LECTURE NOTES – XII

« HYDROELECTRIC POWER PLANTS »

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Chapter 12

PENSTOCK

Penstock Types

In determining the number of penstocks for any particular installation various factors have to be considered. Let us compare by a single penstock and by a system of n penstocks. The fundamental condition of identical discharge can be realized by selecting diameters either,

a) For identical flow velocities,
b) For identical friction losses.

Q = Discharge conveyed in a single penstock,
D = Diameter of the penstock,
V = Flow velocity,
h_L = Headloss,
e = Wall thickness,
G = Weight of the penstock.

a) Identical flow velocities

Dividing the discharge Q among n conduits, the diameter of each pipe should be determined to ensure an identical flow velocity V. With each penstock discharging,

$$Q_n = \frac{Q}{n}$$

The condition of identical velocity is expressed by the relation,

$$V = \frac{Q}{\pi D^2/4} = \frac{Q_n}{\pi D_n^2/4}$$

Where D_n is the diameter of any penstock.

$$D_n = D\sqrt{\frac{Q_n}{Q}} = \frac{D}{\sqrt{n}}$$

The head loss due to friction in case of single penstock installation,

$$h_L = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g} = \frac{fL}{2gD} \cdot \left(\frac{4Q}{\pi D^2}\right)^2$$

$$h_L = \frac{16\, fL}{2g\pi} \cdot \frac{Q^2}{D^5}$$

$$a_1 = \frac{16fL}{2g\pi} = 0.26fL$$

$$h_L = a_1 \frac{Q^2}{D^5}$$

And for n penstocks,

$$h_{L_n} = a_1 \frac{\left(\frac{Q}{n}\right)^2}{\left(\frac{D}{\sqrt{n}}\right)^5} = a_1 \frac{Q^2 n^{5/2}}{n^2 D^5} = a_1 \frac{Q^2\sqrt{n}}{D^5}$$

$$h_{L_n} = h_L\sqrt{n}$$

The wall thickness in case of single penstock arrangement,

$$e = \frac{pD}{2\sigma_{steel}} \rightarrow a_2 = \frac{p}{2\sigma_{steel}}$$

$$e = a_2 D$$

p = Static + water hammer pressure
σ_{steel} = Tensile stress of the steel

For n penstocks,

$$e_n = a_2 D_n = a_2 \frac{D}{\sqrt{n}}$$

$$e_n = \frac{e}{\sqrt{n}}$$

The total penstock weight in case of single penstock installation,

$$G = \gamma_{steel}\pi De \rightarrow a_3 = \gamma_{steel}\pi$$
$$G = a_3 De$$

For one penstock of the n-number system,

$$G_n = a_3 D_n e_n = a_3 \cdot \frac{D}{\sqrt{n}} \cdot \frac{e}{\sqrt{n}}$$

$$G_n = \frac{G}{n}$$

The total weight of the system of n penstocks,

$$nG_n = G$$

b) Identical friction (head) losses

For determining the diameter D_n ensuring a head loss identical with that in the single penstock,

$$h_L = a_1 \frac{Q^2}{D^5} = a_1 \frac{\left(\frac{Q}{n}\right)^2}{D_n^5}$$

$$D_n = \frac{D}{\sqrt[5]{n^2}}$$

The flow velocity in each of the penstocks,

$$V_n = \frac{\frac{Q}{n}}{\frac{\pi D_n^2}{4}} = \frac{V}{\sqrt[5]{n}}$$

The wall thickness of each of the n penstocks,

$$e_n = a_2 D_n = \frac{e}{\sqrt[5]{n^2}}$$

The weight of the each of the n penstocks,

$$G_n = a_3 D_n e_n = \frac{G}{\sqrt[5]{n^4}}$$

The total weight of the penstock system is,,

$$nG_n = \sqrt[5]{n} \cdot G$$

The above results are compiled in the Table.

The alternative based on identical head loss should be considered in the economical analysis, since; energetically this is equivalent to the single penstock arrangement. As can be seen, the theoretical weight increases for several penstocks with $n^{1/5}$ fold. Actually the difference is greater since the weight of couplings and joints does not decrease in proportion with the diameter. The amount of *steel required for solutions involving several penstocks may be significantly higher* than the amount required for a single penstock arrangement. Owing to the increased number supporting piers, the costs of civil engineering construction will also become higher.

On the other hand, *the use of two or more penstocks means added safety of operation* and no complete shutdown will become necessary in case of repair. The number of penstocks should be decided on the basis of thorough economical analysis of different alternatives.

Table. Comparison of single penstock and of multi penstock arrangements

	One penstock	n penstocks for identical	
		Velocity	Head loss
Discharge	Q	$n\frac{Q}{n}=Q$	$n\frac{Q}{n}=Q$
Diameter	D	$D/n^{1/2}$	$D/n^{2/5}$
Velocity	V	V	$V/n^{1/5}$
Head loss	h_L	$h_L n^{1/2}$	h_L
Wall Thickness	e	$e/n^{1/2}$	$e/n^{2/5}$
Total weight	G	G	$Gn^{1/5}$

The penstock is made of steel. As regards the location of the penstock, two different solutions may be discerned which are characteristics of the method of support as well.

1. **Buried penstocks** are supported continuously on the soil at the bottom of a trench backfilled after placing the pipe. The thickness of the cover over the pipe should be about 1.o to 1.2 m.

The advantages of buried pipes are the following:

a) The soil cover protects the penstock against effect of temperature variations,
b) It protects the conveyed water against freezing,
c) Buried pipes do not spoil the landscape,
d) They are safer against rock slides, avalanches and falling trees.

Disadvantages are:

a) Such pipes are less accessible for inspection, faults cannot be determined easily,
b) For large diameters and rocky soils their installation is expensive,
c) On steep hillsides, especially if the friction coefficient of the soil is low, such pipes may slide,
d) Maintenance and repair of the pipe is difficult.

2. **Exposed penstocks** are installed above the terrain surface and supported on piers (briefly called supports or saddles). Consequently, there is no contact between the terrain and the pipe itself, and the support is not continuous but confined piers.

The *advantages* of exposed pipes are the following;

a) The possibility of continuous and adequate inspection during operation,
b) Its installation is less expensive in case of large diameters of rocky terrain,
c) Safety against sliding may be ensured by properly designed anchorages,
d) Such pipes are readily accessible and maintenance and repair operations can be carried out easily.

