LECTURE NOTES - VIII

« WATER RESOURCES »

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CHAPTER 8

FLOW ROUTING

Flow routing is a procedure to determine the time and magnitude of flow (i.e., the flow hydrograph) at a point on a water course from known or assumed hydrographs at one or more points. If the flow is a flood, the procedure is specifically known as *flood routing*. In a broad sense, flow routing may be considered as analysis to trace the flow trough a hydrologic system, given the input. (Chow, et.al., 1988)

8.1. ROUTING EQUATION

Flow routing can be introduced using the black-box concept of systems theory. The input and output functions consist of the runoff hydrographs at the upstream and downstream sections of a channel; the transfer function is the channel routing procedure that is used to translate and attenuate the upstream runoff hydrograph into a downstream hydrograph. The transfer function usually consists of two parts, the routing method and the physical characteristics of the stream reach. In theory, we usually assume that the channel characteristics are known, so the routing method becomes the transfer function.

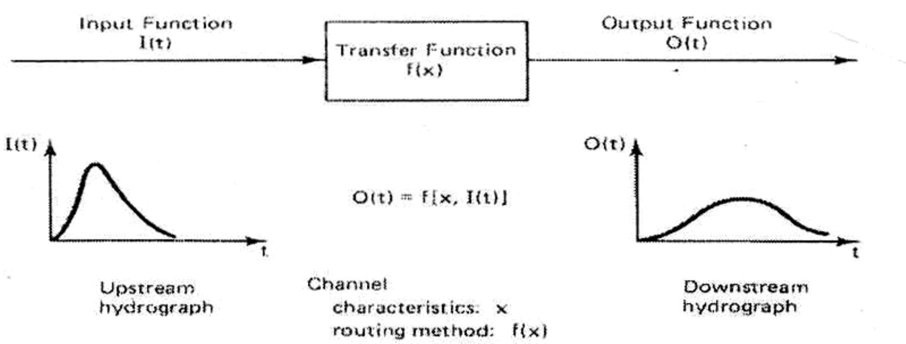


Figure 8.1. Channel routing

For a hydrologic system, input I(t), output Q(t), and storage S(t) are related by continuity equation:

$$\frac{dS}{dt} = I(t) - Q(t) \qquad (8.1)$$

If the flow hydrograph, I(t), is known, Equ. (8.1) cannot be solved directly to obtain the outflow hydrograph, Q(t), because both Q and S are unknown. A second relationship, or *storage function*, is needed to relate S, I, and Q.

$$S = f(I, Q) \tag{8.2}$$

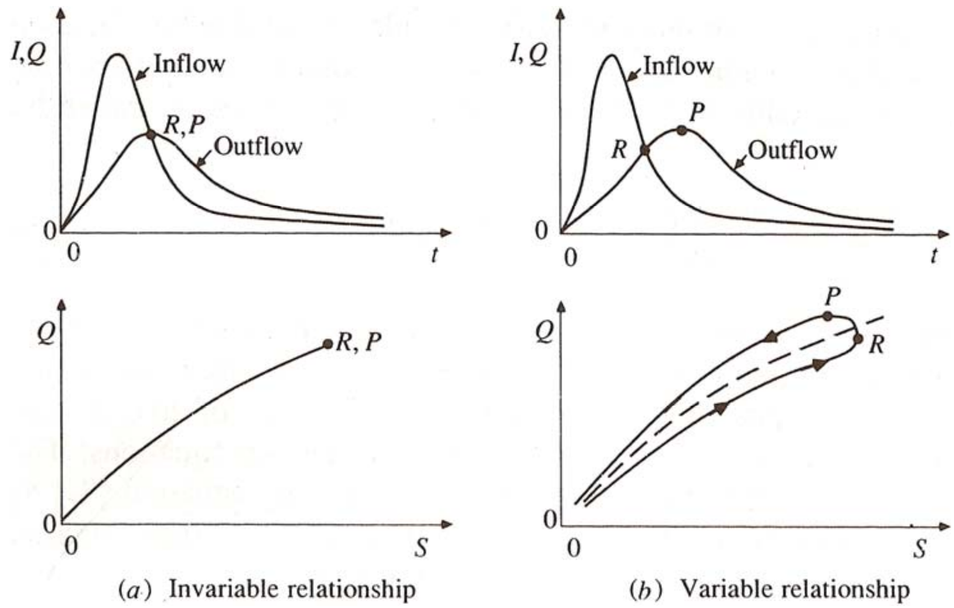


Figure 8.2. Relationships between discharge and storage.

The relationship between the outflow and the storage of a hydrologic system has an influence on flow routing. This relationship may be either *invariable* or *variable*, as shown in Fig. (8.2).

An invariable storage function has the form of,

$$S = f(Q) \tag{8.3}$$

and applies to a reservoir with horizontal water surface. When a reservoir has horizontal water surface, its storage is a function of its water surface elevation, or depth in the pool. Likewise, the outflow discharge is a function of the water surface elevation, or head on the outlet works. By combining these two functions, the reservoir storage and discharge can be related to produce an invariable, single valued storage function, S = f(Q) as shown in Fig. (8.2.a). For such reservoirs, the peak outflow occurs when the outflow hydrograph intersects the inflow hydrograph, because the maximum storage occurs when,

$$\frac{dS}{dt} = I(t) - Q(t) = 0 \rightarrow I(t) = Q(t) \tag{8.4}$$

For this case, the storage and outflow are related by S = f(Q). This is indicated in Fig. (8.2.a) where the points denoting the maximum storage, R, and maximum outflow, P, coincide.

A variable storage-outflow relationship applies to long, narrow reservoirs and to open channel or streams, where the water surface profile may be significantly curved due to the backwater effects. The amount of storage due to backwater depends on the time rate of change of flow through the system. As shown in Fig. (8.2.b), the resulting relationship between the discharge and the system storage is no longer a single-valued function but exhibits a curve usually in the form of a single or twisted loop, depending on the storage. Because of the retarding effect due to backwater, the peak outflow usually occurs later than the time when the inflow and outflow intersect, as indicated in Fig. (8.2.b) where the points R and P do not coincide. If the backwater effect is not very significant, the loop shown in Fig. (8.2.b) may be replaced by an average curve shown by the dashed line. Thus, level pool routing methods can also be applied in an approximate way to routing with an invariable discharge discharge-storage relationship.

The effect of storage is to redistribute the hydrograph by shifting the centroid of the inflow hydrograph to the position of that of the outflow hydrograph centroid in a *time of redistribution*.

In very long channels, the entire flood wave travels a considerable distance and the centroid of its hydrograph may then be shifted by a time longer than the time of redistribution. This additional time may be considered as *time of translation*. As shown in Fig. (8.3), the total *time of flood movement* between the centroids of the outflow and inflow hydrographs is equal to the sum of the time of redistribution and the time of translation. The process of redistribution modifies the shape of the hydrograph, while translation changes its position.

