CHAPTER 3

CONCRETE GRAVITY DAMS

3.1. FORCES ON THE DAM

Concrete gravity dams are designed so that the weight of the dam itself (the gravity force) is sufficient for overturning by the applied forces. The forces that must be considered in the design of the dam are:

1) The hydrostatic forces both upstream and downstream,
2) The hydrostatic uplift,
3) The weight of the dam,
4) Earthquake forces,
5) Ice force.

![Figure 3.1. Forces acting on a section of a concrete gravity dam](image)

**Figure 3.1.** Forces acting on a section of a concrete gravity dam

3.1.1. Hydrostatic Force

Because of the pressure of the water in the reservoir and in the downstream channel, hydrostatic forces will be exerted on the dam. The horizontal upstream force per unit width is $F_{U,H}$.

$$F_{u,H} = \gamma_u h_u^2$$  \hspace{1cm} (3.1)
Here,

\( \gamma_w = \) the specific weight of the water \( \approx 10 \text{ kN/m}^3 \)

\( h_u = \) the vertical distance from the water surface to the base of the dam at the upstream.

The location of the line of action of this force is at 2/3 of the depth below the water surface. If the dam has a sloping surface, there will be a vertical force \( F_{u,v} \).

\( F_{u,v} = \) the weight of the water vertically above the sloping face of the dam, and its line of action is through the centroid of this volume of water.

Similar hydrostatic forces act on the downstream face of the dam.

### 3.1.2. Hydrostatic Lift

After the reservoir is filled, water (under pressure) will seep into the pores of the concrete of the dam and through the pores and fissures of the foundation rock. Once conditions of equilibrium have been established (that is, once the seepage rate is constant), a pressure head gradient will be established in the concrete along the base of the dam. The maximum head is at the heel (upstream limit) of the dam, where \( p/\gamma_w = h_u \), and the minimum head is at the toe of the dam and is equal \( h_D \). Thus the magnitude of the hydrostatic uplift will equal the average uplift pressure and the area of the base section. The line of action of the uplift force will act through the centroid of the pressure prism at the base of the dam.

\[
F_{\text{uplift}} = \eta A \gamma_w \frac{h_u + h_D}{2}
\]  
(3.2)

\( A = \) Area of base of the dam,
\( \gamma_w = \) Specific weight of the water \( \approx 10 \text{ kN/m}^3 \)
\( \eta = \) Area reduction factor (0.5-0.7).

\( F_{\text{uplift}} \) acting through the centroid of the pressure diagram at the distance \( y \) from the heel with \( b \) as the width of the base,

\[
y_1 = \frac{b}{3} \times \frac{2(h_D + h_u)}{(h_D + h_u)}
\]  
(3.3)

Customary practice is to reduce the uplift force by creating a more impervious zone in the rock foundation by boring holes into the foundation rock and pumping cement grout into the holes. (Fig. 3.2).
3.1.3. Weight of a Dam

Self-weight is accounted for in terms of its resultant, $W$, which is considered to act through the centroid of the cross-sectional area $A$ of the dam profile,

$$W = \gamma_c A \text{ (kN/m)}$$  \hspace{1cm} (3.4)

$\gamma_c$ = Specific weight of the concrete, 23.5 kN/m$^3$.

Where crest gates and other ancillary structures or equipment of significant weight are present, they must also be accounted for in determining $W$ and the position of its line action.

3.1.4. Earthquake Forces

When an earthquake occurs, the earth shakes (vibrates), as does the resting on the earth. The dam will be accelerated when the quake occurs so that an inertial force will act through the center of gravity of the dam and in a direction opposite to the acceleration.

The inertial force will be equal to,

$$F_{\text{inertial}} = Ma = \frac{a}{g} M = \alpha W$$ \hspace{1cm} (3.5)
Where,

\[ a = \text{the acceleration due to the quake}, \]
\[ M = \text{Mass of the dam}, \]
\[ W = \text{Weight of the dam}, \]
\[ \alpha = \text{Earthquake coefficient (0.05-0.15)} \]

**Figure 3.3.** Inertial force due to acceleration of a dam during an earthquake.

Besides the inertial effects from an earthquake, the water pressure itself will be increased in a direction toward to reservoir.

Pressure centre (line of action) of this force is \((4h_u/3\pi)\) from the base of the dam.

\[ P_e = 0.555\alpha\gamma_w h_u^2 \]

(3.6)

### 3.2. FORCES, MOMENTS AND STRUCTURAL EQUILIBRIUM

Combination of the applied vertical and horizontal static loads equates to the inclined resultant force, \(R\). This is balanced by an equivalent and opposite reactive resultant force \(R'\), derived from vertical reactions and the reactive horizontal resistance of the foundation. The conditions essential to structural equilibrium and so to stability can therefore be summarized as,

\[ \sum H = \sum V = 0 \]  
\[ \sum M = 0 \]

(3.7)  
(3.8)

\(\sum H\) and \(\sum V\) respectively denote the summation of all active and reactive forces, and \(\sum M\) represents the summation of the moments of those with respect to any point.

The condition represented by \(\sum H = \sum V = 0\) determines that no translational movement is possible. The further condition that \(\sum M = 0\) proscribes any rotational movement, e.g. overturning.
3.3. STABILITY ANALYSIS OF THE DAM

The stability analysis of a given gravity dam cross-section may be carried by the analytical method and carried out in the following steps;

1. Consider unit length of the dam. Calculate all vertical loads acting. They include the weight of the dam, weight of water acting on the inclined faces, uplift force and inertia forces due to vertical acceleration. Find out the algebraic sum, $\sum V$.

