_m \psi_d \omega_r
\end{align*}
\]

(28)

where \( \Delta_d = \{i\} [G] \{i\} \) is the electro-dynamic torque defined by (16); \( T_L \) is the load torque; \( B \) is the friction coefficient, \( p \) is the time derivative and \( J \) is the moment of inertia. From (9), (27), and (28), it is obviously convenient to choose the four currents \( i_d, i_q, \omega_r \) and \( \omega_r \), the rotor torque angle \( \delta \) and the rotor speed \( \omega_r \) as the state-variables. Thus, with necessary rearrangement the above equations can be arranged in state variable form as

\[
\begin{align*}
    \psi_d &= \psi - \omega_r \\
    \psi_q &= \frac{\partial}{\partial J} (T_e - T_L) - B \frac{\partial}{\partial \omega_r} \\
    p d &= \frac{1}{\Delta_d} \left( r_L L_d i_d + L_L i_d \omega_r \right) - L_m r_d \psi_d + L_m \psi_d \omega_r
\end{align*}
\]

(29)
C. Steady-State Synchronous Model

Upon considering synchronous behavior of this system, it is only a matter of setting the derivative terms to zero and \( \omega = \omega \) or \( s = 0 \). Due to \( i_d = i_q = 0 \), in terms of (17) and (19) the system equation in reduced order form is given as

\[
\begin{bmatrix}
-\sqrt{3} V \sin \delta \\
-\sqrt{3} V \cos \delta + \sqrt{3} E_0
\end{bmatrix} = \begin{bmatrix}
r_1 & X_q \\
-\frac{V}{R} & \frac{r_1}{X_q}
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix}
\]

where \( \delta \) is the torque angle. Therefore, (30) can be easily solved to get

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} = \begin{bmatrix}
\frac{\sqrt{3}}{r_1^2 + X_d X_q} \\
\frac{V(X_q \cos \delta - r_1 \sin \delta) - E_0 X_q}{-V(r_1 \cos \delta + X_q \sin \delta) + E_0 r_1}
\end{bmatrix}
\]

Correspondingly, the torque (16) can be reduced to

\[
T_e = P [(L_q - L_d) i_d i_q - L_m i_d i_q]
\]

Since the reluctance torque varies with \( \sin 2\delta \) and the magnet alignment torque varies with \( \sin \delta \), there are ranges of \( \delta \) in which these torques are additive, resulting in a higher pull-out torque than would be obtained with either by itself [23]. The value of \( \delta_0 \) can be approximately derived by manipulating (33) as

\[
\delta_0 = \cos^{-1} \left( \frac{E_0}{V(1 - X_d/X_q)} \right)
\]

IV. SIMULATION OF SYNCHRONOUS PERFORMANCE

Since modern permanent magnets, such as NdFeB, have linear second quadrant B-H characteristics over the normal operating range, they can be modeled as simple current sheets surrounding a material having a permeability equal to the recoil permeability of the material. Since the stator magnetomotive force (MMF) is in alignment with the phase which is carrying maximum current, it is convenient to choose the axis of A phase as a time-axis. Then, the initial phase angle \( \beta \) of the stator current \( I_1 \) is also the space angle between the stator and the rotor MMF's.

For a PM motor fed from a current source, the finite element analysis is relatively straightforward. Different operating modes can be simulated just by shifting the angle \( \beta \). For a voltage source supply, however, the current \( I_m \) satisfying the prescribed terminal voltage \( V_0 \) for a specific angle \( \beta \) is not available until field solutions are obtained. Therefore, the following Newton-Raphson iterative algorithm will be applied to evaluate the updated value of \( I_m^{(i+1)} \) in the \( i \)th iteration loop

\[
I_m^{(i+1)} = I_m^{(i)} + (V_0 - V_c^{(i)})/V_{r}^{(i)}
\]

where \( V_c^{(i)} \) is the calculated value and \( V_r^{(i)} \) is its derivative. This procedure will be repeated until certain convergence criterion is satisfied.