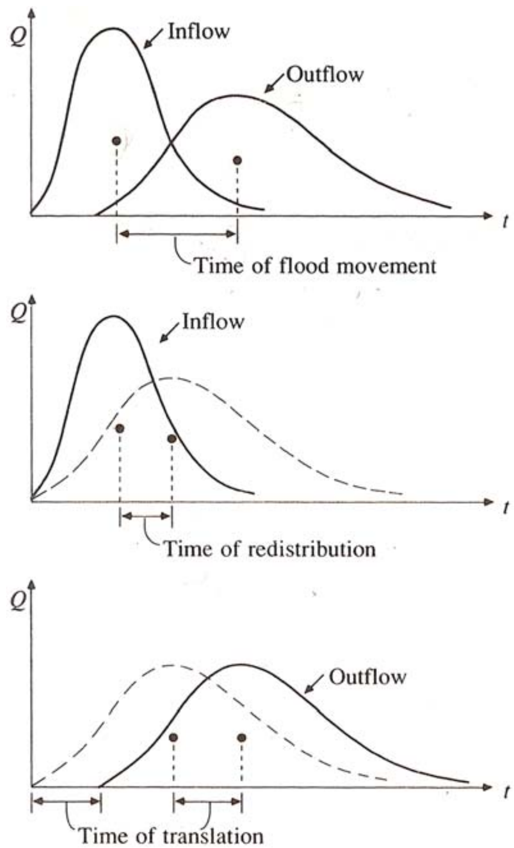


Figure 8.3. Conceptual interpretation of the time of flood movement

8.2. FLOOD ROUTING ON A HORIZONTAL POOL

Flood routing on a horizontal pool is a procedure for calculating the outflow hydrograph from a reservoir with a horizontal water surface, given its inflow hydrograph and storage-outflow characteristics.

The time horizon is broken into intervals of duration Δt, indexed by j, that is, t=0, Δt, 2Δt, ……, jΔt, (j+1)Δt,…….., and the continuity Equ. (8.1) is integrated over each time interval, as shown in Fig. (8.4).

For the j-th time interval,

$$\int_{S_j}^{S_{j+1}} dS = \int_{j\Delta t}^{(j+1)\Delta t} I(t)dt - \int_{j\Delta t}^{(j+1)\Delta t} Q(t)dt \tag{8.5}$$

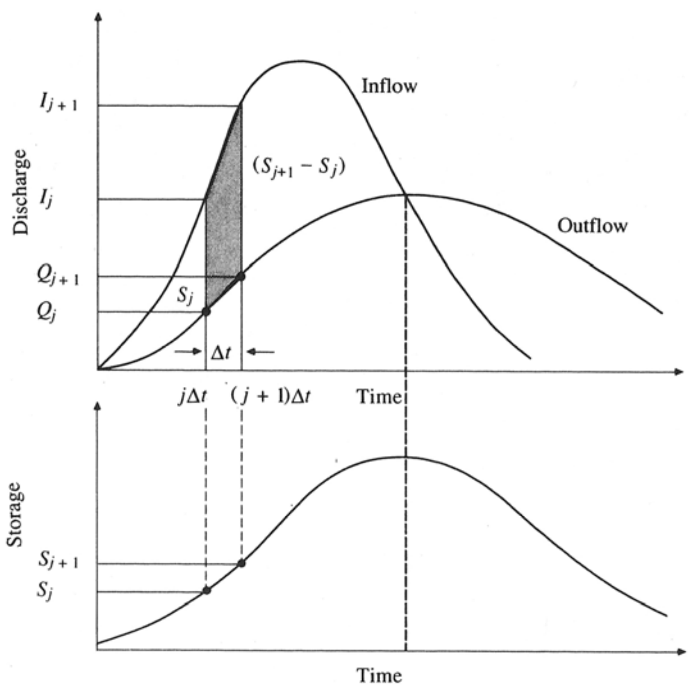


Figure 8.4. Change of storage during a routing period Δt.

The inflow values at the beginning and end of the j-th time interval are I_j and I_{j+1}, respectively, and the corresponding values of the outflow are Q_j and Q_{j+1}. Here, both inflow and outflow data a sample data. If the variation of inflow and outflow over the interval is approximately linear, $(S_{j+1} - S_j)$, can be found by rewriting Equ. (8.5) as,

$$S_{j+1} - S_j = \frac{I_j + I_{j+1}}{2}\Delta t - \frac{Q_j + Q_{j+1}}{2}\Delta t \tag{8.6}$$

The values of I_j and I_{j+1} are known because they are prespecified. The values of Q_j and S_j are known at the j-th time interval from calculation from the previous time interval. Hence, Equ. (8.6) contains two unknowns, Q_{j+1} and S_{j+1}, which are isolated by multiplying Equ. (8.6) through by $(2/\Delta t)$, and rearranging the result produces;

$$\left(\frac{2S_{j+1}}{\Delta t}+Q_{j+1}\right)=\left(I_j+I_{j+1}\right)+\left(\frac{2S_j}{\Delta t}-Q_j\right) \tag{8.7}$$

In order to calculate the outflow, Q_{j+1}, from Equ. (8.7), a *storage-outflow function* relating $(2S/\Delta t+Q)$ and Q is needed. The method for developing this function using elevation-storage and elevation-outflow relationships is shown in Fig. (8.5).

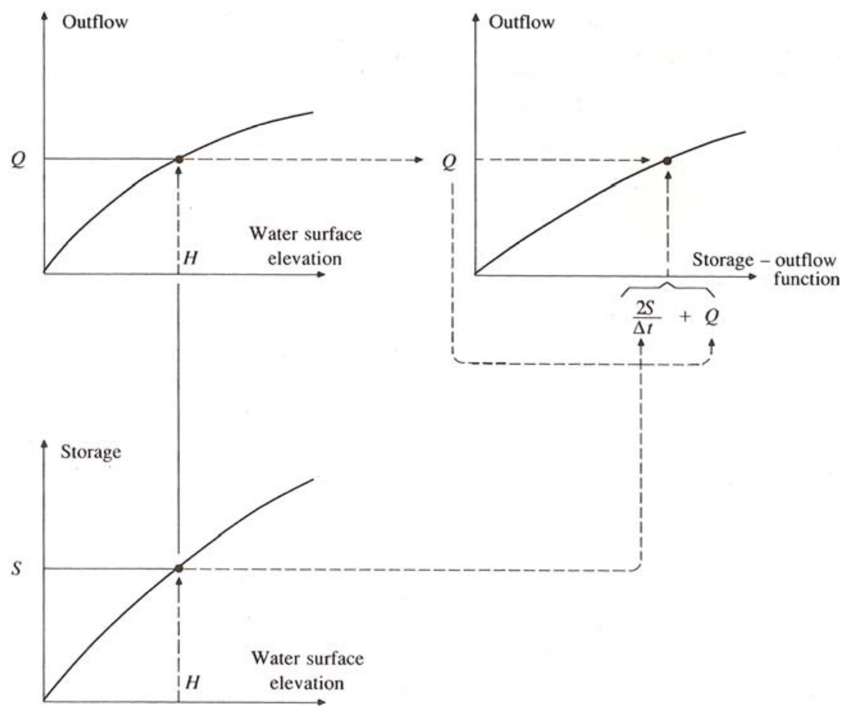
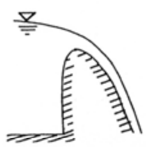
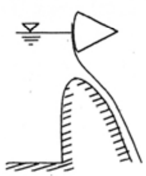
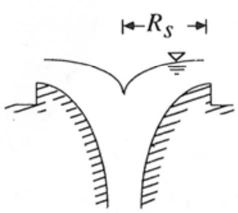


Figure 8.5. Development of the storage-outflow function for pool routing on the basis of storage-elevation and elevation-outflow curves.