The *disadvantages* are;

a) Full exposure to external variations in temperature,
b) The water conveyed may freeze,
c) Owing to the spacing of supports and anchorages significant longitudinal stresses may develop especially in pipes of large diameters designed for low internal pressures.

As a general rule, buried pipes are applied only on mildly sloping terrain where the top layers do not consist of rock. *The exposed arrangement is more frequently applied*. The main advantage of exposed penstocks is the possibility of continuous inspection during operation.

Concrete blocks holding the pipeline may be simple *supporting piers* permitting slight longitudinal movement of the pipe, or *anchor blocks* which do not permit movement of the pipe. Anchorages are usually installed at angle joints, while supporting piers are spaced rather closely (6 to 12 m) depending on the beam action of the pipe and the supporting capacity of the soil.

In order to reduce the longitudinal stresses due to the temperature variations and other causes, rigid joints between pipe sections should in some places be substituted by elastic ones.

Large power penstocks subject to heads of several hundred meters may be constructed of *banded steel pipes*.

Simple steel pipes are used for,

$$pD < 10000 (kg/cm)$$

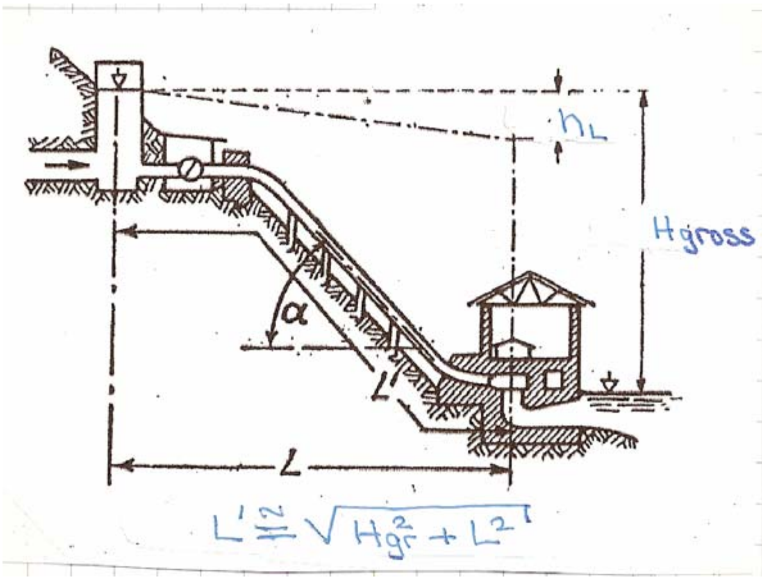
Banded steel pipes for,

$$pD > 10000 (kg/cm)$$

Where p (kg/cm^2) internal pressure, and D (cm) pipe diameter.

Penstock Hydraulic Calculations

Practical empirical equations used to find out the diameter of a penstock will be given.



Maximum velocity in the penstocks may be taken as V_{max} = 6 m/sec. Using the head loss condition,

$$h_L = \frac{V^2 L' n^2}{R^{4/3}} \leq 0.05 H_{gross}$$

Ludin – Bundschu has given empirical equations to compute the economical pipe inner diameter by depending on the head shown in the Figure,

$$H_{gross} < 100m \rightarrow D = \sqrt[7]{0.05Q^3} \qquad \text{(m)}$$

$$H_{gross} > 100m \rightarrow D = \sqrt[7]{\frac{5.2Q^3}{H_{gross}}} \qquad \text{(m)}$$

Example: Calculate the inner diameter of the penstock for a hydroelectric power plant for Q = 15 m^3/sec discharge, and H = 120 m head. Water surface oscillations in the surge tank will not be taken into account.

Solution:

a) Choosing the velocity in the penstock as V_{max} = 6 m/sec,

$$A = \frac{Q}{V} = \frac{15}{6} = 2.5m^2$$

$$A = \frac{\pi D^2}{4} \rightarrow D = 2\sqrt{\frac{A}{\pi}}$$

$$D = 2\sqrt{\frac{2.5}{\pi}} \cong 1.80m$$

Hydraulic radius = $R = \frac{D}{4} = \frac{1.80}{4} = 0.45m$

The slope angle of the penstock will be assumed as α = 45 ^{0} and the length of the penstock will be,

$$L' = \sqrt{120^2 + 120^2} \cong 170m$$

Manning coefficient = n = 0.014

The head loss,

$$h_L = \frac{V^2 L' n^2}{R^{4/3}} = \frac{6^2 \times 170 \times 0.014^2}{0.45^{4/3}} = 3.48m$$

$$0.05H = 0.05 \times 120 = 6m > 3.48m$$

V_{max} = 6 m/sec velocity may be accepted.

b) Using empirical diameter equations,

$$H = 120m > 100m \rightarrow D = \sqrt[7]{\frac{5.2Q^3}{H}}$$

$$D = \sqrt[7]{\frac{5.2 \times 15^3}{120}} = 2.04m$$

$$R = \frac{D}{4} = \frac{2.04}{4} = 0.51m$$

$$V = \frac{4Q}{\pi D^2} = \frac{4 \times 15}{\pi \times 2.04^2} = 4.59\, m/\sec$$

$$h_L = \frac{4.59^2 \times 170 \times 0.014^2}{0.51^{4/3}} = 1.72m$$

The head gained by increasing the pipe inner diameter,

$$\Delta H = 3.48 - 1.72 = 1.76m$$

If the plant runs 180 days by 24 hours daily, the gained energy will be,

$$\Delta E = 8QHT$$

$$\Delta E = 8 \times 15 \times 1.76 \times 180 \times 24 = 912384kwh$$

Forces Acting on Pipes

Pipes must be designed to withstand stresses created by internal and external pressures, changes in momentum of the flowing liquid, external loads, and temperature changes, and to satisfy the hydraulic requirements of the project.

1. Internal Forces

The internal pressure within a conduit is caused by static pressure and water hammer. Internal pressure causes circumferential tension in the pipe walls which is given approximately by,

$$\sigma = \frac{pr}{t} \qquad (1)$$

Where,

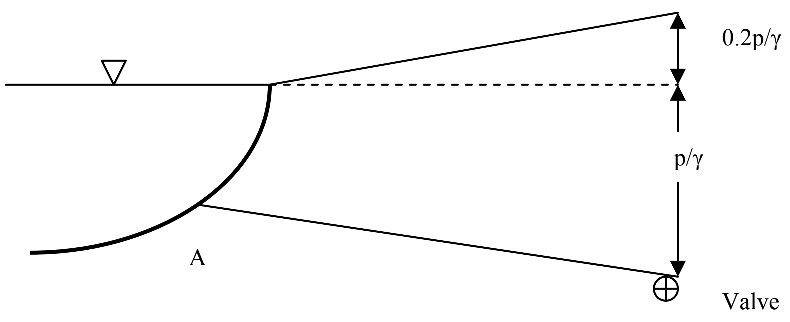
σ = Tensile stress,
p = Static + water hammer pressure
r = Internal radius of the pipe,
e = Wall thickness.