2. Find out the sum of horizontal forces $\sum H$, and the horizontal pressure due to hydrodynamic pressure (earthquake effect).

3. Find out the sum of overturning moments $\sum M^-$ and the sum of righting moments $\sum M^+$ at the toe. Also find the algebraic sum of all the moments,

$$\sum M = \sum M^+ - \sum M^- \quad (3.9)$$

4. Find out the location (i.e. the distance $x$) of the resultant force $R$ from toe, by relation,

$$x = \frac{\sum M}{\sum V} \quad (3.10)$$

5. Find out the eccentricity $e$ of the resultant $R$ from the centre of the horizontal cross-section at the foundation as,

$$e = \frac{b}{2} - x \quad (3.11)$$

$b = \text{Base width of the cross-section}$.

The stability analysis checks,

1. For resistance to overturning,
2. For resistance to sliding,
3. To make sure that allowable normal stresses in the concrete are not exceeded.

3.3.1. Resistance to Overturning

If the dam is too thin, it may not have enough weight to resist the action of the water pressure and may fail by tipping in the downstream direction about its toe. If this were to happen, the line of action of the resultant applied forces would lie outside the pivot point, as shown in Fig. (3.4). We might then conclude that a dam would be safe from overturning if a rule were adopted stating that the line of action of the resultant should lie inside the toe of the dam (the broken line in the Fig. 3.4).
A simplistic factor of safety with respect to overturning, $F_0$, can be expressed in terms of the moments operating about the downstream toe of any horizontal plane. $F_0$ is then defined as the ratio of the summation of all restoring (i.e. positive) moments, $\sum M_+$ to the summation of all overturning moments, $\sum M_-$, thus,

$$F_0 = \frac{\sum M_+}{\sum M_-}$$  \hspace{1cm}(3.12)$$

It may be noted that $\sum M_-$ is inclusive of the moment generated by uplift load.

Values of $F_0$ in excess of 1.25 may generally be regarded as acceptable, but $F_0>1.5$ is desirable.

### 3.3.2. Resistance to Sliding

The forces that tend to cause sliding are the pressure of the water on the face of the dam, horizontal earthquake forces, ice forces, and wave forces. If these forces are less than the resistance to shear at the base of the dam, or any other horizontal section through the dam, then the dam will not slide. The applied horizontal forces are determined by the methods we discussed in the previous section.

Sliding Factor, $f$, is expressed as a function of the resistance to simple sliding over the plane considered. It is the ratio of the summation of all horizontal load components, $\sum H$, to the summation of all vertical loads, $\sum V$, on the plane considered, i.e. for a horizontal plane,

$$f = \frac{\sum H}{\sum V}$$  \hspace{1cm}(3.13)$$

$f$ on a horizontal plane should not be permitted to exceed 0.75 for the specified normal load combination.

The allowable friction factor, $f$, for rock is best determined by laboratory analyses. However, the values shown in Table were given by the U.S. Bureau of Reclamation as a guide for preliminary analysis.
Table 3.1. Representative friction factors for foundation materials (USBR, 1960)

<table>
<thead>
<tr>
<th>Material</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound rock, clean and irregular surface</td>
<td>0.80</td>
</tr>
<tr>
<td>Rock, some jointing and laminations</td>
<td>0.70</td>
</tr>
<tr>
<td>Gravel and coarse sand</td>
<td>0.40</td>
</tr>
<tr>
<td>Sand</td>
<td>0.30</td>
</tr>
<tr>
<td>Shale</td>
<td>0.30</td>
</tr>
</tbody>
</table>

- For silt and clay, testing is required.

3.3.3. Determination of Maximum Normal Stress

Shearing stress at the dam is calculated by,

\[
\beta = \frac{\sum V \times f + A \times \tau}{\sum H} \geq 5
\]  (3.14)

where,

\( \sum V = \text{Total vertical forces}, \)

\( f = \text{Sliding factor}, \)

\( A = \text{Base area of the dam}, \)

\( \tau = \text{Shear resistance of the concrete}, \)

\( \sum H = \text{Total horizontal forces}. \)

In order to calculate the normal stress distribution at the base, or at any cross-section, let \( H \) be the total horizontal force, \( V \) be the total vertical force, and \( R \) be the resultant force cutting the base at an eccentricity \( e \) from the centre of the base of width \( b \).

![Figure 3.5.](image-url)
Thus,

Direct stress $= \frac{V}{b \times 1}$ \hfill (3.15)

Bending stress $= \pm \frac{M_y}{I} = \pm \frac{V e}{2} \frac{b}{1 \times b^3} \pm \frac{6V e}{12}$ \hfill (3.16)

Hence the total stress $\sigma$ is given by,

$$\sigma = \frac{V}{b} \left( 1 \pm \frac{6e}{b} \right)$$ \hfill (3.17)

The positive sign will be used for calculating normal stress at the toe, since bending stress will be compressive there, and negative sign will be used for calculating normal stress at the heel.