For obtaining synchronous parameters, the conventional method considers neither \( E_0 \) changed with load nor the interaction between the d-axis and q-axis [23]-[25]. Therefore, a novel method, called the loading method can be applied to determine the saturated parameters. From field analysis, it is not difficult to get the d-axis and q-axis fundamental fluxes

\[
\begin{align*}
T_{e1} &= \frac{3 P E_0 V}{\omega X_d} \\
T_{e2} &= \frac{3 P V^2}{\omega} \left( \frac{X_q - X_d}{X_d X_q} \right)
\end{align*}
\]
per pole. These are denoted as $a_1$ and $b_1$ respectively. The inner torque angle and the induced phase voltage are obtained by the following expressions

$$\delta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$$  \hspace{1cm} (37)
$$E_i = 4.44f\phi_mN_xk_\phi k_{dp1}$$  \hspace{1cm} (38)

where $\phi_m = \sqrt{a_1^2 + b_1^2}$ is the resultant fundamental flux per pole; $k_{dp1}$ is the fundamental winding factor and $k_\phi$ is the skew slot factor. Further equating the voltage components in the $d$-axis and $q$-axis, one gets

$$E_i \cos \delta_1 = E_0 + I_1 \cos \beta X_{md}$$  \hspace{1cm} (39)
$$E_i \sin \delta_1 = I_1 \sin \beta X_{mq}.$$  \hspace{1cm} (40)

From (40), the $q$-axis magnetizing reactance $X_{mq}$ can be easily obtained. However, from (39), it is inadequate to evaluate two unknown quantities $E_0$ and $d$-axis magnetizing reactance $X_{md}$. To this end, a small displacement from the operating point is applied and then the corresponding load field calculation is used to get the following equation

$$E_i' \cos \delta_1' = E_0 + I_1' \cos \beta X_{md}'.$$  \hspace{1cm} (41)

In fact, this approach does not demand extra field calculation. After the load field investigation covers the whole range of operation, it has become possible to draw three curves $E_i(\beta)$, $\delta_1(\beta)$, and $I_1(\beta)$ by the curve fitting scheme. As a result, for any particular operating point, the required information for both (39) and (41) can be obtained by reading up the three ready-made curves. Practical application shows that this scheme not only simplifies the computation, but also offers a satisfying result as long as the selected displacement point from the curves is not unreasonably too far from the chosen operating point.

A laboratory 1-hp, 4-pole interior-magnet type motor with the configuration shown in Fig. 3 is employed to illustrate the model [26]. The calculated values of the load-dependent parameters of $X_d, X_q$, and $E_0$, by using the loading method, are shown in Fig. 4 by dashed lines. In the same figure, included by solid lines are the corresponding experimental values. The operating point where the $d$-axis armature reaction is changing from magnetizing to demagnetizing region is around 48° of torque angle. It is noted that at the torque angle around 25°, there exists a dip on the $E_0$ curve and a crest on the $X_d$ curve. The reason for this is due to the fact that this operating point lies in the magnetizing region of $d$-axis armature reaction, and the direction of the armature reaction flux in the inter-pole ferromagnetic bridge of the magnets is opposite to the direction of the leakage flux of the magnets. Even so, under the most of its operation range, the small magnet leakage flux can easily make this narrow ferromagnetic bridge highly saturated. Thus, the main path of the $d$-axis component of armature reaction is across the magnet or the inter-pole magnetic barrier, resulting in a very large reluctance. However, under certain load conditions (for this particular example, around 25°), these two flux components in the opposite direction turn out to be somewhat balanced and make the magnetic bridge become least saturated.

As a result, the leakage flux of the magnets across the bridge increases rapidly. At the same time, the unsaturated bridge also provides a shortcut for the $d$-axis component of the armature reaction flux. Consequently, at this particular load condition, $X_d$ increases notably with the considerable reduction of $E_0$.