The relationship between water surface elevation and reservoir storage can be derived by planimetering topographic maps or from field surveys. The elevation-discharge relation is derived from hydraulic equations relating head and discharge, such as those shown in Table (8.1), for various types of spillways and outlet works. The value of Δt is taken as the time interval of the inflow hydrograph. For a given value of water surface elevation, the values of S and discharge Q determined [parts (a) and (b) of Fig. (8.5)], then the value of $(2S/\Delta t+Q)$ is calculated and plotted on the horizontal axis of a graph with the value of the outflow Q on the vertical axis [part (c) of Fig. (8.5)].

In routing the flow through time interval j, all terms on the right side of Equ. (8.7) are known, and so the value of $\left(2S_{j+1}/\Delta t + Q_{j+1}\right)$ can be computed. The corresponding value of Q_{j+1} can be determined from the storage-outflow function $(2S/\Delta t+Q)$ versus Q, either graphically or by linear interpolation of tabular values. To set up the data required for the next interval, the value of $\left(\dfrac{2S_{j+1}}{\Delta t} - Q_{j+1}\right)$ is calculated by,

Table 8.1. Spillway discharge equations

Spillway type	Equation	Notation
Uncontrolled over-flow ogee crest 	$Q = CLH^{3/2}$	Q = discharge, cfs C = variable coefficient of discharge L = effective length of crest H = total head on the crest including velocity of approach head.
Gate controlled ogee crest 	$Q = \frac{2}{3}\sqrt{2g}CL\left(H_1^{3/2} - H_2^{3/2}\right)$	H_1 = total head to bottom of the opening H_2 = total head to top of the opening C = coefficient which differs with gate and crest arrangement
Morning glory spill-way 	$Q = C_o(2\pi R_s)H^{3/2}$	C_o = coefficient related to H and R_s R_s = radius of the over-flow crest H = total head
Culvert (submerged inlet control)	$Q = C_d WD\sqrt{2gH}$	W = entrance width D = height of opening C_d = discharge coefficient

Source: Design of Small Dams, Bureau of Reclamation, U. S. Department of the Interior, 1973.

$$\left(\frac{2S_{j+1}}{\Delta t}-Q_{j+1}\right)=\left(\frac{2S_{j+1}}{\Delta t}+Q_{j+1}\right)-2Q_{j+1} \tag{8.8}$$

The computation is repeated for subsequent periods.

EXAMPLE 8.1: The river is diverted to a tunnel by a cofferdam during the construction of a dam. The relationship between the water stage at the entrance of the tunnel, the storage volume S and the tunnel capacity is given in Table (8.2). The 25-yr flood hydrograph to be used in the design of the cofferdam is shown in Table (8.3). During a flood, the flood wave is dampened in the pool before it enters the tunnel. Determine the maximum water surface elevation in the pool.

Solution : Using the relationship between the water volume stored in the pool S and the outflow Q, the values of ($2S/\Delta t+Q$) are computed and given in Table (8.2) for Δt = 6hr= 6×3600= 21600 sec.

Table 8.2. Storage-outflow function

Water Stage (m) (1)	Discharge Q (m^3/sec) (2)	Storage S ($10^6 m^3$) (3)	($2S/\Delta t+Q$) (m^3/sec) (4)
395	900	1.00	993
400	1350	2.00	1535
405	1850	2.75	2105
413	2550	20.00	4402
420	3100	44.50	7220
430	3950	121.00	15150
440	4650	285.20	31060
450	5300	567.00	57800

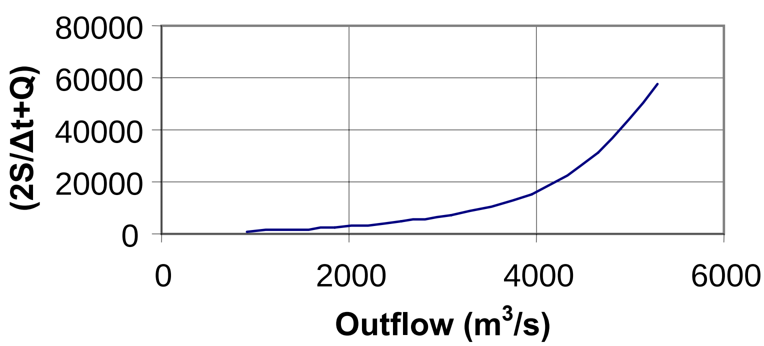


Figure 8.6. Storage-outflow function

Assuming that the reservoir is empty initially, Equ. (8.7) is applied with time steps $\Delta t = 6$ hr. Computations are shown in Table (8.3) with the following steps.

1) Inflow value at time j, I_j, is added to the I_{j+1} of the column (3) and written in column (4).
2) Using Equ. (8.7),

$$\left(\frac{2S_{j+1}}{\Delta t}+Q_{j+1}\right)=\left(I_j+I_{j+1}\right)+\left(\frac{2S_j}{\Delta t}-Q_j\right)$$
$$\left(\frac{2S_2}{6\times 3600}+Q_2\right)=1060+0=1060\,m^3/\text{sec}$$

and written in column (6).

3) The outflow value of Q_{j+1} corresponding to $\left(\frac{2S_{j+1}}{\Delta t}+Q_{j+1}\right)$ is calculated from Table (8.2) by interpolation as,

$$\frac{2S_2}{\Delta t}+Q_2=1060\,m^3/s=x$$
$$Q=y$$

Two pairs of values around $\frac{2S}{\Delta t}+Q=1060\,m^3/s$ are selected as (x_1, y_1) and (x_2, y_2) from Table (8.3). They are $(x_1, y_1) = (993, 900)$ and $(x_2, y_2) = (1535, 1350)$. The value of $y = Q_2$ for $x = 1060$ is, by linear interpolation,

$$Q_2=900+\frac{(1350-900)}{(1535-993)}\times(1060-993)=956\,m^3/s$$

4) $2Q_2$ is subtracted from $\left(\frac{2S_2}{\Delta t}+Q_2\right)$ to obtain $\left(\frac{2S_2}{\Delta t}-Q_2\right)$ by using Equ. (8.8).

$$\left(\frac{2S_2}{\Delta t}-Q_2\right)=1060-2\times 956=-852$$

The calculations for subsequent time intervals are performed in the same way with the results tabulated in the Table (8.3).

Table 8.3. Routing of flow through a detention reservoir by the level pool method.

Time Index j (1)	Time (hr) (2)	Inflow I (m^3/sec) (3)	I_j+I_{j+1} (m^3/sec) (4)	$\frac{2S_{j+1}}{\Delta t}-Q_{j+1}$ (5)	$\frac{2S_{j+1}}{\Delta t}+Q_{j+1}$ (6)	Outflow Q (m^3/sec) (7)
1	0	0	-	0	-	0
2	6	1060	1060	-852	1060	956
3	12	2410	3470	-1394	2618	2006
4	18	4240	6650	-202	5216	2709
5	24	5930	10170	3178	9968	3395
6	30	8290	14220	9300	17398	4049
7	36	8840	17130	17538	26430	4446
8	42	8290	17130	25192	34668	4738
9	48	7410	15700	31114	40892	4889
10	54	6560	13970	35102	45084	4991
11	60	5540	12100	37118	47202	5042
12	66	4580	10120	37152	47238	5043
13	72	3670	8250	35404	45402	4999

Largest discharge is 5043 (m^3/s), which corresponds to the stage using the values of the Table (8.3),

$(z_1, y_1) = (440, 4650)$, $(z_2, y_2) = (450, 5300)$

$$z = 440 + \frac{(450-440)}{(5300-4650)} \times (5043-4650) = 446m$$

Thus, the highest water elevation in the pool to be used in the design of the cofferdam should be greater than 446 m.