Thus, normal stresses at the toe and heel are,

$$\sigma_{\text{toe}} = \frac{V}{b} \left( 1 + \frac{6e}{b} \right)$$ \hfill (3.18)

$$\sigma_{\text{heel}} = \frac{V}{b} \left( 1 - \frac{6e}{b} \right)$$

The maximum compressive stress occurs at the toe and for safety; this should not be greater than the allowable compressive stress $\sigma_{\text{con}}$ for the foundation material. Hence, from strength point of view,

$$\frac{V}{b} \left( 1 + \frac{6e}{b} \right) \leq \sigma_{\text{con}}$$ \hfill (3.19)

When the eccentricity $e$ is equal to $b/6$, we get,

$$\sigma_{\text{toe}} = \frac{V}{b} \left( 1 + \frac{6 \times b}{b} \right) = \frac{2V}{b}$$ \hfill (3.20)

The corresponding stress at the heel in that circumstance will evidently be zero.
Tension

The normal stress at the heel is

\[ \sigma_{\text{heel}} = \frac{V}{b} \left( 1 - \frac{6e}{b} \right) \]  

(3.21)

It is evident that if \( e > \frac{b}{6} \), the normal stress at the heel will be \( \sigma_{\text{heel}} < 0 \) (tensile). No tension should be permitted at any point of the dam under any circumstance for moderately high dams. For no tension to develop, the eccentricity should be less than \( b/6 \). In other words, the resultant force should always lie within the middle third.

3.4. ELEMENTARY PROFILE OF A GRAVITY DAM

In the absence of any force than the forces due to water, an elementary profile will be triangular in cross-section, having zero width at the water level, where water pressure is zero, and a maximum base width \( b \), where the maximum water pressure acts. Thus, the cross-section of the elementary profile is of the same shape as the hydrostatic pressure distribution.
We shall consider the following forces acting on the elementary profile of a gravity dam:

1) Weight of the dam = W

\[ W = \frac{1}{2} \gamma_c bH \] (Acting \( \frac{2}{3} \) from the toe)

\( \gamma_c = \) Specific weight of dam material

2) Hydrostatic force = \( F_H \)

\[ \tilde{F}_H = \frac{1}{2} \gamma_w H^2 \] (Acting \( \frac{H}{3} \) from the base)

3) Uplift force = \( F_{up} \)

\[ \uparrow F_{up} = \frac{1}{2} \eta \gamma_w bH \]

\( \eta = \) area reduction factor (0.5 – 0.7)

### 3.4.1. Base Width of Elementary Profile

The base width \( b \) of the elementary profile is to be found under two criteria:

#### 3.4.1.1. Stress Criterion

*When reservoir is empty, for no tension to develop, the resultant should act at the inner “third point”, \( M_1 \). For the reservoir full condition, for no tension to develop, the resultant \( R \) must pass through the outer “third point”, \( M_2 \).*

Taking the moment of all forces about \( M_2 \) and equating to zero (since the moment of \( R \) about \( M_2 \) is zero), we get

\[ \frac{1}{2} \gamma_w H^2 H + \frac{1}{2} \eta \gamma_w bH \frac{b}{3} - \frac{1}{2} \gamma_c bH \frac{b}{3} = 0 \]

Multiplying all the terms by \( \left( \frac{6}{\gamma_w H} \right) \),
\[
\frac{1}{6} \gamma_w H^3 \times \frac{6}{\gamma_w H} + \eta \gamma_w b^2 H \times \frac{6}{\gamma_w H} - \frac{\gamma_w b^2 H}{\gamma_w H} = 0
\]

\[H^2 + \eta b^2 - \frac{\gamma_e}{\gamma_w} b^2 = 0\]

\[\gamma = \rho g \rightarrow \gamma_e = \rho_e g \rightarrow \gamma_w = \rho_w g\]

\[\gamma_e = \frac{\rho_c}{\rho_w} = \rho_c\]

\[\gamma_w = \rho_w\]

\[\rho_c = \text{Specific mass (density) of dam material}\]

\[\rho_w = \text{Specific mass of water} = 1 \text{ ton/m}^3\]

\[H^2 + b^2(\eta - \rho_c) = 0\]

\[b^2 = \frac{H^2}{(\rho_c - \eta)}\]

\[b = \frac{H}{\sqrt{\rho_c - \eta}} \quad (3.22)\]

The above equation can also be alternatively derived by considering the force triangles. Thus,

\[F_H \frac{H}{3} + F_{wp} \frac{b}{3} = W \frac{b}{3}\]

\[\left( W - F_{wp} \right) \frac{b}{3} = F_H \frac{H}{3}\]

\[W - F_{wp} = \frac{H}{b}\]

\[b^2(\gamma_e - \eta \gamma_w) = H^2 \gamma_w\]

\[b^2(\rho_c g - \eta \rho_w g) = H^2 \rho_w g\]

\[\rho_w = 1(\text{ton/m}^3)\]

\[b = \frac{H}{\sqrt{\rho_c - \eta}}\]

If uplift is not considered, \(\eta = 0\),

\[b = \frac{H}{\sqrt{\rho_c}} \quad (3.23)\]
3.4.1.2. Sliding Criterion

For no sliding to occur, horizontal force causing sliding should be balanced by the frictional forces opposing the same. Hence,

\[ F_{ih} = f(W - F_{wp}) \]

\[ \frac{1}{2} \gamma_w H^2 = f \left( \frac{1}{2} b H \gamma_c - \frac{1}{2} \eta b H \gamma_w \right) \]

\[ \frac{1}{2} \gamma_w H^2 = \frac{1}{2} f b H (\gamma_c - \eta \gamma_w) \]

\[ b = \frac{\rho_w g H}{f(\rho_c - \eta)} \]

\[ b = \frac{H}{f(\rho_c - \eta)} \]  \hspace{1cm} (3.24)