It can also be noted from Fig. 4(c) that the magnet excitation voltage $E_0$ varies apparently with load. The trend of $E_0$ increase is also attributed to the interaction between the magnet flux and the armature reaction flux. When the $d$-axis armature reaction changes from magnetizing to zero, and from less demagnetizing to heavy demagnetizing with the increase of the load, the main path of the magnet flux in the iron part becomes less saturated. In contrast, the leakage path of magnet flux in the inter-pole ferromagnetic bridge turns to be more and more saturated due to the same direction of these two flux components in the bridge region. As a result, both these two factors lead to the increase of the main magnet flux; thus, the increase of the value $E_0$.

Figs. 5 and 6 show the comparisons between the simulated and measured torque and efficiency characteristics, respectively, when the PM motor was fed from a voltage source. It is shown from these figures that the excellent agreements between the simulated and tested results were achieved, except for some discrepancy of the torque values under heavy loads. This discrepancy is mainly due to the impact of neglecting the thermally induced resistance variation in stator windings, which would appreciably degrade the capability of torque output with the considerable temperature rise at heavy load conditions.

V. SIMULATION OF STARTING PERFORMANCE

Since the finite element analysis considered in this paper is limited to 2-D, the end effects due to the rotor end-ring resistance are treated by a modification of the rotor bar
conductivity, and the end ring leakage flux will be accounted for by end ring leakage inductance. Time-phasor is introduced to make finite element analysis be independent of the time factor. However, the introduction of time-phasor requires the reluctivity to be constant throughout the time cycle. In fact, the reluctivity will vary nonlinearly with the flux density. Therefore, equivalent reluctivity is introduced to handle this difficulty in terms of equivalent magnetic energy [26].

The discrete eddy-current finite element model with current source supply can be expressed as [17]

\[
([K] + j \omega_r [H]) \{A\} = \{P\}. \tag{42}
\]

For a voltage source supply, however, the current density \( J \) is unknown before field solutions are available. In general, the impressed voltage will not balance the summation of the calculated induced voltage and impedance voltage drops under the assumed current set. The lack of balance is usually used to obtain an improved estimate of the correct current set, and to proceed in an iterative manner until satisfactory balance is achieved. In this paper, an attempt is made to eliminate this iterative process.

The average value of the electromotive force \( e_c \), over a coil side of cross-sectional area \( s_c \), is given by

\[
e_c = -j \omega N l_i e_f \left( \sum_{i=1}^{n_c} \sum_{j=1}^{3} A_{ij} \right) \frac{3}{3s_c} \tag{43}
\]

where \( s_c \) is half the value of the slot area for two-layer windings, or just the value of the whole slot area for single-layer windings; the summation for \( i \) ranges over all the elements \( n_c \) in the area \( s_c \). Depending upon the details of the discretization and the connections of the phase windings, the induced voltage in phase \( A \) winding is given as

\[
E_A = -j \frac{\omega N l_i e_f 2P}{3s_c a_1} (D)'_A \{A\} \tag{44}
\]

where \( a_1 \) is the number of parallel circuits; \((D)'_A = \{d_{A1}, \ldots, d_{Ah}, \ldots, d_{An}\}\) is the area vector related to all the

![Figure 4](image)

**Fig. 4.** Variation of \( X_d, X_q, \) and \( E_0 \) with load angle \( \delta \).

![Figure 5](image)

**Fig. 5.** Simulated and measured results of torque characteristics.

![Figure 6](image)

**Fig. 6.** Simulated and measured results of efficiency characteristics.
nodes in the subregion of phase A windings. The definition of any component \( d_{Ak} \) is given by

\[
d_{Ak} = \sum_{e=1}^{E_k} \Delta_{ke}
\]

where \( \Delta_{ke} \) denotes the area of element \( e \), one of whose vertexes is just the numbered node \( k.E_k \) is the number of total elements related to node \( k \). Similarly, one can obtain induced voltage \( E_{B} \) in B-phase, and \( E_{C} \) in C-phase.