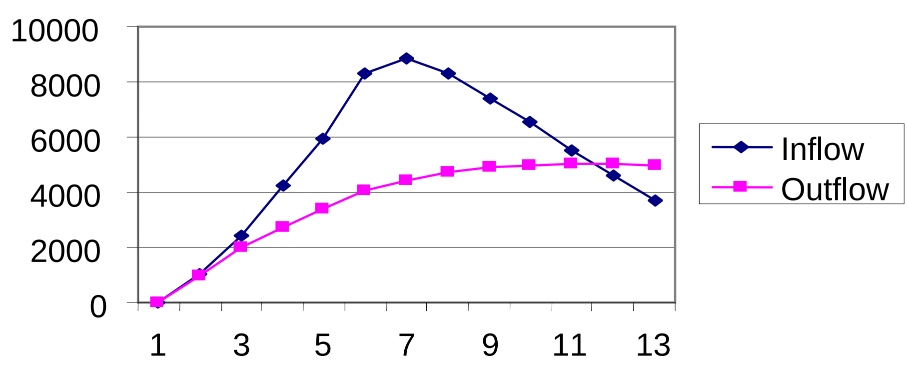


Figure 8.7. Routing of flow through a detention reservoir.

EXAMPLE 8.2. A reservoir for detaining flood flows is 4000 m^2 in horizontal area, and has a 1.50 m diameter reinforced concrete pipe as the outlet structure. The headwater-discharge relation for the outlet pipe is given in columns (1) and (2) of Table (8.4). Use the level pool routing method to calculate the reservoir outflow from the inflow hydrograph given in columns (2) and (3) of Table (8.4). Assume that reservoir is initially empty.

Table 8.4. Development of storage-outflow function for a detention reservoir. Time interval, Δt=10 min.

Elevation (m) (1)	**Discharge Q (m^3/sec) (2)**	**Storage S (m^3) (3)**	**($2S/\Delta t$)+Q (m^3/sec) (4)**
0	0	0	0
0.15	0.09	600	2.09
0.30	0.23	1200	4.23
0.45	0.48	1800	6.48
0.60	0.85	2400	8.85
0.75	1.22	3000	11.22
0.90	1.70	3600	13.70
1.05	2.21	4200	16.21
1.20	2.75	4800	18.75
1.35	3.31	5400	21.31
1.50	3.88	6000	23.88
1.65	4.42	6600	26.42
1.80	4.90	7200	28.90
1.95	5.38	7800	31.38
2.10	5.80	8400	33.80
2.25	6.17	9000	36.17
2.40	6.54	9600	38.54
2.55	6.85	10200	40.85
2.70	7.16	10800	43.16
2.85	7.48	11400	45.48
3.00	7.79	12000	47.79

Solution: The inflow hydrograph is specified at 10-min intervals, so $\Delta t=10\times60=600$ s. For all elevations, the horizontal area of the reservoir water surface is 4000 m^2, and the storage is calculated as S = 4000× (depth of water). For example, for a depth of 0.15 m, $S=0.15\times4000=600$ m^3, as shown in column (3) of Table (8.4). The corresponding value of $(\frac{2S}{\Delta t}+Q)$ can then be determined. For a depth of 0.15 m, the discharge is given in column (2) of Table (8.4) as 0.09 m^3/sec, so the storage-outflow function value is,

$$\frac{2S}{\Delta t}+Q=\frac{2\times 600}{600}+0.09=2.09\,m^3/sec$$

as shown in column (4) of Table (8.4). The storage-outflow function is plotted in Figure (8.8).

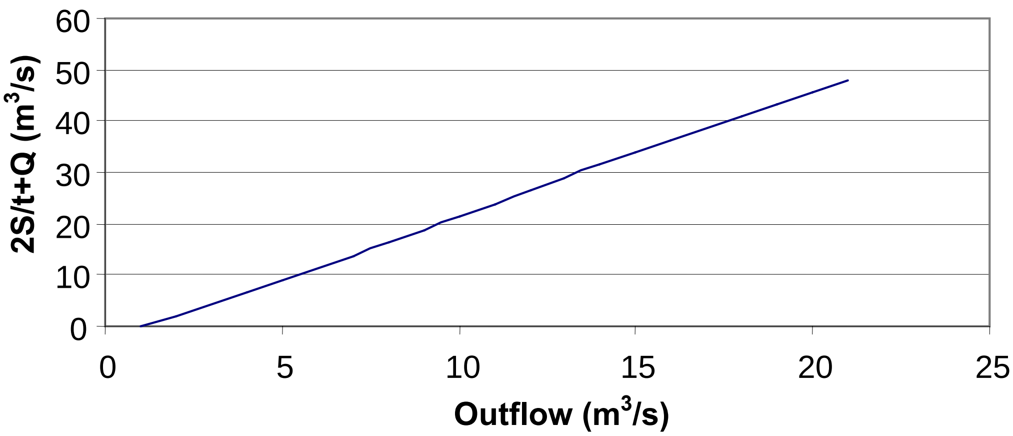


Figure 8.8. Storage-outflow function for a detention reservoir

The flow routing calculations are carried out using Equ. (8.7). For the first time interval, S_1=Q_1=0 because reservoir is initially empty; hence,

$$\frac{2S_1}{\Delta t}-Q_1=0$$

also. The inflow values are I_1=0 and I_2=1.70, so (I_1+I_2)=0+1.70=1.70 m^3/s. The value of the storage-outflow function at the end of the time interval is calculated from Equ. (8.7) with j=1,

$$\left(\frac{2S_2}{\Delta t}+Q_2\right)=I_1+I_2+\left(\frac{2S_1}{\Delta t}-Q_1\right)$$

$$\left(\frac{2S_2}{\Delta t}+Q_2\right)=1.70+0=1.70\,m^3/sec$$

Table 8.5. Routing of flow through a detention reservoir by the level pool method.

Time Index j (1)	Time (min) (2)	Inflow (m^3/s) (3)	I_j+I_{j+1} (m^3/s) (4)	$2S_j/\Delta t-Q_j$ (m^3/s) (5)	$2S_{j+1/\Delta t}+Q_{j+1}$ (m^3/s) (6)	Outflow (m^3/s) (7)
1	0	0	-	0	-	0
2	10	1.70	1.70	1.55	1.70	0.073
3	20	3.40	5.10	4.32	5.34	0.51
4	30	5.10	8.50	9.76	12.82	1.53
5	40	6.80	11.90	14.88	21.66	3.39
6	50	8.50	15.30	19.88	30.18	5.15
7	60	10.20	18.70	25.48	38.58	6.55
8	70	9.10	19.30	30.02	44.78	7.38
9	80	7.90	17.00	31.71	47.09	7.69
10	90	6.80	14.70	31.21	46.41	7.60
11	100	6.60	13.40	29.89	44.61	7.36
12	110	4.50	11.10	27.25	40.99	6.87
13	120	3.40	7.90	23.13	35.15	6.01
14	130	2.30	5.70	19.05	28.83	4.89
15	140	1.10	3.40	15.33	22.45	3.56
16	150	0	1.10	11.91	16.43	2.26
17	160	-	0	9.21	11.91	1.35
18	170	-	-	7.39	9.21	0.91
19	180	-	-	6.15	7.39	0.62
20	190	-	-	5.27	6.15	0.44
21	200	-	-	4.57	5.27	0.35
22	210	-	-	4.03	4.57	0.27