3.4.2. Stresses Developed in the Elementary Profile

The normal stress,

\[ \sigma = \frac{W}{b} \left( 1 \pm \frac{6e}{b} \right) \]

For elementary profile,

\[ V = (W - F_{wp}) \] \quad and \quad \[ e = \frac{b}{6} \]

Hence, for full reservoir case, the normal stress at the is,

\[ \sigma_{hoe} = \frac{(W - F_{wp})}{b} (1 + 1) = \frac{2(W - F_{wp})}{b} \]

\[ \sigma_{hoe} = \frac{2}{b} \left( \frac{1}{2} b H \gamma_c - \frac{1}{2} \eta b H \gamma_w \right) \]

\[ \sigma_{hoe} = \frac{2}{b} \times \frac{1}{2} \times b H (\gamma_c - \eta \gamma_w) \]

\[ \sigma_{hoe} = H (\gamma_c - \eta \gamma_w) \]  \hspace{1cm} (3.24)

The corresponding stress at the heel will be,
\[ \sigma_{\text{heel}} = \frac{W - F_{\text{up}}}{b}(1 - 1) = 0 \]  

(3.25)

For elementary profile,

\[ V = (W - F_{\text{up}}) \quad \text{and} \quad e = \frac{b}{6} \]

Hence, for full reservoir case, the normal stress at the is,

\[ \sigma_{\text{hoe}} = \frac{(W - F_{\text{up}})(1 + 1)}{b} = \frac{2(W - F_{\text{up}})}{b} \]

\[ \sigma_{\text{hoe}} = \frac{2}{b} \left( \frac{1}{2} b H \gamma_c - \frac{1}{2} \eta b H \gamma_w \right) \]

\[ \sigma_{\text{hoe}} = \frac{1}{2} \times b H \left( \gamma_c - \eta \gamma_w \right) \]

\[ \sigma_{\text{hoe}} = H (\gamma_c - \eta \gamma_w) \]  

(3.24)

The corresponding stress at the heel will be,

\[ \sigma_{\text{heel}} = \frac{W - F_{\text{up}}}{b}(1 - 1) = 0 \]  

(3.25)

**EXAMPLE 3.1:** Using the flexure formula, prove that the resultant force acting on a gravity dam must pass within the middle one third of the base of the dam to ensure that none of the concrete is in tension along the base.

**Solution:** Consider the resultant force \( R \) as shown. Resolve this force into components \( R_y \) and \( R_x \) acting on the base. Let \( x \) be the distance from the centroid of the base to where the resultant force intersects the base. There the flexure formula will yield the normal stress at any point on the base.

\[ \sigma = \frac{F}{A} \pm \frac{M}{W} = \frac{F}{A} \pm \frac{Me}{I} \]

Where,

\[ P = R_y \]

\[ A = h \quad \text{(considering unit length of dam)} \]

Taking moment with respect to the centroid O of the base,
\[ M = R_y x \]

\[ I = \frac{hb^3}{12} = 1 \times \frac{b^3}{12} \quad \text{(for unit length of the dam)} \]

\[ \sigma = \frac{R_y}{b} \pm \frac{R_y x b}{b^3} \frac{b}{12} \]

However, for the condition of zero tensile stress \( \sigma = 0 \); therefore,

\[ 0 = \frac{R_y}{b} \pm \frac{6R_y x}{b^2} \]

Solving for \( x \) yields,

\[ x = \pm \frac{b}{6} \]

Thus, for \( x = \pm b/6 \) (limits of middle third of the base) the normal stress will be zero at the heel or toe but for any other position within the middle third the normal stress will be positive.

**EXAMPLE 3.2:**

a) Determine the normal stresses at the heel and toe of the concrete dam using the simple flexure formula for solving the stress. Consider weight of the dam, 2/3 hydrostatic uplift, and hydrostatic force on the face of the dam in your analysis. \( \gamma_c = 23.5 \text{ kN/m}^3 \). \( \gamma_w = 10 \text{ kN/m}^3 \).
Solution: Forces will be calculated per unit length of dam.

Weight of the dam:

\[ W_1 = \gamma_c A_1 = 23.5 \times 4 \times 47 = 4418 \text{kN/m} \]
\[ W_2 = \gamma_c A_2 = 23.5 \times \frac{41 \times 47}{2} \approx 22642 \text{kN/m} \]

Hydrostatic force:

\[ F_x = \frac{1}{2} \gamma_w H^2 = \frac{1}{2} \times 10 \times 45^2 = 10125 \text{kN/m} \]

Uplift force:

\[ F_{up} = \frac{2}{3} \times \frac{1}{2} \gamma_w H^2 \]
\[ F_{up} = \frac{2}{3} \times 10125 = 6750 \text{kN/m} \]

Applying flexure equation:

\[ \sigma = \frac{\sum F}{A} \pm \frac{\sum Me}{I} \]

\[ \sum F = W_1 + W_2 - F_{up} \]
\[ \sum F = 4418 + 22642 - 6750 = 20310 \text{kN/m} \]
\[ A = 1 \times 45 = 45 \text{m}^2 \]
\[ \sum M = -F_s \frac{H}{3} + W_i \left( \frac{45}{2} - \frac{4}{2} \right) + W_2 \left( \frac{41 \times 2}{3} - \frac{45}{2} \right) - F_{up} \left( \frac{45 \times 2}{3} - \frac{45}{2} \right) \]
\[ \sum M = -10125 \times 15 + 4418 \times 20.5 + 22642 \times 4.83 - 6750 \times 7.5 = \]
\[ \sum M = -2570kNm \]