Using motor conventions, the relationship between the terminal voltage and induced voltage will be given as

\[
\sqrt{2}V_i = -E_i + I_iZ \quad (i = A, B, C)
\]

where \( Z \) is the total phase impedance including external impedance except the slot leakage reactance, tooth top leakage reactance and harmonic leakage reactance. The induced voltage \( E_i \) from the field analysis enables the presence of the slot and tooth top leakage flux and harmonic fluxes to be included implicitly. The current density flowing in phase-windings can be derived as

\[
J_i = \frac{\sqrt{2}N_iV_i}{s_{a1}Z} - j\omega \frac{2PN_i^2\ell_{cf}}{3s_{a1}^2a_i^4Z} \{D\} \{A\} \quad (i = A, B, C).
\]

Equation (47) is the expression of equivalent current source for voltage source supply. It is recognized that the value of calculated current density is not only the function of the impressed voltage source, but also related to vector potentials to be solved. Therefore, the discrete finite element model for the voltage source can be expressed as

\[
[K] + j\omega[H] + j\omega[T] \{A\} = \{P_v\} + \{P\}
\]

where \([K]\), \([H]\), and \([P]\) are the same as in (42) for the current source case. Matrix \([T]\) and vector \([P_v]\) corresponding to the voltage source supply are defined as

\[
[T] = \frac{2PN_i^2\ell_{cf}}{9s_{a1}^2a_i^4Z} \{\{D\}_A \{D\}_B \{D\}_C \}
\]

\[
\{P_v\} = \frac{\sqrt{2}}{3} \frac{N_i}{s_{a1}} \{V_A \{D\}_A + V_B \{D\}_B + V_C \{D\}_C\}.
\]

In starting directly across the line at rated values of voltage and frequency, the motor must pull into synchronism from some subsynchronous speed to which the motor has just been accelerated by the asynchronous cage torque [27]. Based on the eddy-current finite element solutions and using the established lumped parameter model [28], starting performance investigation is easy to carry out.

The conventional d-axis and q-axis subtransient equivalent circuits can assist one to obtain starting parameters. The d-q-axis rotor resistances and leakage reactances \( r_d, r_q, x_{d}, x_{q} \) may change notably with the rotor speed due to skin effect and nonlinearity. Stator leakage reactances \( x_{1} \) is the combination of stator end winding leakage reactance \( x_{1a} \) and \( x_{1b} \) which are the sum of slot leakage reactance, tooth top leakage reactance, and harmonic leakage reactance. The reason to separate \( x_{1a} \) with \( x_{1a} \) is that the former comes from the analytical solution, while the latter may be determined directly from the field solution.

The procedure for obtaining starting parameters is given as follows: first, the voltage-source finite element model is applied to get the starting current and stator leakage reactance \( x_{1s} \). Then, with the 50 obtained starting currents, the finite element model for the current source supply is employed to calculate d-q axis eddy-current fields, respectively. Analysis will start from slip \( = 0 \), since in this case the rotor branch circuit can be regarded as open circuit. Thus, one can conveniently figure out \( X_{rd} \) or \( X_{mq} \). Then, one can proceed to evaluate the remainder parameters in terms of the induced voltage for different slips. Since in starting process, the magnets will not induce eddy-current in rotor part, the rotor starting parameters can be determined in the absence of magnets. However, the effects of magnets on magnetic saturation have to be taken into account. In the first case for determining starting currents, an auxiliary real vector potential \( A^{(k)} \) is introduced and defined by

\[
A^{(k)} = (|A^{(k)}| + |A_m|) \cdot A^{(k)}/|A^{(k)}|
\]

where \( A^{(k)} \) is the complex vector potential solution from the kth iteration; \( A_m \) is the real vector potential solution due to magnet excitation alone, which was previously stored in computer memory. In terms of \( A^{(k)} \), one can obtain the flux density by \( \vec{B}^{(k)} = \nabla \times A^{(k)} \). After getting the amplitude \( B_m^{(k)} \), the equivalent reluctivity \( \nu^{(k+1)} \) for the next iteration can be determined from the equivalent magnetization curves. For the second case with the aim to determine d-q axis parameters from corresponding d-axis or q-axis field analysis, an auxiliary real vector potential \( A^{(k)} \) is introduced and defined by

\[
A^{(k)} = Re(A^{(k)}) + A_m.
\]

In terms of the computed \( B_m^{(k)} \), the required reluctivity \( \nu^{(k+1)} \) for the next iteration can be evaluated directly from the fundamental magnetization curve.