Pipes are chosen to supply this condition. If the steel pipe is chosen, the thickness of its wall may be calculated by,

$$e = \frac{pr}{\sigma_{steel}} \qquad (2)$$

2. Water Hammer

When a liquid flowing in a pipeline is abruptly stopped by the closing a valve, dynamic energy is converted to elastic energy and a series of positive and negative pressure waves travel back and forth in the pipe until they are damped our by friction. This phenomenon is known as *water hammer*.



Figure…Static + water hammer pressure

This results in a pressure rise which causes a portion of the pipe surrounding the element to stretch. A pressure in excess of hydrostatic cannot be maintained at the junction of pipe and reservoir, and the pressure at A drops to normal as some of the water in the pipe flows back into the reservoir.

The velocity c (*celerity*) of a pressure wave in any medium is the same as the velocity of sound in that medium and is given by,

$$c = \left(\frac{E_w}{\rho_w} \right)^{1/2} \qquad (3)$$

E_w = the modulus of elasticity of the water,
ρ_w = the specific mass of water.

c is about 1440 m/sec for water under ordinary conditions. The velocity of a pressure wave created by water hammer is less than 1440 m/sec because of the elasticity of pipe. The velocity of a pressure wave in a water pipe usually ranges from 600 to 1200 m/sec for normal pipe dimensions and materials. If longitudal extension of the pipe is prevented while circumferential stretching takes place freely, the velocity of a pressure wave c_p is given by,

$$c_p = \left(\frac{E_w}{\rho_w} \right)^{0.5} \left(\frac{1}{1 + \frac{ED}{E_p e}} \right)^{0.5} \qquad (4)$$

Where,

E_p = the modulus of elasticity of the pipe walls,
D = the pipe diameter,
e = the wall thickness.

If the valve is closed instantaneously, a pressure wave travels up the pipe with the velocity c_p. In a short interval of time dt, an element of water of length $c_p dt$ is brought to rest. Applying Newton's second law and neglecting friction,

$$F = m \frac{dV}{dt}$$

$$Fdt = MdV$$

$$-Adpdt = \rho A c_p dt dV$$

Since velocity is reduced to zero, dV = -V and dp equals the pressure p_h caused by water hammer. Hence,

$$p_h = \rho c_p V \qquad (5)$$

The total pressure at the valve immediately after closure is,

$$p_{total} = p_h + p \qquad (6)$$

If the length of the pipe is L, the wave travels from valve to reservoir and back in time,

$$t = \frac{2L}{c_p} \qquad (7)$$

This is the time that a positive pressure will be maintained at the valve. If the valve is closed gradually, a series of small pressure waves is transmitted up the pipe. These waves are reflected at the reservoir and return down the pipe as waves of normal pressure. If the valve is completely closed before the reflected wave returns from the reservoir, the pressure increase is, $p_h = \rho c_p V$. If the closure time $t_c > t$, negative pressure waves will be superimposed on the positive waves and the full pressure p_h' developed by gradual closure of the valve is given approximately by,

$$p_h' \approx \frac{t}{t_c} p_h = \frac{2L}{c_p t_c} \rho c_p V = \frac{2L\rho V}{t_c} \qquad (8)$$

Example: Water flows at 2 m/sec from a reservoir into a 100 cm diameter steel pipe which is 2500 m long and has a wall thickness e = 2.5 cm. Find the water hammer pressure developed by closure of a valve at the end of the line if the closure time is a) 1 sec , b) 8 sec.

$E_w = 2\times10^9$ (N/m^2), $E_{steel} = 2\times10^{11}$ (N/m^2), $\rho_{water} = 1000$ kg/m^3

Solution:

$$c_p = \left(\frac{E_w}{\rho}\right)^{0.5} \left(\frac{1}{1+\frac{E_w D}{E_{steel} e}}\right)^{0.5}$$

$$c_p = \left(\frac{2\times10^9}{1000}\right) \times \left(\frac{1}{1+\frac{2\times10^9\times1}{2\times10^{11}\times0.025}}\right)^{0.5}$$

$$c_p = 1414\times0.714 = 1010\, m/\sec$$

$$t = \frac{2L}{c_p} = \frac{2500\times2}{1010} = 4.95\sec$$

a) If t_c = 1 sec < 4.95 sec,

$$p_h = \rho c_p V = 1000\times1010\times2 = 2.02\times10^6\, kPa$$

b) If t_c = 8 sec > 4.95 sec,

$$p_h' \cong \frac{t}{t_c} p_h = \frac{4.95}{8}\times2.02\times10^6 = 1.25\times10^6\, kPa$$

Water hammer pressures can be greatly reduced by use of slow-closing valves, automatic relief valves, air chambers, and surge tanks. If the velocity of flow is increased suddenly by the opening of a valve or starting a pump, a situation opposite to water hammer develops.

For practical purposes,

$$p_h = 0.20p \qquad (9)$$

may be taken.

3) Forces at Bends and Changes in Cross-Section

A change in the direction or magnitude of flow velocity is accompanied by a change in momentum comes from the pressure variation within the fluid and from forces transmitted to the fluids from the pipe walls.