Consider normal stress at heel and toe;

\[ \sigma_{\text{heel}} = \frac{19159}{45} + \frac{-2570 \times \frac{45}{2}}{45^3} \cdot \frac{12}{12} \]
\[ \sigma_{\text{heel}} = 425.76 - 7.61 = 418.15kN/m^2 \]
\[ \sigma_{\text{toe}} = 425.76 + 7.61 = 433.37kN/m^2 \]

b) What would be the stresses if earthquake forces were also included? Consider only horizontal acceleration toward reservoir and take earthquake coefficient \( \alpha = 0.10 \).

Solution: Earthquake acceleration is taken as \( a = 0.10g \). Assuming acceleration direction toward reservoir there will be an added clockwise moment due to the acceleration of the mass of the dam and there will be an added force due to the additional pressure of the water on the dam. Assume no change in uplift force.

Added moment from mass of the dam itself;

\[ W_1 \rightarrow F = ma \]
\[ F_1 = \frac{W_1}{g} \times 0.10g = 0.10W_1 \]

Moment arm: \( \frac{47}{2}m \)

\[ \text{Added moment:} - \frac{47}{2} \times 0.10 \times 4418 = -10382kNm \]

\[ W_2 \rightarrow F_2 = 0.10W_2 \]

\[ \text{Added moment:} - \frac{47}{3} \times 0.10 \times 22642 = -35472kNm \]

Increased water pressure;
\[
P_c = 0.555\alpha y_w h_u^2
\]
\[
P_c = 0.555 \times 0.10 \times 10 \times 45^2 = 1124 kN/m^2
\]

Added moment from the added hydrostatic force;

Pressure centre (line of action) of this force: \( y = \frac{4h_u}{3\pi} = \frac{4 \times 45}{3\pi} = 19.10m \)

\[
M = -1124 \times 19.10 = -21468 kNm
\]

Total moment (including earthquake forces):

\[
\sum M = -2570 - 10382 - 35472 - 21468 = -69892 kNm
\]

Normal stresses at the heel and toe of the dam;

\[
\sigma = \frac{P}{A} \pm \frac{M e}{I}
\]

\[
\sigma = 425.76 \pm \frac{-69892 \times 45}{45^3 \times 12} = 425.76 \pm 207.09
\]

\[
\sigma_{\text{heel}} = 632.85 kN/m^2
\]

\[
\sigma_{\text{toe}} = 218.67 kN/m^2
\]

**EXAMPLE 3.3:** A concrete dam with a triangle shape has a base width 72 m and height 75 m. Specific weight of the concrete, \( \gamma_c = 23.5 \) kN/m\(^3\). Specific weight of water, \( \gamma_w = 10 \) kN/m\(^3\). Area reduction factor, \( \eta = 0.70 \). Sliding factor, \( f = 0.80 \). The strength of the rock foundation, \( \sigma = 2 \) kN/m\(^2\). Unit shear resistance of the concrete, \( \tau = 1500 \) kN/m\(^2\).

**Solution:** The problem will be solved for unit width of dam to the figure. The primary forces on the concrete dam are,

a) Hydrostatic force,

\[
F_{U,H} = \frac{1}{2} \gamma_w h_u^2
\]

\[
F_{U,H} = \frac{1}{2} \times 10 \times 75^2 = 28125 kN/m
\]
b) Hydrostatic uplift,

\[ F_{up} = \eta A \gamma_w \frac{h_u}{2} \]

\[ F_{up} = 0.70 \times 72 \times 10 \times \frac{75}{2} = 18900 \text{kN/m} \]

e) Weight of the dam,

\[ W = \gamma_c A \]

\[ W = 23.5 \times \frac{75 \times 72}{2} = 63450 \text{kN/m} \]

Stability of the dam will be checked,

a) Resistance to overturning;

Moments of the primary forces with respect to O,

\[ F_O = \sum M_+ = \frac{W \times 48}{F_{U,H} \times 25 + F_{up} \times 48} \]

\[ F_O = \frac{63450 \times 48}{28125 \times 25 + 18900 \times 48} = 1.89 \]

\[ F_O = 1.89 > 1.50 \]

The dam is resistant for overturning.
b) Resistance to sliding,

\[
f = \frac{\sum H}{\sum V} = \frac{F_{u,h}}{W - P_{up}}
\]

\[
f = \frac{28125}{63450 - 18900} = 0.68
\]

\[f = 0.68 < f_{cr} = 0.80\]

The dam is resistant against sliding.

c) Shearing stress at the bottom of the dam,

\[
\beta = \frac{\sum Vf + A\tau}{\sum H} \geq 5
\]

\[
\beta = \frac{(W - P_{up}) \times f + b \times \tau \times 1}{F_{u,h}}
\]

\[
\beta = \frac{(63450 - 18900) \times 0.80 + 72 \times 1500}{28125} = 5.11
\]

\[\beta = 5.11 > 5.00\]

Determination the vertical stresses and major principal stresses at either faces when the dam is full and empty;

Hydrostatic force = 28125 kN/m

Moment arm = \(\frac{75}{3} = 25m\)

