LECTURE NOTES – XI

« HYDROELECTRIC POWER PLANTS »

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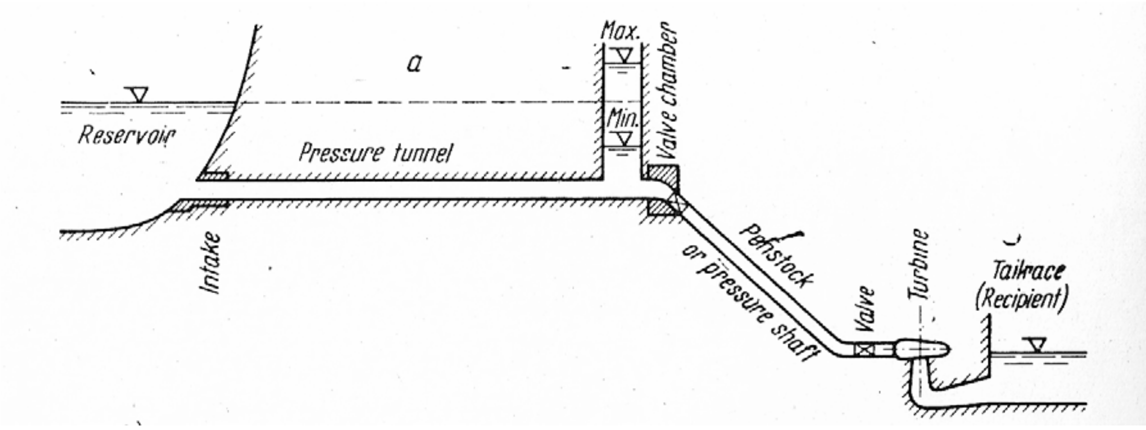
Chapter 11

The Surge Tank

The surge tank is located between the almost horizontal or slightly inclined conduit and steeply sloping penstock and is designed as a chamber excavated in the mountain.



Surge tanks serve as a threefold purpose;

1. Upon the rapid closure of the turbine, water masses moving in the pressure tunnel and in the penstock are suddenly decelerated. Owing to the inertia of moving masses, F = ma, high overpressures develop at the lower end of the penstock, which are propagating upwards in the penstock in the form of pressure wave. The magnitude of the so-called *water hammer*, caused by the moving masses by closure, will depend upon the dimensions and elastic properties of the conduit. The overpressure due to water hammer travels along the closed conduit and is not relieved until a free water surface is reached.

   An important function of the surge tank can be summarized like this. The turbines to the reservoir is practically interrupted by the surge tank to prevent the pressure wave due to the water hammer at the free water surface and to free the pressure tunnel from excessive pressures.

2. The surge provides protection to the penstock against damage of water hammer. The overpressure depends upon the length of the penstock (the closed conduit). The surge tank, by interrupting the closed system of the penstock and of the pressure tunnel, reduces the overpressure due to water hammer.

3. The third purpose of the surge tank is to provide water supply to the turbines in case of starting up. The amount of water required during these changes in operating conditions is supplied by the surge tank installed in the conduit. The capacity thereof should be selected to ensure the required water supply during the most unfavorable increase in demand, until the water mass in the tunnel

has attained the necessary velocity. Air should be prevented from entering the penstock even in case of the deepest downsurge in the chamber.

The height of the surge tank is governed by the highest possible water level that can be expected during operation. Variations in demand initiated by a rapid opening or closure of the valve or turbine are followed with a time lag by the water masses moving in the tunnel. Upon the rapid and partial closure of the valve following a sudden load decrease, water masses in the penstock are suddenly decelerated, and one part of the continuous supply from the tunnel fills the surge tank. The water surface in the surge chamber will be raised to above static level. In case of rapid opening, the flow in the tunnel is smaller than the turbine demand to supply water to the turbine. The water surface in the chamber will start to drop to below of the steady-state level. To establish steady-flow conditions, the water surface will again start to rise from the low point, but owing to the inertia of moving water, will again rise over the steady-level. The cycle is repeated all over again with amplitudes reduced by friction, i.e. the oscillation is damped. The phenomenon described is the *water surface oscillation*. The maximum amplitude of water surface oscillation can be observed when the water demand is suddenly stopped.

A wide variety of types has been developed in practice for the surge tank. According to the hydraulic design, the following groups can be distinguished.

1. *Simple surge tanks* designed as basins, which may be provided with overfall.
2. *Special surge tanks:*

   Surge tanks with expansion chambers, which may be provided with overfall.
   Surge tank with upper expansion chamber.
   Surge tank with lower expansion chamber.
   Double-chamber surge tank.

3. *Restricted-orifice type (throttled) surge tanks:*
   Simple restricted-orifice surge tank.
   Differential (Johnson type) surge tank.
   Double-chamber, restricted-orifice surge tank.