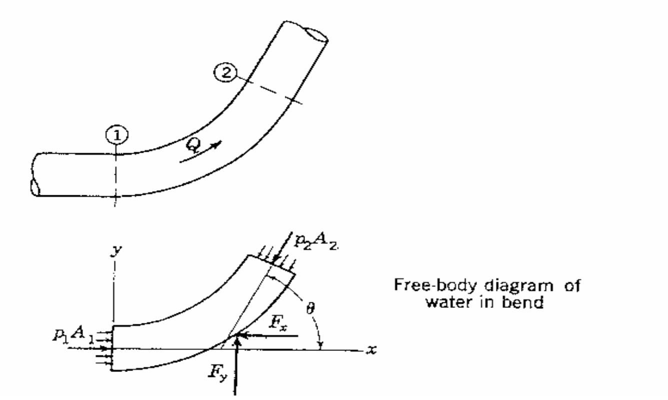
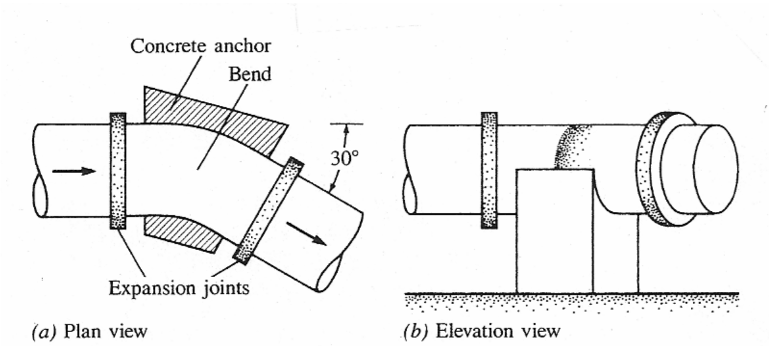


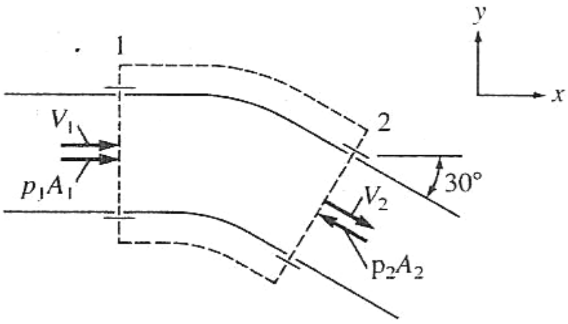
Figure . Forces at a horizontal pipe bend

Where p_1 and p_2 and V_1 and V_2 represent the pressure and average velocity in the pipe at sections 1 and 2, respectively. For a horizontal pipe bend of uniform section, $V_1 = V_2$ and $p_1 \approx p_2$.

Example: A 1 m diameter pipe has a 30^0 horizontal bend in it, and carries water at a rate of 3 m^3/sec. If we assume the pressure in the bend is uniform at 75 kPa gauge pressure, the volume of the bend is 1.8 m^3, and the metal in the bend weighs 4 kN, what forces must be applied to the bend by the anchor to hold the bend in place? Assume expansion joints prevent any force transmittal through pipe walls of the pipes entering and leaving the bend.



Solution: Consider the control volume shown in figure, and first solve for the x component of force,



$$\sum F_x = \rho Q(V_2 - V_1)$$
$$p_1A_1 - p_2A_2\cos 30^0 + F_{anchor,x} = 1000\times 3\times\left(V_2\cos 30^0 - V_1\right)$$

Where,

$$p_1 = p_2 = 75000Pa$$
$$A_1 = A_2 = \frac{\pi D^2}{4} = 0.785m^2$$
$$V_1 = V_2 = \frac{Q}{A} = \frac{3}{0.785} = 3.82\,m/\sec$$

$$F_{anchor,x} = 1000\times 3.82\times\left(\cos 30^0 - 1\right) + 75000\times 0.785\times\left(\cos 30^0 - 1\right)$$
$$F_{anchor,x} = -9423N$$

Solve for F_y:

$$\sum F_y = \rho Q\left(V_2\sin 30^0 - V_1\right)$$
$$p_2A_2\sin 30^0 + F_{anchor,y} = \rho Q\left(-3.82\sin 30^0\right)$$
$$F_{anchor,y} = -1000\times 3\times 3.82\times\sin 30^0 - 75000\times 0.785\times\sin 30^0$$
$$F_{anchor,y} = -35168N$$

Solve for F_z:

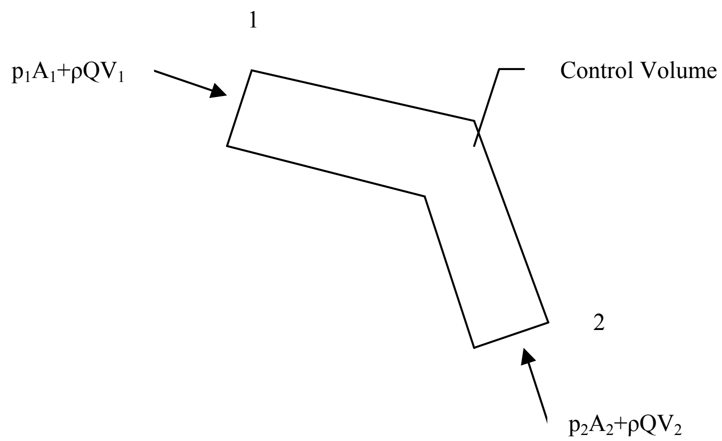
$$\sum F_z = \rho Q\left(V_{2z} - V_{1z}\right)$$
$$W_{bend} + W_{water} + F_{anchor,z} = 0$$
$$F_{anchor,z} = 4000 + 1.8\times 9810 = 21680N$$

Then the total force that the anchor will have to exert on the bend will be,

$$F_{anchor} = -9420\vec{i} - 35168\vec{j} + 21658\vec{k}$$

Example: This 130-cm overflow pipe from a small hydroelectric plant conveys water from the 70-m elevation to the 40-m elevation. The pressures in the water at the bend entrance and exit are 20 kPa and 25 kPa, respectively. The bend interior volume is 3 m^3, and the bend itself weighs 10 kN. Determine the force that a thrust block must exert on the bend to secure it if the discharge is 15 m^3/sec.