The value of Q_{j+1} is found by linear interpolation given $\left(\frac{2S_{j+1}}{\Delta t}+Q_{j+1}\right)$. If there is a pair of values (x,y) with known pairs of values (x_1,y_1) and (x_2,y_2), then the interpolated value of y corresponding to a given value of x in the range $(x_1 \le x \le x_2)$ is,

$$y = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)} \times (x - x_1)$$

In this case, $x = \frac{2S}{\Delta t} + Q$, and y=Q. Two pairs of values around $\frac{2S}{\Delta t} + Q = 1.70$ are selected from Table (8.5); they are $(x_1,y_1) = (0, 0)$ and $(x_2,y_2) = (2.09,0.09)$. The value of y for x=1.70 is, by linear interpolation,

$$y = 0 + \frac{(0.09-0)}{(2.09-0)} \times (1.70-0) = 0.07\, m3^3/sec$$

So, Q_2=0.73 m^3/s, and the value of $\left(\frac{2S_2}{\Delta t} - Q_2\right)$ needed for the next iteration is found by using Equ. (8.8) with j=2,

$$\left(\frac{2S_2}{\Delta t} - Q_2\right) = \left(\frac{2S_2}{\Delta t} + Q_2\right) - 2Q_2$$

$$\left(\frac{2S_2}{\Delta t} - Q_2\right) = 1.70 - 2 \times 0.073 = 1.55$$

Proceeding the next interval,

$$(I_2 + I_3) = 1.70 + 3.40 = 5.10\, m^3/sec$$

and the routing is performed with j=2 as,

$$\left(\frac{2S_3}{\Delta t} + Q_3\right) = (I_2 + I_3) + \left(\frac{2S_2}{\Delta t} - Q_2\right)$$

$$\left(\frac{2S_3}{\Delta t} + Q_3\right) = 5.10 + 1.55 = 6.55\, m^3/sec$$

For x=6.55, from Table (8.4). (x_1,y_1) = (6.48,0.48) and (x_2,y_2) = (8.85,0.85),

$$Q_3 = y = 0.48 + \frac{(0.85-0.48)}{(8.85-6.48)} \times (6.65-6.48) = 0.51\, m^3/sec$$

$$\left(\frac{2S_3}{\Delta t} - Q_3\right) = 5.34 - 2 \times 0.51 = 4.32\, m^3/sec$$

The calculations for subsequent time intervals are performed in the same way, with the results tabulated in Table (8.5) and plotted in Fig. (8.9). The peak flow is 10.20 m^3/sec and occurs at 60 min; the detention reservoir reduces the peak flow to 7.69 m^3/sec and delays it until 80 min. The outflow is maximized at the point where the inflow and outflow are equal, because storage is also maximized at that time, and there is a single-valued function relating storage and outflow for a reservoir with a level pool.

The maximum depth in the storage reservoir is calculated by interpolation from Table (8.4) as 2.95 m at the peak discharge of 7.69 m^3/sec. If this depth is too great, or if the discharge of 7.69 m^3/sec in the 1.50 m outlet pipe is too large, either the outlet structure or the surface area of the basin must be enlarged.

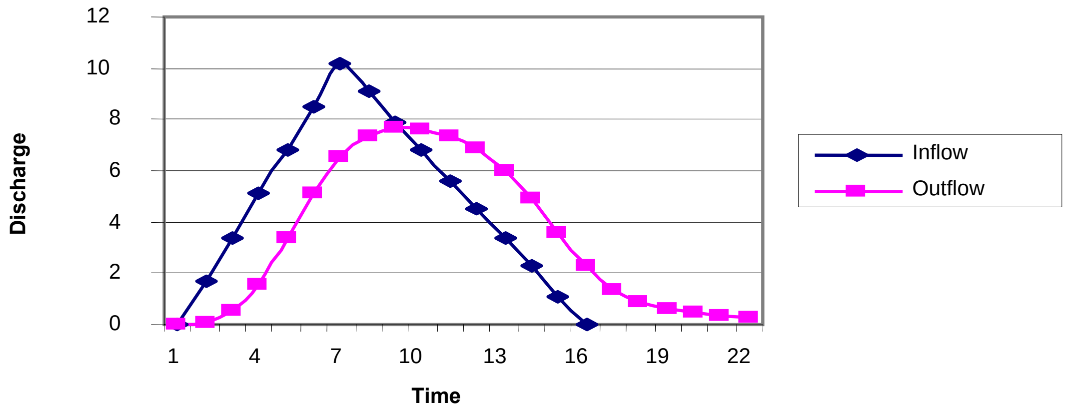


Figure 8.9. Routing of flow through a detention reservoir.

EXAMPLE 8.3. The following data pertains to an inflow hydrograph whose flows have been recorded at 6 hours interval on a certain stream.

Table 8.6

Time (h)	Inflow (m^3/sec)
0	42
6	45
12	57
18	88
24	147
30	210
36	272
42	340
48	350
54	338
60	314
66	288
72	263
78	240
84	198
90	170
96	143
102	120

This flood approaches a reservoir with uncontrolled spillway with elevation area and elevation outflow data as shown in Table (8.7).

Table 8.7

Elevation (m)	Area (km^2)	Outflow (m^3/sec)
100	4.05	0
100.3	4.12	15
100.6	4.20	42
100.9	4.25	77
101.2	4.28	119
101.5	4.36	167
101.8	4.45	217
102.1	4.53	272
102.4	4.60	334
102.7	4.69	405

The flood water level reaches the crest level (elevation 100.00m) of the spillway at 4 hours after the inflow recording station.

Determine the maximum reservoir level and maximum discharge over the spillway. Draw inflow and routed hydrograph indicating the reduction in peak flow and peak lag introduced due to routing.

Solution:

1) Draw the elevation–outflow curve as shown in Figure (8.10),

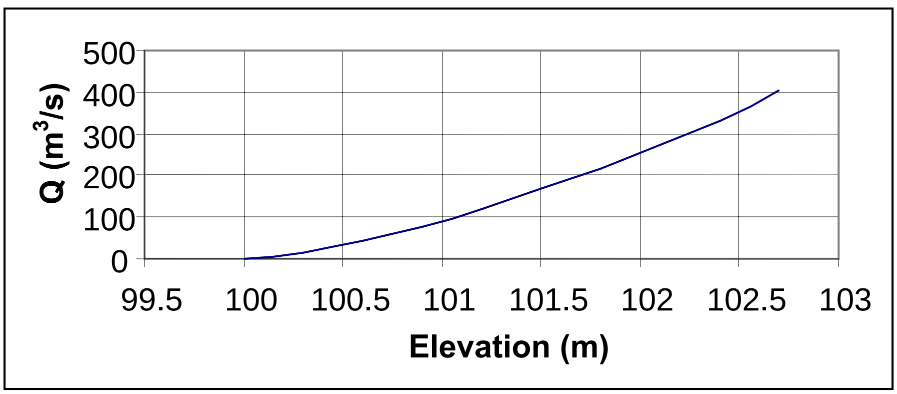


Figure 8.10. Elevation-outflow curve.

2) The storage between elevations (100-100.3) m is calculated by,

$$S = \frac{A_{100} + A_{100.3}}{2} \times (100.0 - 100.3)$$

$$S = \frac{(4.05 + 4.12) \times 10^6}{2} \times 0.30 = 1.23 \times 10^6\, m^3$$

Storages between elevations are computed and given in column (4) at Table (8.8).