Once the accurate field-based starting parameters become available, the starting performance prediction is readily carried out by using the lumped parameter model. Fig. 7(a) shows the resulting average torque as well as its individual components. In this case, all parameters are chosen as constant values at \( s = 0 \). It can be seen that the cage torque component is similar to the typical characteristics of induction motor except with a dip at slip slightly less than 0.5. This is caused by the electromagnetic unсимmetry between the d-axis and q-axis. It is the existence of this dip that leads to the second dip on...
Fig. 7. Average torque components. (a) Using constant parameter. (b) Using varying parameter.

Fig. 8. Simulated and measured results of speed versus time. (a) Simulation. (b) Measured (300 r/min/div., 0.5 s/div.).

the resultant torque curve which has been pointed out in the literatures [3], and is usually considered as one of the main concerns for PM motors. Fortunately, the concern may turn out to be uncalled-for. This is attributed to the skin effect which may favorably make rotor dynamic parameters much less unsymmetrical. Fig. 7(b) shows the results using varying dynamic parameters. The dip of cage torque component has disappeared. When the magnet torque is combined with cage torque component, a dip is produced in the resulting average torque curve. This dip may be the main concern for the successful starting, but not the previous understanding that there exist two tips.

Figs. 8, 9, and 10 show the simulation and measured results of speed versus time, torque versus time and current versus time, respectively, with a coupled load of about twelve times the rotor inertia. It is shown that the agreement between the measured and simulated results is excellent. Synchronization of a PM motor is complex because of the interaction of the inertial torque with the components of synchronizing torque. It is to be noted that the effect of magnet flux level in PM synchronous machines is important. The higher the magnet flux, the better the pull-in and pull-out torques will be, but the starting current and input kVA constraints will require the magnet flux level to be kept within certain limits.

The results obtained from the field-circuit combined approach are useful in order to ensure that a brushless PM synchronous motor is well designed to start and synchronize properly. The maximum current demand is important with many types of inverter fed drives, and the time of run-up to stable synchronization needs to be specified for some applications involving high coupled inertia loads. The torque levels throughout the run-up, and particularly during the pull-in period can be deciding factors in the suitability of a brushless PM synchronous motor for a particular application.

VI. CONCLUSION

Two-dimensional magnetic field calculation has been successfully applied to determine the load-dependent direct and quadrature axis reactance parameters and excitation voltage, in which the effect of mutual interaction between the direct and quadrature axis reactances as well as excitation voltage is included. Time varying eddy current field calculation is used to get the starting parameters. In order to simplify the time-varying field calculation, time phasor is used to separate the
Fig. 9. Simulated and measured results of torque versus time. (a) Simulation. (b) Measured (2.5 N-m/div., 0.5 s/div.).

Fig. 10. Simulated and measured results of starting current. (a) Simulation. (b) Measured (10 A/div., 0.5 s/div.).

time variable from the field equation based on the introduction of equivalent reluctivity in terms of equivalent magnetic energy. For voltage source inverter supply, corresponding finite element formulates are derived with complete elimination of the literature process for the starting current. Due to the availability of the field based accurate operational parameters, it has become possible to use the lumped parameter circuit model to obtain the complete starting and synchronous performances. The field circuit combined technique has been successfully applied to a laboratory 1-hp permanent magnet synchronous motor. There exists excellent agreement between the simulated and experimental results.

REFERENCES


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