Solution:



The geometric location of the bend in space, (x,y,z)= (0, 13, 60) m

Velocity and pressure vectors at cross-sections 1 and 2 respectively,

$$\vec{V} = \left(\frac{Q}{A}\right)\left(x\vec{i} + y\vec{j} + z\vec{k}\right)$$

$$l_1 = \sqrt{0^2 + 13^2 + 10^2} = 16.40m$$

$$\vec{V}_1 = \left(\frac{Q}{A}\right)\left(0\vec{i} + \frac{13.00}{16.40}\vec{j} - \frac{10.00}{16.40}\vec{k}\right)$$

$$\vec{V}_1 = \left(\frac{Q}{A}\right)\left(0.793\vec{j} - 0.61\vec{k}\right)$$

$$\vec{F}_{p_1} = (p_1A_1)\left(0.793\vec{j} - 0.61\vec{k}\right)$$

$$l_2 = \sqrt{13^2 + 19^2 + 20^2} = 30.50m$$

$$\vec{V}_2 = \left(\frac{Q}{A}\right)\left(\frac{13.0}{30.5}\vec{i} + \frac{19.0}{30.5}\vec{j} - \frac{20.0}{30.5}\vec{k}\right)$$

$$\vec{V}_2 = \left(\frac{Q}{A}\right)\left(0.426\vec{i} + 0.623\vec{j} - 0.656\vec{k}\right)$$

$$\vec{F}_{p_2} = (p_2A_2)\left(-0.426\vec{i} - 0.623\vec{j} + 0.656\vec{k}\right)$$

$$\text{Weight} = -3 \times 9810\vec{k}$$

Using momentum equation;

$$\sum \vec{F} = \rho Q(\vec{V}_2 - \vec{V}_1)$$

$$\sum F_x = \rho Q(V_{2,x} - V_{1,x})$$

$$F_{block,x} - 0.426 p_2 A = \rho Q\left(0.426\frac{Q}{A} - 0\right)$$

$$p_2 A = 25000 \times \frac{\pi \times 1.30^2}{4} = 33183N$$

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4 \times 15}{\pi \times 1.3^2} = 11.30\, m/\sec$$

$$F_{block,x} = 0.426 \times 33182 + 1000 \times 15 \times 0.426 \times 11.30$$

$$F_{block,x} = 86343N$$

$$F_{block,y} + 0.793 \times p_1 A - 0.623 \times p_2 A = \rho QV(0.623 - 0.793)$$

$$p_1 A = 20000 \times \frac{\pi \times 1.30^2}{4} = 26547N$$

$$F_{block,y} = 1000 \times 15 \times 11.30 \times (0.623 - 0.793) - 0.793 \times 26547 + 0.623 \times 33183$$

$$F_{block,y} = -29193N$$

$$F_{block,z} - 0.61 \times p_1 A + 0.656 \times p_2 A - W = \rho QV(-.656 + 0.61)$$

$$F_{block,z} = 1000 \times 15 \times 11.30 \times (0.610 - 0.656) + 0.610 \times p_1 A - 0.656 \times p_2 A + 3 \times 9810 + 10000$$

$$F_{block,z} = 26059N$$

Then the total force vector which the thrust block exerts on the bend to hold it in place is,

$$\vec{F} = 86343\vec{i} - 29193\vec{j} + 26059\vec{k}$$

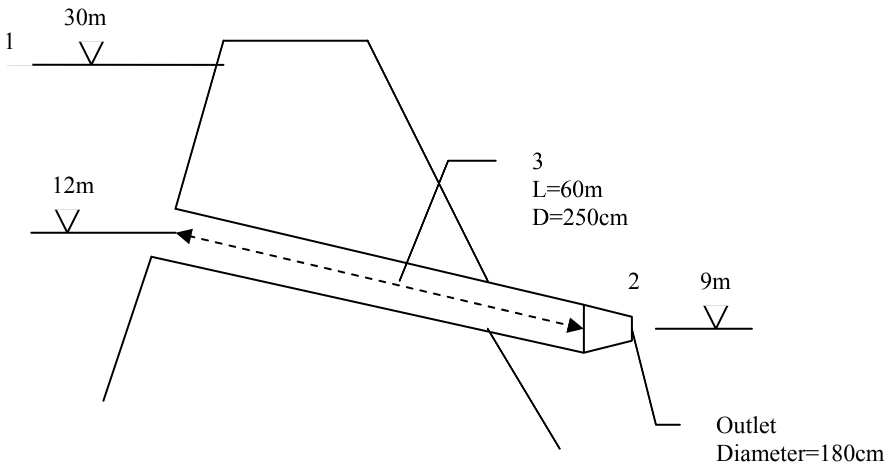
Example: The sluiceway is steel lined and has nozzle at its downstream end. What discharge may be expected under the given conditions? What force will be exerted on the joint that joins the nozzle and sluiceway lining? f = 0.01.

Solution: First write energy equation from water surface in reservoir to outlet of sluices;

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$0 + 0 + 30 = 0 + \frac{V_2^2}{2g} + 9 + \frac{f}{d} * \frac{V_3^2}{2g} L$$

$$A_2 = \frac{\pi \times 1.80^2}{4} = 2.54m^2 \rightarrow V_2 = \frac{Q}{2.54}$$



$$A_3 = \frac{\pi \times 2.50^2}{4} = 4.91m^2 \rightarrow V_3 = \frac{Q}{4.91}$$

$$30 = \frac{Q^2}{2.54^2} \times \frac{1}{19.62} + 9.0 + \frac{0.01}{2.50} \times \frac{Q^2}{4.91^2} \times \frac{1}{19.62} \times 60$$

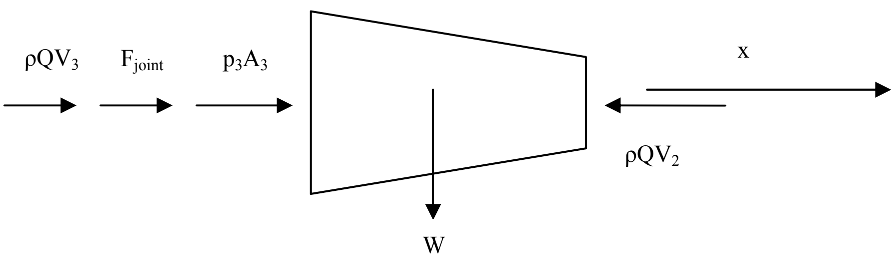
$$21 = 0.0079Q^2 + 0.00051Q^2$$

$$Q \cong 50\, m^3/\text{sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{50.0}{2.54} = 19.69\, m/\text{sec}$$