3) Computing the storage for the elevations given in Table (8.8), calculating the storages as in step 2, taking the outflow discharges from the Table (8.7), $(2S/\Delta t + Q)$ values are calculated and given in Table (8.8) in column (6) for Δt=6 hr= 6×3600=21600 sec.

4) Assuming that there is no flow over the spillway, inflow at time j, I_j, is added to the I_{j+1} of the column (3) and written in column (4) at Table (8.9).

$$I_1 + I_2 = 42 + 45 = 87\, m^3/sec$$

5) Using Equ. (8.7),

$$\left(\frac{2S_{j+1}}{\Delta t} + Q_{j+1}\right) = \left(I_j + I_{j+1}\right) + \left(\frac{2S_j}{\Delta t} - Q_j\right)$$

$$\left(\frac{2S_2}{6 \times 3600} + Q_2\right) = 87 + 0 = 87\, m^3/sec$$

and written in column (6).

6) The outflow Q_{j+1} corresponding to $\left(\frac{2S_{j+1}}{\Delta t} + Q_{j+1}\right)$ is calculated from table by interpolation as,

$$\frac{2S_2}{\Delta t} + Q_2 = 87\, m^3/\text{sec} = x$$
$$Q = y$$

Two pairs of values around x = 87 m^3/s are selected as (x_1,y_1) = (0,0) and (x_2,y_2) = (129,14.9). The value of y = Q_2 for x = 87 is, by linear interpolation,

$$Q_2 = 0 + \frac{(14.9 - 0)}{(129 - 0)} \times (87 - 0) = 10\, m^3/sec$$

7) $2Q_2$ is subtracted from $\left(\frac{2S_2}{\Delta t}+Q_2\right)$ to obtain $\left(\frac{2S_2}{\Delta t}-Q_2\right)$ by using Equ. (8.8),

$$\left(\frac{2S_2}{\Delta t}-Q_2\right)=87-2\times 10=67\,m^3/sec$$

The calculations for subsequent time intervals are performed in the same way with the results tabulated in Table (8.9).

Table 8.8. Development of storage-outflow function for a detention reservoir. Time interval Δt = 6hr.

Elevation (m) (1)	**Outflow (m^3/sec)** (2)	**Area between Contours A (km^2)** (3)	**Storage between contours** $S=\frac{(A_1+A_2)}{2}\times(H_2-H_1)$ **(10^6m^3)** (4)	**Cumulative Storage above crest (10^6m^3)** (5)	$(2S/\Delta t+Q)$ **(m^3/sec)** (6)
100	0	4.05	-	-	-
100.30	14.9	4.12	1.23	1.23	129
100.60	42.2	4.20	1.25	2.45	269
100.90	77.3	4.25	1.27	3.72	422
101.20	119	4.28	1.28	5.00	582
101.50	167	4.36	1.30	6.30	750
101.80	217	4.45	1.32	7.62	923
102.10	272	4.53	1.35	8.97	1103
102.40	334	4.60	1.37	10.34	1291
102.70	405	4.69	1.39	11.73	1491

-

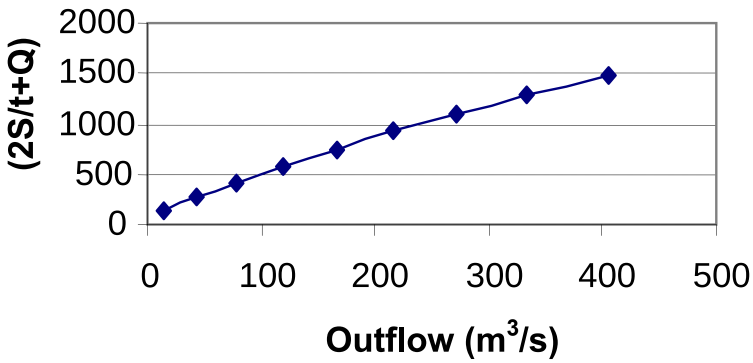


Figure 8.11. Storage-outflow function
Table 8.9. Routing of flow calculations.

Time Index j (1)	**Time (hr) (2)**	**Inflow (m^3/sec) (3)**	**I_j+I_{j+1} (m^3/sec) (4)**	$\frac{2S_{j+1}}{\Delta t}-Q_{j+1}$ **(5)**	$\frac{2S_{j+1}}{\Delta t}+Q_{j+1}$ (6)	**Outflow Q (m^3/sec) (7)**
1	0	42	-	-	-	0
2	6	45	87	67	87	10
3	12	57	102	123	169	23
4	18	88	145	184	268	42
5	24	147	235	265	419	77
6	30	210	357	362	622	130
7	36	272	482	456	844	194
8	42	340	612	546	1068	261
9	48	350	690	604	1236	316
10	54	338	688	624	1292	334
11	60	314	652	618	1276	329
12	66	288	602	598	1220	311
13	72	263	551	575	1149	287
14	78	240	503	550	1078	264
15	84	198	438	514	988	237
16	90	170	368	472	882	205
17	96	143	313	431	785	177
18	102	120	263	392	694	151

Maximum discharge over spillway from column (7), Q_{max} =334 m^3/sec,

Maximum reservoir level corresponding to Q_{max} = 334 m^3/sec by using Table (8.8) is 102.4 m.

Reduction in peak discharge, $I_{max} - Q_{max} = 350 - 334 = 16\, m^3/sec$

Lag time between the maximum inflow and outflow discharges

Peak lag = 54 – 48 = 6 hours.

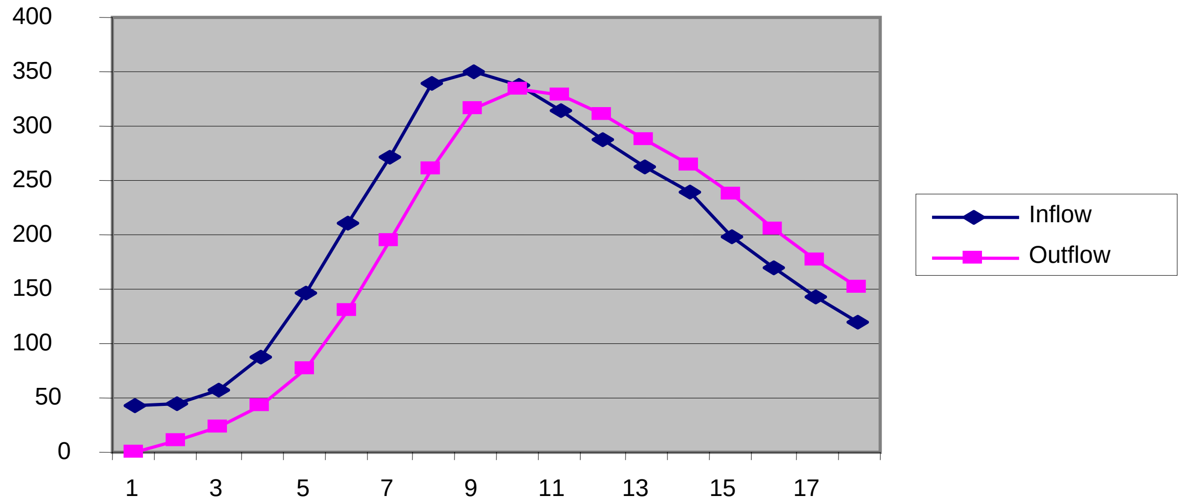


Figure 8.12. Routing of flow hydrographs.