Weight = 63450 kN/m

Moment arm = \(48 - \frac{72}{2} = 12m\)

\(M^*\) = Moments relative to the centroid of the plane of the dam bottom.

\[
\sum M^* = 63450 \times 12 - 28125 \times 25 = 58275 kNm
\]

\[
\sum V = 63450 kN/m \quad \text{(Excluding uplift force)}
\]

\[
e = \frac{\sum M^*}{\sum V} = \frac{58275}{63450} = 0.92m
\]
e lies upstream of the centroid.

\[
\sigma_{up} = \sum \frac{V}{b} \left(1 - \frac{6e}{b}\right)
\]

\[
\sigma_{up} = \frac{63450}{72} \left(1 - \frac{6 \times 0.92}{72}\right) = 814 \text{kN/m}^2
\]

\[
\sigma_{down} = \sum \frac{V}{b} \left(1 + \frac{6e}{b}\right)
\]

\[
\sigma_{down} = \frac{63450}{72} \left(1 + \frac{6 \times 0.92}{72}\right) = 948 \text{kN/m}^2
\]

When the dam is empty,

Hydrostatic force = 0

Weight of the dam = 63450 kN/m

Moment arm = \(48 - \frac{72}{2} = 12 \text{m}\)

\[\sum M^* = 63450 \times 12 = 761400 \text{kNm}\]

\[e = \frac{\sum M^*}{\sum V} = \frac{761400}{63450} = 12 \text{m}\]

\[
\sigma_{up} = \frac{\sum V}{b} \left(1 - \frac{6e}{b}\right)
\]

(No tensile stresses)

\[
\sigma_{up} = \frac{63450}{72} \left(1 - \frac{6 \times 12}{72}\right) = 0
\]

\[
\sigma_{down} = \frac{\sum V}{b} \left(1 + \frac{6e}{b}\right)
\]

\[
\sigma_{down} = \frac{63450}{72} \left(1 + \frac{6 \times 12}{72}\right) = 1763 \text{kN/m}^2
\]

**EXAMPLE 3.4:** Considering earthquake forces (horizontal acceleration), in addition to the hydrostatic pressure, determine the base width of the elementary profile of a concrete gravity dam so that the resultant force passes through the outer third parts.

Height of the dam = H = 100 m, specific mass of concrete = \(\rho_c = 2.4 \text{ ton/m}^3\), specific mass of the water = \(\rho_w = 1 \text{ ton/m}^3\), area reduction factor for uplift = \(\eta = 0.70\), earthquake coefficient = \(\alpha = 0.20\).
Solution:

The various forces acting on the dam shown are shown on the Figure.

$b =$ Base width of elementary dam ABC

a) Vertical forces

1) Force due to self-weight of the dam;

$$W = \frac{1}{2}bH\gamma_w = \frac{1}{2} \times b \times 100 \times 2.4 \times 9.81 \approx 1177b$$

2) Force due to uplift;

$$F_{up} = \frac{1}{2}bH\eta_w\gamma_w = \frac{100 \times 0.70 \times 1 \times 9.81}{2} \times b \approx 343b$$

Hence,

$$\sum V = W - F_{up} = 117b - 343b = 834b$$

Moment arm to point $M_2 = \frac{b}{3}$
b) Horizontal forces

1) Hydrostatic force;

\[
\vec{F}_H = \frac{1}{2} \gamma_w H^2 = \frac{1 \times 9.81 \times 100^2}{2} = 49050kN
\]

Moment arm = \( \frac{H}{3} \)

2) Earthquake forces;

\[
\vec{F} = a W = a \frac{1}{2} b H \gamma_e
\]

\[
\vec{F}_{\text{earth}} = 0.20 \times \frac{1}{2} \times 100 \times 2.4 \times 9.81 \times b \cong 235b
\]

Moment arm = \( \frac{H}{3} \)

3) Hydrodynamic pressure;

\[
P_e = 0.555 \times \alpha \gamma_w H^2
\]

\[
P_e = 0.555 \times 0.20 \times 1 \times 9.81 \times 100^2 = 10889kN
\]

Moment arm = \( \frac{4H}{3 \pi} = \frac{4 \times 100}{3 \pi} = 42.44m \)

If the resultant of all forces has to pass through the outer third part \( M_2 \), moment of all these forces at this point must be zero.

\[
\sum V \times \frac{b}{3} - F_H \times \frac{H}{3} - F_{\text{earth}} \times \frac{H}{3} - P_e \times 42.44 = 0
\]

\[
834b \times \frac{b}{3} - 49050 \times \frac{100}{3} - 235b \times \frac{100}{3} - 10889 \times 42.44 = 0
\]

\[
278b^2 - 7833b - 2097129 = 0
\]

\[
b_{1,2} = \frac{7833 \pm \sqrt{7833^2 + 4 \times 278 \times 2097129}}{2 \times 278}
\]

\[
b_{1,2} = \frac{7833 \pm 48922}{556}
\]

Positive root will be taken.

\[
b \cong 102m
\]
Putting $\alpha = 0$ when no earthquake acts, the value of $b$ reduces to,
\[
\sum V \times \frac{b}{3} = F_u \times \frac{H}{3}
\]
\[
\frac{834b^2}{3} = 49050 \times \frac{100}{3} = 1635000
\]
\[
b \approx 77m
\]
\[
b = \frac{H}{\sqrt{\rho_c - \eta}} = \frac{100}{\sqrt{2.4 - 0.7}} \approx 77m
\]

**EXAMPLE 3.5:** A concrete dam cross-section is given in the Figure. Check the stability of the dam for,

a) Reservoir empty,
b) Reservoir full,
c) Reservoir empty with earthquake effect,
d) Reservoir full with earthquake effect.