$$V_3 = \frac{50.0}{4.91} = 10.18\, m/\text{sec}$$

Now consider the force at the joint,



$$\sum F_x = \rho Q (V_2 - V_3)$$

$$p_3 A_3 + F_{joint} = 1000 \times 50 \times (19.69 - 10.18)$$

$$F_{joint} = -p_3 A_3 + 475500$$

Writing energy equation at the joint,

$$\frac{p_3}{\gamma} + \frac{V_3^2}{2g} = \frac{V_2^2}{2g}$$

$$\frac{p_3}{\gamma} = \frac{\left(19.69^2 - 10.18^2\right)}{19.62} = 14.48m$$

$$p_3 = 14.48 \times 9810 = 142035 Pa$$

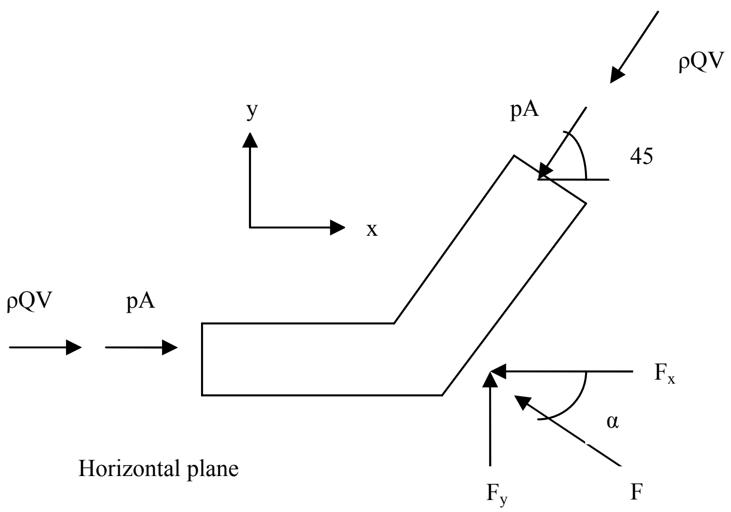
$$F_{joint} = -142035 \times 4.91 + 475500$$

$$F_{joint} = -222891 N$$

In addition a force will have to be applied at the joint to resist the weight of the nozzle and weight of water in the nozzle. Depending upon the length of the nozzle this may be as much as 30% or 40% of the force calculated above and it will act upward.

Example: A pipe 40 cm in diameter has a 135^0 horizontal bend in it. The pipe carries water under a pressure of kPa gage at a rate of 0.40 m^3/sec.What is the magnitude and direction of horizontal external force necessary to hold the bend in place under the action of water?

Solution:



$$A = \frac{\pi D^2}{4} = \frac{\pi \times 0.40^2}{4} = 0.126 m^2$$

$$V = \frac{Q}{A} = \frac{0.40}{0.126} = 3.17\, m/\sec$$

$$\sum F_x = 0$$

$$\rho QV + pA - F_x - \left(pA + \rho QV\right)\cos 45^0 = 0$$

$$F_x = \left(\rho QV + pA\right)\left(1 - \cos 45^0\right)$$

$$F_x = \left(1000 \times 0.40 \times 3.17 + 90000 \times 0.126\right) \times \left(1 - \cos 45^0\right)$$

$$F_x = 3694 N$$

$$\sum F_y = 0$$

$$F_y = \left(pA + \rho QV\right)\sin 45^0$$

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$$F_y = (11340 + 1268)\sin 45^0$$

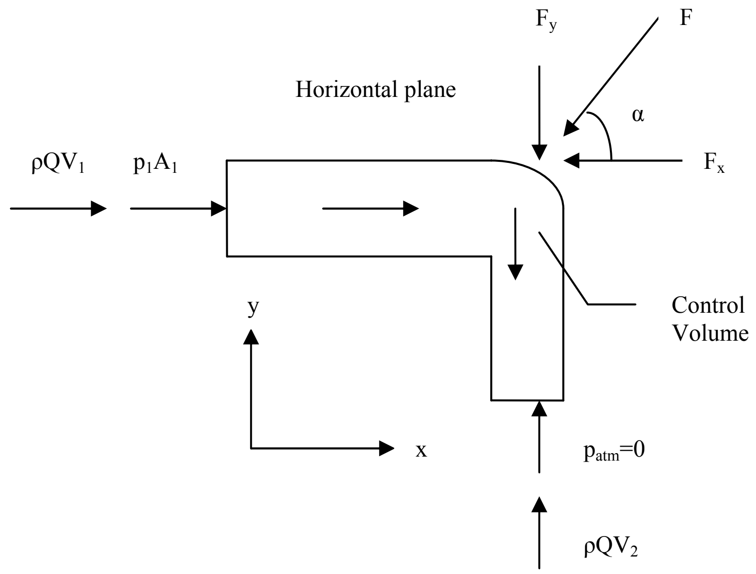
$$F_y = 8914N$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{3694^2 + 8914^2} = 9649N$$

$$\cos\alpha = \frac{F_x}{F} = \frac{3694}{8914} = 0.383 \rightarrow \alpha =$$

Example: A 90^0 horizontal bend narrows from a 60 cm diameter upstream to a 30 cm downstream. If the bend is discharging water into the atmosphere and pressure upstream is 170 kPa gage, what is the magnitude and direction of the resultant horizontal force to hold the bend in place?

Solution:



$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{A_2} + \frac{V_2^2}{2g}$$

$$V_1A_1 = V_2A_2$$

$$V_1\frac{\pi}{4}D_1^2 = V_2\frac{\pi}{4}D_2^2$$

$$\frac{V_2}{V_1} = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{0.60}{0.30}\right)^2 = 4$$

$$V_2 = 4V_1$$

$$0 + \frac{170000}{9810} + \frac{V_1^2}{2g} = 0 + 0 + \frac{16V_1^2}{2g}$$

$$17.33 = \frac{V_1^2}{2g}$$

$V_1 = 4.76\,m/\sec \rightarrow V_2 = 4\times 4.76 = 19.04\,m/\sec$

$$Q = \frac{\pi\times 0.60^2}{4}\times 4.76 = 1.35\,m^3/\sec$$

$$F_x = p_1 A_1 + \rho Q V_1$$

$$F_x = 170000\times\frac{\pi\times 0.60^2}{4} + 1000\times 1.35\times 4.76 = 54468N$$

$$F_y = \rho Q V_2 = 1000\times 1.35\times 19.04 = 25704$$

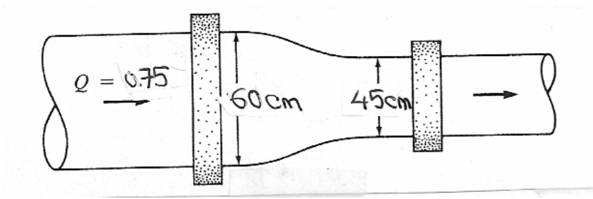
$$F = \sqrt{54468^2 + 25704^2} = 60228N$$

$$\cos\alpha = \frac{F_x}{F} = \frac{54468}{60228} = 0.904 \rightarrow \alpha =$$

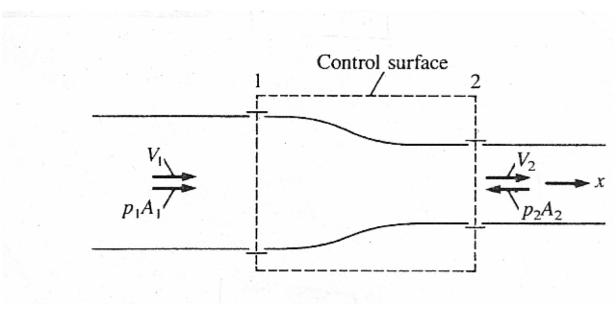
4) Transitions

The fitting between two pipes of different size is a *transition*. Because of the change in flow area and change in pressure, a longitudal force will act on the transition. To determine the force required to hold the transition in place, the energy, momentum, and continuity equations will be applied.