8.3. HYDROLOGIC RIVER ROUTING

The *Muskingum method* is a commonly used hydrologic routing method for handling a variable discharge-storage relation. This method models the storage volume of flooding in a river channel by a combination of wedge and prism storages. During the advance of a flood wave, inflow exceeds outflow, producing a *wedge of storage*. During recession, outflow exceeds inflow, resulting a negative wedge. In addition, there is a *prism of storage* which is formed by a volume of constant cross section along the length of prismatic channel.

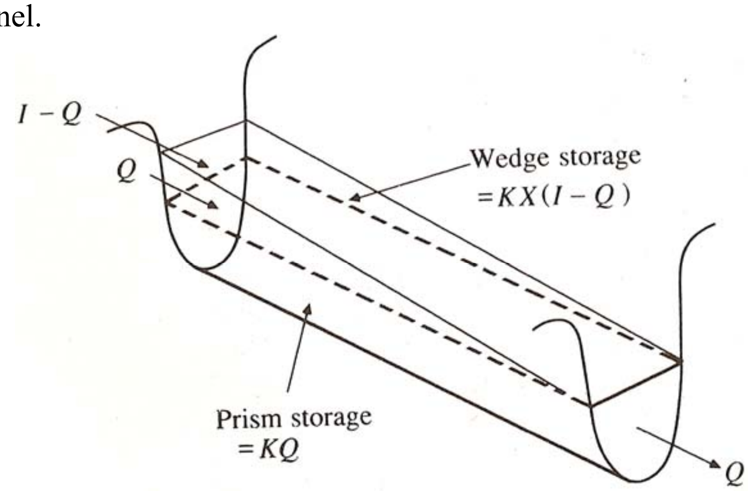


Figure 8.12. Prism and wedge storages in a channel reach

Assuming that cross-sectional area of the flood flow is directly proportional to the discharge at the section, the volume of prism storage is equal to KQ where K is a proportionality coefficient, and the volume of wedge storage is equal to KX(I-Q), where X is weighing factor having the range ($0 \le X \le 0.5$). The total storage is therefore the sum of two components,

$$S = KQ + KX(I - Q) \tag{8.9}$$

which can be arranged to give the storage function for the Muskingum method.

$$S = K[XI + (1 - X)Q] \tag{8.10}$$

and represents a linear model for routing flow in streams.

The value of X depends on the shape of the modeled wedge storage. The value of X ranges from 0 for reservoir–type storage to 0.5 for a full wedge. When X = 0, there is no wedge and hence no backwater; this is the case for a level-pool reservoir. In this case, Equ. (8.10) results in a linear-reservoir model, S =KQ. In natural streams, X is between 0 and 0.3 with a mean value 0.2. *The parameter K is the time of travel of the flood wave through the channel reach.* For hydrologic routing, the values of K and X are assumed to be specified and constant throughout the range of flow.

The values of storage at time j and j+1 can be written, respectively, as,

$$S_j = K\lfloor XI_j + (1 - X)Q_j \rfloor \tag{8.11}$$

and

$$S_{j+1} = K\lfloor XI_{j+1} + (1 - X)Q_{j+1} \rfloor \tag{8.12}$$

Using Equs. (8.11) and (8.12), the change in storage over time interval Δt (Fig.8.12) is,

$$S_{j+1} - S_j = K\{\lfloor XI_{j+1} + (1 - X)Q_{j+1} \rfloor - \lfloor XI_j + (1 - X)Q_j \rfloor\} \tag{8.13}$$

The change in storage can also be expressed, using Equ. (8.6) as,

$$S_{j+1} - S_j = \frac{(I_j + I_{j+1})}{2}\Delta t - \frac{(Q_j + Q_{j+1})}{2}\Delta t \tag{8.14}$$

Combining Equs. (8.13) and (8.14) and simplifying gives,

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j \tag{8.15}$$

which is routing equation for the Muskingum method where,

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X)+\Delta t} \tag{8.16}$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X)+\Delta t} \tag{8.17}$$

$$C_3 = \frac{2K(1-X)-\Delta t}{2K(1-X)+\Delta t} \tag{8.18}$$

Note that,

$$C_1 + C_2 + C_3 = 1 \tag{8.19}$$

If observed inflow and outflow hydrographs are available for a river reach, the values of K and X can be determined. Assuming various values of X and using known values of the inflow and outflow, successive values of the numerator and denominator of the following expression for K, derived from Equs. (8.13) and (8.14), can be computed as,

$$K = \frac{0.5\Delta t\left[\left(I_{j+1}+I_j\right)-\left(Q_{j+1}+Q_j\right)\right]}{X\left(I_{j+1}-I_j\right)+(1-X)\left(Q_{j+1}-Q_j\right)} \tag{8.19}$$

The computed values of the numerator and denominator are plotted for each time interval, with the numerator on the vertical axis and denominator on the horizontal. This usually produces a graph in the form of a loop. The value of X that produces a loop closest to a single line is taken to be the correct value for the reach, and K, according to Equ. (8.19), is equal to the slope of the line. Since K is the time required for the incremental flood wave to traverse the reach, its value may also be determined as the observed time of travel of peak flow trough the reach.

EXAMPLE 8.4: The inflow hydrograph to a river reach is given in columns (1) and (2) of Table (8.10). Determine the outflow hydrograph from this reach if K = 2.3 h, X = 0.15, and Δt= 1 h. The initial outflow is 85 m^3/sec.

Solution: Determine the coefficients C_1, C_2, and C_3 using Equs. (8.16) – (8.18):

$$C_1 = \frac{1-2\times 2.3\times 0.15}{2\times 2.3\times(1-0.15)+1} = \frac{0.31}{4.91} = 0.0631$$

$$C_2 = \frac{1 + 2\times 2.3\times 0.15}{4.91} = \frac{1.69}{4.91} = 0.3442$$
$$C_3 = \frac{2\times 2.3\times (1-0.15) - 1}{4.91} = \frac{2.91}{4.91} = 0.5927$$

Check to see that the sum of the coefficients C_1, C_2, and C_3 is equal to 1.

$$C_1 + C_2 + C_3 = \frac{0.31 + 1.69 + 2.91}{4.91} = \frac{4.91}{4.91} = 1.0$$

For the first time interval, the outflow is determined using values for I_1= 93 m^3/sec and I_2 = 137 m^3/sec from Table (8.10), the initial outflow is Q_1 = 85 m^3/sec, and using Equ. (8.15) with j=1.

$$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1$$
$$Q_2 = \frac{0.31\times 137 + 1.69\times 93 + 2.91\times 85}{4.91} = \frac{446.99}{4.91} = 91 m^3/sec$$

as shown in columns (3) to (6) of Table (8.10). Computations for the following time intervals use the same procedure with j =2, 3,……to produce the results shown in Table 10. The inflow and outflow hydrographs are plotted in Figure (8.13). It can be seen that the outflow lags the inflow by approximately 2.3 h, which was the value of K used in the computations and represents the travel time in the reach.