Analysis of dam is to be carried out under the following conditions.

1) Horizontal effect of the earthquake will be considered with $\alpha = 0.10$.
2) Specific weight of the concrete $= \gamma_c = 24$ kN/m$^3$,
   Specific weight of the water $= \gamma_w = 10$ kN/m$^3$,
   Reduction factor for uplift $= \eta = 2/3$,
   Shear stress for concrete $= \tau = 1400$ kPa
   Friction factor for sliding $= f = 0.70$. 

40  A.Bulu
Solution:
1) Vertical forces;

a) Self-weight of the dam,

\[ W_1 = \gamma_c \times 7 \times 90 = 24 \times 7 \times 90 = 15120 kN \]

Moment arm to toe = \( 58 + \frac{7}{2} = 61.50 m \)

\[ M_1^+ = 15120 \times 61.50 = 929880 kNm \]

\[ W_2 = \frac{58 \times 89}{2} \times 24 = 61944 kN \]

Moment arm = \( \frac{2}{3} \times 58 = 38.67 m \)

\[ M_2^+ = 61944 \times 38.67 = 2395374 kNm \]

\[ W_3 = \frac{8 \times 28}{2} \times 24 = 2688 kN \]

Moment arm = \( \frac{8}{3} + 7 + 58 = 67.67 m \)
\[ M_3 = 2688 \times 67.67 = 181897 \text{kNm} \]

\[ \downarrow \sum W = 15120 + 61944 + 2688 = 79752 \text{kN} \]
\[ \sum M^+ = 929880 + 2395374 + 181897 = 3507151 \text{kNm} \]

b) Weight of the superimposed column of water,
\[ \downarrow F_{w} = 8 \times \frac{89 \times 61}{2} \times 10 = 6000 \text{kN} \]

Moment arm to toe O may be calculated by dividing the trapezoidal pressure diagram to a triangle and a rectangle,
\[ 61 \times 8 \times 10 \times 4 + \frac{28 \times 28}{2} \times \frac{8}{3} = 6000 \times x \]
\[ x = 3.30 \text{m} \]

Moment arm = \((8.00 - 3.30) + 7.0 + 58.0 = 69.7 \text{m} \)
\[ M_4^+ = 6000 \times 69.70 = 418200 \text{kNm} \]

\[ \downarrow \sum V = 70752 + 6000 = 85752 \text{kN} \]
\[ \sum M^+ = 3507151 + 418200 = 3925351 \text{kNm} \]

c) Uplift force,
\[ \uparrow F_{up} = \gamma \eta \frac{73 \times 89}{2} = 10 \times \frac{2}{3} \times \frac{73 \times 89}{2} = 21657 \text{kN} \]

Moment arm = \(\frac{2}{3} \times 73 = 48.67 \text{m} \)
\[ M_1^- = 21657 \times 48.67 = 1054046 \text{kNm} \]

\[ \uparrow \sum V = 85752 - 21657 = 64095 \text{kN} \]

2) Horizontal forces;
a) Hydrostatic force,
\[ \bar{F}_{H} = \frac{1}{2} \gamma \eta H^2 = \frac{1}{2} \times 10 \times 89^2 = 39605 \text{kN} \]
Moment arm = \( \frac{89}{3} = 29.67m \)

\[ M_2^- = 39605 \times 29.67 = 1175080kNm \]

b) Hydrodynamic pressure of water due to the earthquake,

\[ \bar{P}_c = 0.555 \times \alpha \times \gamma_w \times H^2 \]
\[ \bar{P}_c = 0.555 \times 0.10 \times 10 \times 89^2 = 4396kN \]

Moment arm = \( \frac{4H}{3\pi} = \frac{4 \times 89}{3\pi} = 37.77m \)

\[ M_3^- = 4396 \times 37.77 = 166037kNm \]

c) Inertial force due to the earthquake,

\[ \bar{F}_1 = \alpha W_1 = 0.10 \times 15120 = 1512kN \]

Moment arm = \( \frac{90}{2} = 45m \)

\[ M_4^- = 1512 \times 45 = 68040kNm \]

\[ \bar{F}_2 = \alpha W_2 = 0.10 \times 61944 \approx 6194kN \]

Moment arm = \( \frac{89}{3} = 29.67m \)

\[ M_5^- = 6194 \times 29.67 = 183775kNm \]

\[ \bar{F}_3 = \alpha W_3 = 0.10 \times 2688 \approx 269kN \]

Moment arm = \( \frac{28}{3} = 9.33m \)

\[ M_6^- = 269 \times 9.33 = 2510kNm \]

a) Stability analysis when the reservoir is empty,

\[ \sum V = W_1 + W_2 + W_3 = 79753kN \]
\[ \sum M = M_1^+ + M_2^+ + M_3^+ = 929880 + 2395374 + 181897 \]
\[ \sum M^+ = 3507151kNm \]
Distance where the resultant force acts from toe is,

\[ \bar{x} = \frac{\sum M}{\sum V} = \frac{3507151}{79752} \approx 44m \]