Example: Water flows through the contraction at a rate of 0.75 m^3/sec. The head loss coefficient for this particular contraction is 0.20 based on the velocity head in the smaller pipe. What longitudal force (such as from an anchor) must be applied to the contraction to hold it in place? We assume the upstream pipe pressure is 150 kPa, and expansion joints prevent force transmittal between the pipe and the contraction.



Solution: Let the x direction be in the direction of flow, and let the control surface surround the transition as shown in the figure.



First solve for p_2 with the energy equation,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g\gamma}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

Where,

$$\frac{p_1}{\gamma}=\frac{150000}{9810}=15.30m$$

$$V_1=\frac{Q}{A_1}=\frac{4\times 0.75}{\pi\times 0.60^2}=2.65\,m/\sec$$

$$V_2=\frac{Q}{A_2}=\frac{4\times 0.75}{\pi\times 0.45^2}=4.72\,m/\sec$$

$$z_1=z_2$$

$$h_L=0.20\frac{V_2^2}{2g}=0.20\times\frac{4.72^2}{19.62}=0.23m$$

$$\frac{p_2}{\gamma}=15.30+\frac{2.65^2}{19.62}-\frac{4.72^2}{19.62}-0.23=14.29m$$

$$p_2=9810\times 14.29=140kPa$$

The anchor force,

$$\sum F_x=\rho Q(V_2-V_1)$$

$$p_1A_1-p_2A_2+F_{anchor}=1000\times 0.75\times(4.72-2.65)$$

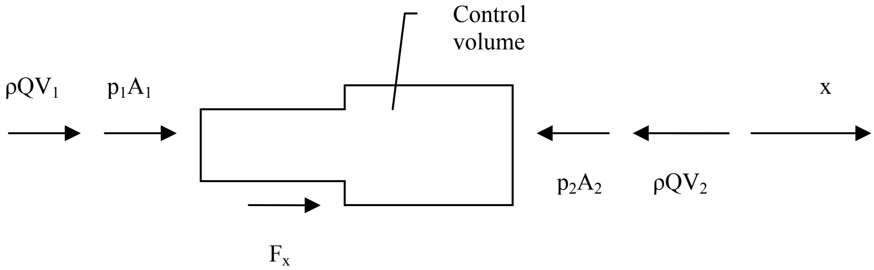
$$F_{anchor}=140000\times\frac{\pi\times 0.60^2}{4}-150000\times\frac{\pi\times 0.45^2}{4}+1553$$

$$F_{anchor}=-18637N$$

Then anchor must exert a force of 18637 N in the negative x direction on the transition.

Example: A 50 cm diameter pipe expands to a 60 cm diameter pipe. These pipes are horizontal, and the discharge of water from the smaller size to the larger is 0.80 m^3/sec. What horizontal force is required to hold the transition in place if the pressure in the 50 cm pipe is 70 kPa? Also, what is the head loss?

Solution:



$$A_1 = \frac{\pi \times 0.50^2}{4} = 0.1963m^2 \rightarrow V_1 = \frac{0.80}{0.1963} = 4.08\, m/\sec$$

$$A_2 = \frac{\pi \times 0.60^2}{4} = 0.2826m^2 \rightarrow V_2 = \frac{0.80}{0.2826} = 2.83\, m/\sec$$

$$h_L = \frac{(V_1 - V_2)^2}{19.62} = \frac{(4.08 - 2.83)^2}{19.62} = 0.08m$$

Writing energy equation between sections 1 and 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$\frac{p_2}{\gamma} = \frac{70000}{9810} + \frac{4.08^2}{19.62} - \frac{2.83^2}{19.62} - 0.08$$

$$\frac{p_2}{\gamma} = 7.50m \rightarrow p_2 = 9810 \times 7.50 = 73575Pa$$

Writing the momentum equation for the transition,

$$\sum F_x = \rho Q (V_2 - V_1)$$

$$p_1 A_1 - p_2 A_2 + F_x = 1000 \times 0.80 \times (2.83 - 4.08)$$

$$F_x = 73575 \times 0.2826 - 70000 \times 0.1963 - 1000 \times 0.80 \times 1.25$$

$$F_x = 3694N$$

5) Temperature Stress and Strain

Temperature stresses develop when temperature changes occur after the pipe is installed and rigidly held in a place. For example, if a pipe is strained from expanding when the temperature changes ΔT^0, the pipe would be subjected to a compressive longitudal reflection of,

$$\Delta L = \alpha L \Delta T \qquad (10)$$

α = Coefficient of thermal expansion

Then the resulting effective longitudal strain would be,

$$\varepsilon = \frac{\Delta L}{L} = \alpha \Delta T \quad (11)$$

And the resulting temperature stress would be,

$$\sigma = E\varepsilon = E\alpha\Delta T \quad (12)$$

E = Bulk modulus of elasticity of the pipe material.

Example: Find the longitudal stress in a steel pipe caused by temperature increase of 30^0C. Assume that longitudal expansion is prevented. For steel, E = 210×10^6 (kN/m^2, kPa), α = 11.7×10^{-6}.

Solution:

$$\sigma = 210\times10^6 \times 11.7\times10^{-6} \times 30 = 73710\left(kN/m^2, kPa\right)$$

To eliminate the temperature stress, expansion joints are used. These joints can be placed at regular intervals and must allow the pipe to expand a distance ΔL, where L is the spacing between expansion joints.