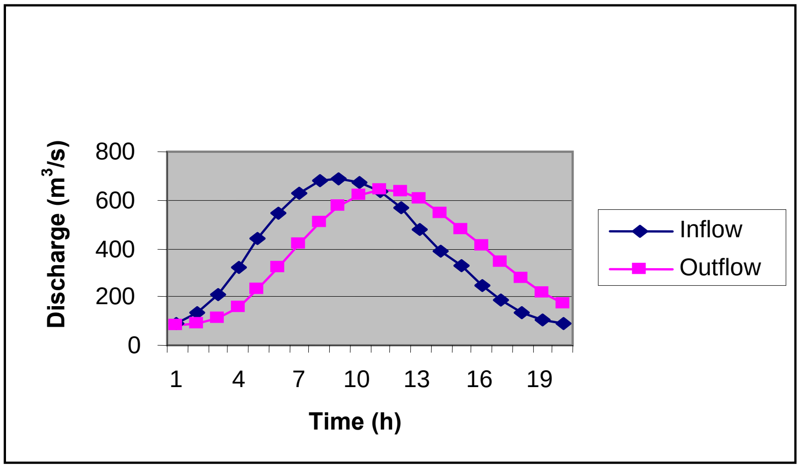


Figure 8.13. Routing of flow through a river reach by Muskingum method.

Table 8.10. Flow routing through a river reach by the Muskingum method

Routing period j (h) (1)	**Inflow I (m^3/sec) (2)**	**C_1I_{j+1} C_1=0.0631 (3)**	**C_2I_j C_2=0.3442 (4)**	**C_3Q_j C_3=0.5927 (5)**	**Outflow Q (m^3/sec) (6)**
1	93	-	-	-	85
2	137	8.64	32.01	50.38	91
3	208	13.12	47.16	53.94	114
4	320	20.19	71.59	67.57	159
5	442	27.89	110.14	94.24	232
6	546	34.45	152.14	137.51	324
7	630	39.75	187.93	192.03	420
8	678	42.78	216.85	248.93	509
9	691	43.60	233.37	301.68	579
10	675	42.59	237.84	343.17	624
11	634	40.01	232.34	369.84	642
12	571	36.03	218.22	380.51	635
13	477	30.10	196.54	376.36	603
14	390	24.61	164.18	357.40	546
15	329	20.76	134.24	323.61	479
16	247	15.59	113.24	283.90	413
17	184	11.61	85.02	244.79	341
18	134	8.46	63.33	202.11	274
19	108	6.81	46.12	162.40	215
20	90	5.68	37.17	127.43	170

EXAMPLE 8.5: The infow flood hydrograph observed at the entrance of a stream reach is given in Table (8.11). Determine the outflow hydrograph at the exit, 18 km from the entrance, by Muskingum method. The flood wave propagates in the stream with a velocity of 2 m/sec. Muskingum coefficient X will be taken as 0.25 and time step is Δt = 2hr.

Table 8.11.

Time (hr)	0	1	2	3	4	5	6	7	8	9	10
Inflow (m^3/sec)	10	12	18	28.5	50	78	107	134.5	147	150	146
Time (hr)	11	12	13	14	15	16	17	18	19	20	21
Inflow (m^3/sec)	129	105	78	59	45	33	24	17	12	10	10

Solution: The computation step will be taken as Δx = 6 km. Since K is the time it takes the flood wave to pass through the reach,

$$K = \frac{\Delta x}{V} = \frac{6000}{2} = 3000\,\text{sec}$$

The coefficients C_1, C_2, and C_3 are calculated by using Equs. (8.16)-(8.18);

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X)+\Delta t}$$
$$C_1 = \frac{2\times3600 - 2\times3000\times0.25}{2\times3000\times(1-0.25)+2\times3600} = \frac{5700}{1170} = 0.487$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X)+\Delta t}$$
$$C_2 = \frac{2\times3600 + 2\times3000\times0.25}{2\times3000\times(1-0.25)+2\times3600} = \frac{8700}{11700} = 0.744$$

$$C_3 = \frac{2K(1-X)-\Delta t}{2K(1-X)+\Delta t}$$
$$C_3 = \frac{2\times3000\times(1-0.25)-2\times3600}{2\times3000\times(1-0.25)+2\times3600} = \frac{-2700}{11700} = -0.231$$

Check to see that the sum of coefficients C_1, C_2, and C_3 is equal to 1.

$$C_1 + C_2 + C_3 = \frac{5700+8700-2700}{11700} = 1$$

For the first reach Δx = 6 km and Δt = 2 hr, the outflow is calculated using values for I_1 = 10 m^3/sec, I_2 = 18 m^3/sec, and Q_1 = 10 m^3/sec, and using Equ. (8.15) with j=1.

$$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1$$
$$Q_2 = 0.487\times18 + 0.744\times10 - 0.231\times10 = 14\,m^3/sec$$

The hydrograph ordinates are calculated at Δx = 6 km reach with Δt = 2 hr steps and given in the column (3) of Table (8.12). This is taken as the inflow hydrograph and the outflow hydrograph at section Δx = 12 km is derived similarly and tabulated in column (4). The same steps are carried on for the reach Δx = 18 km and the values written in column (5).

Table 8.12

Time (hr) (1)	Δx (km)			
	0 (2)	6 (3)	12 (4)	18 (5)
0	10	10	10	10
2	18	14	12	11
4	50	34	24	18
6	107	81	59	42
8	147	132	111	88
10	146	150	145	133
12	105	125	139	145
14	59	78	99	118
16	33	42	56	74
18	17	23	30	39
20	10	12	16	21
22	10	10	10	12
24	10	10	10	10
26	10	10	10	10
28	10	10	10	10

The peak discharge of the flood hydrograph at section Δx = 18 km is 145 m^3/s, a reduction of 5 m^3/s with respect to the peak of 150 m^3/s at the entrance section. Lag time is,

Lag time = 12 – 9 = 3 hr.

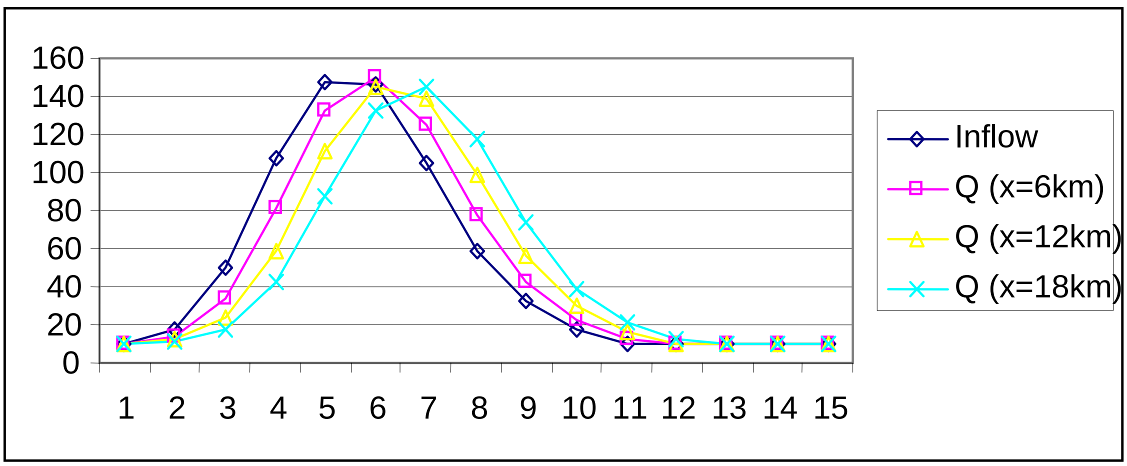


Figure 8.14. Flow routing with Muskingum method