Its distance \( e \) from the center of the base is,

\[ e = \frac{b}{2} = \frac{73}{2} - 44 = -7.50m \quad \text{(At the left of the center)} \]

\[ \sigma = \frac{\sum V}{b} \left( 1 + \frac{6e}{b} \right) = \frac{79752}{73} \left( 1 \pm \frac{6 \times (-7.5)}{73} \right) \]
\[ \sigma = 1092.49 \times (1 \pm 0.62) \]

\[ \sigma_{\text{toe}} = 415 \text{kN/m}^2 = \text{kPa} \]
\[ \sigma_{\text{heel}} = 1700 \text{kN/m}^2 = \text{kPa} \]

i) Factor of safety against overturning;

Reservoir is empty; there will be no overturning force.

ii) Factor of safety against sliding;

Reservoir is empty; therefore there is no sliding force.

iii) Shear friction factor;

Reservoir is empty.

b) Stability analysis when the reservoir is full,

\[ \uparrow \sum V = 79752 + 6000 - 21657 = 64095kN \]
\[ \sum M = \sum M^+ - \sum M^- = 3925351 - 1054046 - 1175080 \]
\[ \sum M^+ = 1696225kNm \]

Distance where the resultant acts from toe is,

\[ \bar{x} = \frac{\sum M}{\sum V} = \frac{1696225}{64095} = 26.46m \]

Its distance \( e \) from the center of the base is,
\[
e = \frac{b}{2} - \bar{x} = \frac{73}{2} - 26.46 \geq 10 m
\]

\[
\sigma = \sum \frac{V}{b} \left(1 \pm \frac{6e}{b}\right) = \frac{64095}{73} \left(1 \pm \frac{6 \times 10}{73}\right)
\]

\[
\sigma = 878 \times (1 \pm 0.82)
\]

\[
\sigma_{soe} = 1598 kN/m^2 = kPa
\]

\[
\sigma_{heel} = 158 kN/m^2 = kPa
\]

i) Factor of safety against overturning,

\[
F_0 = \frac{\sum M^+}{\sum M^-} = \frac{3925351}{2229126} = 1.76 > 1.50
\]

The dam is safe against overturning.

ii) Factor of safety against sliding,

\[
f = \frac{\sum H}{\sum V} = \frac{39605}{64095} = 0.62 < 0.70
\]

The dam is safe against sliding.

iii) Shear friction factor,

\[
\beta = \frac{\sum Vf + A \tau}{\sum H} \geq 5
\]

\[
\beta = \frac{64095 \times 0.70 + 73 \times 1 \times 1400}{39605} = 3.71 < 5
\]

The dam is unsafe for shear friction.

c) Stability analysis when the reservoir is empty with earthquake effect,

\[
\sum V = 79752 kN
\]

\[
\sum M = \sum M^+ - M^- = 3507151 - 68040 = 3439111 kNm
\]
\[ \bar{x} = \frac{\sum M}{\sum V} = \frac{343911}{79752} = 43.12m \]

\[ e = \frac{b}{2} - \bar{x} = \frac{73}{2} - 43.12 = -6.62m \]

\[ \sigma = \frac{79752}{73} \times \left(1 \mp \frac{6 \times 6.62}{73}\right) \]

\[ \sigma = 1092.49 \times (1 \mp 0.54) \]

\[ \sigma_{hoe} = 502kPa \]

\[ \sigma_{heel} = 1682kPa \]

i) Factor of safety against overturning,

\[ F_0 = \frac{\sum M^+}{\sum M^-} = \frac{3507151}{68040} = 52 > 1.50 \quad \text{(safe)} \]

ii) Factor of safety against sliding,

\[ f = \frac{\sum H}{\sum V} = \frac{1512}{79752} = 0.02 < 0.70 \quad \text{(safe)} \]

iii) Shearing factor,

\[ \beta = \frac{79752 \times 0.70 + 73 \times 1400}{1512} = 105 > 5 \quad \text{(safe)} \]

d) Stability analysis when the reservoir is full with earthquake effect,

\[ \sum V = 64095kN \]

\[ \sum M = 3507151 + 418200 - 1054046 - 1175080 - 166037 - 68040 - 183775 - 2510 \]

\[ \sum M^+ = 1275863 \]
\[
\bar{x} = \frac{\sum M}{\sum V} = \frac{1275863}{64095} = 19.91m
\]
\[
e = \frac{b}{2} - \bar{x} = \frac{73}{2} - 19.91 = 16.60m
\]
\[
\sigma = \frac{64095}{73} \times \left(1 \pm \frac{6 \times 16.60}{73}\right)
\]
\[
\sigma_{oc} = 2076kPa
\]
\[
\sigma_{heel} = -320kPa \quad \text{(Tensile, not safe)}
\]

i) Overturning check,
\[
F_0 = \frac{\sum M^+}{\sum M^-} = \frac{3925351}{2649488} = 1.48 \approx 1.50 \quad \text{(Not safe)}
\]

ii) Sliding check,
\[
f = \frac{\sum H}{\sum V} = \frac{39605 + 4396 + 1512 + 6194 + 269}{64095} \quad \text{(Not safe)}
\]
\[
f = \frac{51976}{64095} = 0.81 > 0.70
\]

iv) Shear friction factor,
\[
\beta = \frac{64095 \times 0.70 + 73 \times 1400}{51976} = 2.83 < 5 \quad \text{(Not safe)}
\]

The dam cross-section is therefore, unsafe for the present loading conditions.