6) Steel Pipe Weight

The area of the material of a steel pipe for a 1 m length with a wall thickness e,

$$A = \frac{\pi}{4}\left[(D+2e)^2 - D^2\right]$$

$$A = \frac{\pi}{4}\left(D^2 + 4eD + 4e^2 - D^2\right)$$

$$A = \pi e(D+e) = \pi eD\left(1 + \frac{e}{D}\right)$$

If the ration $\left(\frac{e}{D} \cong 0\right)$ is taken as zero,

$$A = \pi eD \quad (13)$$

The weight of the L length pipe with specific mass (density) of steel ρ_{steel} = 7800 (kg/m^3) is,

$$G = \pi eDL\rho_{steel}g$$

$$G = \pi eDL \times 7800 \times 9.81$$

$$G = 240388eDL \text{ (Newton)} \quad (14)$$

Pipe wall thickness e due to the static and water hammer pressure is,

$$p = p_{static} + p_{hammer} \cong 0.12 p_{static} = 0.12 p_{st} \qquad (15)$$

$$\sigma = \frac{p_s D}{2e} \rightarrow e = \frac{p_s D}{2\sigma}$$

The working stress of the steel is, σ_{st} = 11000 (kN/m^2),

$$e = \frac{0.12 p_{st} D \times 9810}{2 \times 11000 \times 10^3}$$

$$G = 240388 \times \frac{0.12 \times 9810}{2 \times 110000 \times 10^3} \times p_{st} D^2$$

$$G = 1.29 p_{st} D^2 L \qquad (16)$$

Where, p_{st} = Static pressure (N/m^2), D = Internal pipe diameter (m), L = Pipe length (m).

7) External Pressure

When the pipe is empty, it must have a wall thickness enough to resist atmospheric pressure. This minimum wall thickness is calculated by Allievi equation as,

$$e_{\min} = 0.008D \qquad (17)$$

In which e_{min} is the minimum thickness of the pipe wall in m and D is the pipe inner diameter in m as well.

8) Longitudal Bending

Pipes should normally be designed to resist some bending in the longitudal direction even they are to be buried or they are laid on saddles.

The maximum span which a simply supported pipe could accommodate is calculated below. The maximum stress is,

$$\sigma = \frac{M}{W}$$

Where M is the bending moment and W is the section modulus. For a pipe whose wall thickness e, is small in comparison with the internal diameter D,

$$W = \frac{\pi D^2}{4} e$$

If bending moment is,

$$M = \frac{pL^2}{8}$$

Where p is the steel pipe weight with water in it for unit length and L is the span,

$$\sigma_{st} = \frac{\frac{pL^2}{8}}{\frac{\pi D^2}{4}e} = \frac{pL^2}{2\pi D^2 e}$$

$$L^2 = \frac{2\pi D^2 e\sigma_{st}}{p}$$

If the pipe of specific weight γ_{st} is conveying water with specific weight γ_w,

$$p = \frac{\pi D^2}{4}\gamma_w + \gamma_{st}\pi De$$

$$L = \sqrt{\frac{8De\sigma_{st}}{\gamma_w D + 4\gamma_{st}e}} \qquad (18)$$

σ_{st} = Working stress of the steel.

Example: Calculate the maximum permissible simply supported span L for the 1 m diameter steel pipe with a wall thickness e = 0.012 m. σ_{st} = 110000 kN/m^2, γ_w = 9810 N/m^3, γ_{st} = 80000 N/m^3.

Solution: The weight of the 1 m length of pipe with the water in it is,

$$p = \gamma_w \frac{\pi D^2}{4} + \gamma_{st}\pi De$$

$$p = 9810 \times \frac{\pi \times 1.0^2}{4} + 80000 \times \pi \times 1.0 \times 0.012$$

$$p = 10721(N/m)$$

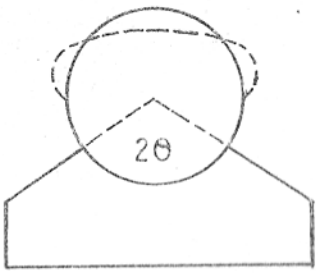
Maximum permissible span,

$$\sigma_{st} = \frac{10721 \times L^2}{2 \times \pi \times 1.0^2 \times 0.012}$$

$$L^2 = \frac{110 \times 10^6 \times 2 \times \pi \times 1.0^2 \times 0.012}{10721}$$

$$L \cong 28m$$

The angle of bottom support, 2θ, is normally 120^0 for concrete saddles. In order to prevent the deflection during the filling of the pipe, it is recommended to take the bottom support angle, 2θ, as given below.



$D \leq 3$ m → $2\theta = 120^0$
$3 < D \leq 4$ m → $2\theta = 180^0$
$4 < D \leq 5$ m → $2\theta = 210^0$
$D > 5$ m → $2\theta = 240^0$

9) Freezing Effects

The water in the pipe can freeze whenever there is no flow in it and the outside temperature drops to the low values. These precautions are recommended for the side effects,

1. The water velocity in the pipe should be kept grater than 0.50 m/sec.
2. A discharge of 1 m^3/hour for 1 m^2 of pipe perimeter is to be supplied,
3. A minimum required discharge may be calculated by,

$$Q_{\min} = \frac{0.434k(\pi DL)}{\gamma_w(Ln\theta_0 - Ln\theta_1)} \qquad (19)$$

Where, $\theta_0 = t_0 - T_0$, $\theta_1 = t_1 - T_1$. t_0 is the water temperature in ^{0}C at the inlet of the pipe which can be taken as +4^{0}C for the reservoirs, T_0 is the outside temperature at the inlet, t_1 is the temperature of the water at the outlet (^{0}C) and T_1 is the outside temperature at the outlet. Generally, $T_0 = T_1$. The coefficient k = 1/200 for water. $\gamma_w \approx 10000$ N/m^3.

Example: Calculate the minimum required discharge for a pipe of 1 m diameter and 1000 m length if the temperature of air drops to -40^{0}C.

Solution:

$$\theta_0 = t_0 - T_0 = 4 + 40 = 44^0 C$$
$$\theta_1 = t_1 - T_1 = 0 + 40 = 40^0 C$$

$$Q_{\min} = \frac{0.434 \times \dfrac{1}{200} \times \pi \times 1.0 \times 1000}{1000 \times (Ln44 - Ln40)} = 0.007\, m^3/\sec$$

Using the perimeter related solution,

$$P = \pi D \times 1 = 3.14 m^2$$
$$Q = 1 \times 3.14\, m^3/hour = 0.00087\, m^3/\sec$$

These equations are empirical equations and have to be used cautiously. The experience of design engineers is very important when using empirical equations.