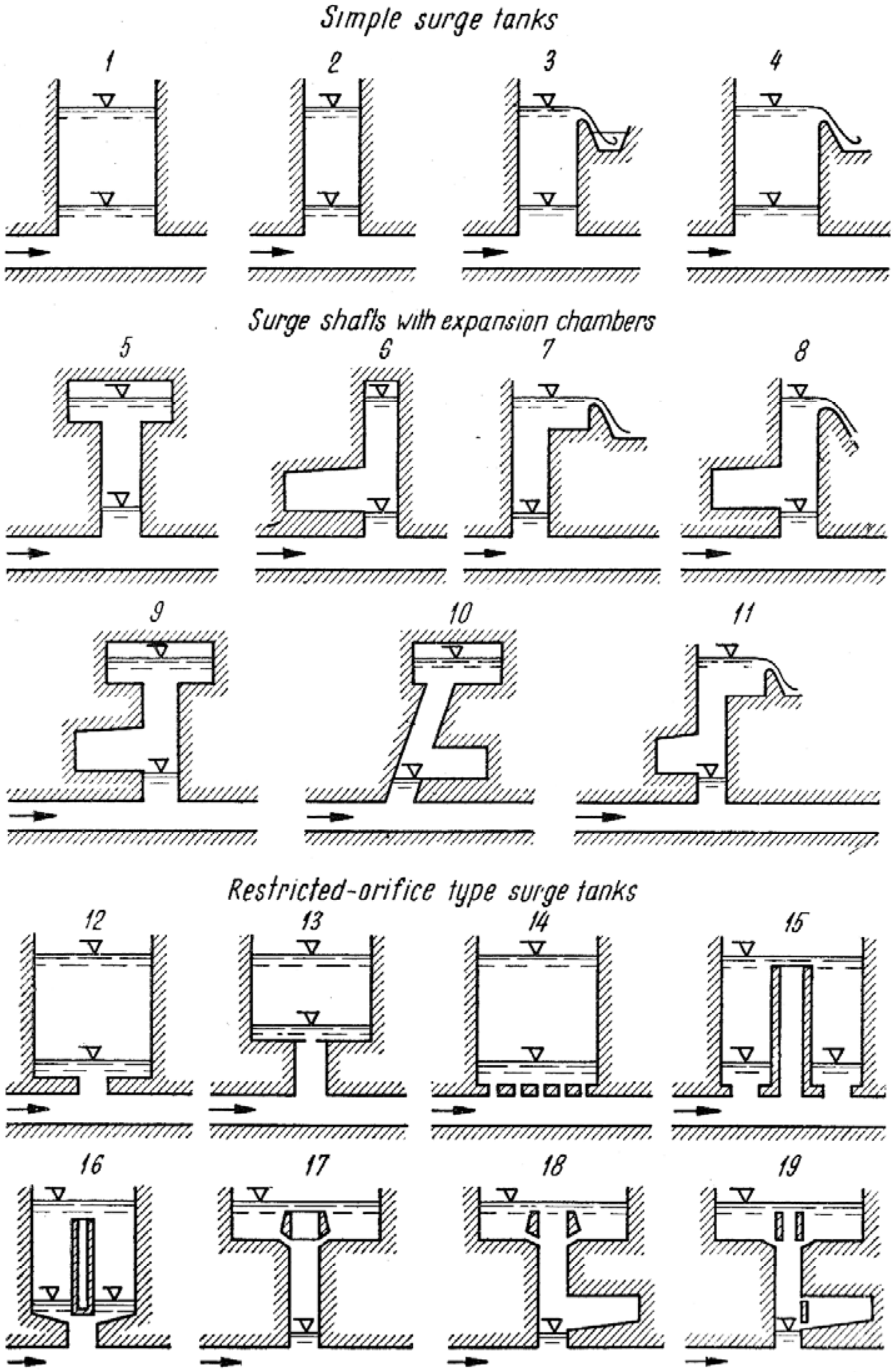
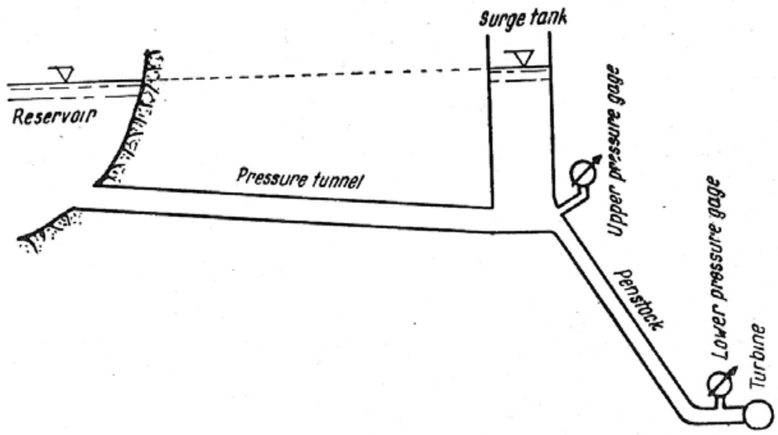


Fig. 3/84. **Surge tanks: 1. simple tank, 2. simple shaft, 3. spilling shaft, 4. overflow type tank, 5. shaft with upper expansion chamber, 6. shaft with lower chamber, 7. shaft with overflow type upper chamber, 8. overflow type shaft with lower chamber, 9. shaft with two chambers, 10. inclined shaft with two chambers, 11. overflow type tank with two chambers, 11—12. simple, restricted-orifice type tanks, 14. tank with grate type throttling, 15—16. differential (Johnson type) tanks, 17. differential tank with upper chamber, 18. double-chamber differential tank, 19. double-chamber differential tank with throttled lower chamber**

Water Surface Oscillations in Simple Surge Tank

The oscillating movement starts as soon as the pressure wave due to a change in the turbine reaches the surge tank after traveling the length penstock.



Placing manometers at the upper and lower end of the penstock, it will be seen that the two react differently to sudden changes in turbine discharges. The lower manometer will be the first to indicate the pressure wave starting from the lower end of the penstock. The upper manometer will indicate the low-frequency oscillations and will also show the water level fluctuations at the same cycle with the surge tank. Waves are damped by roughness conditions.

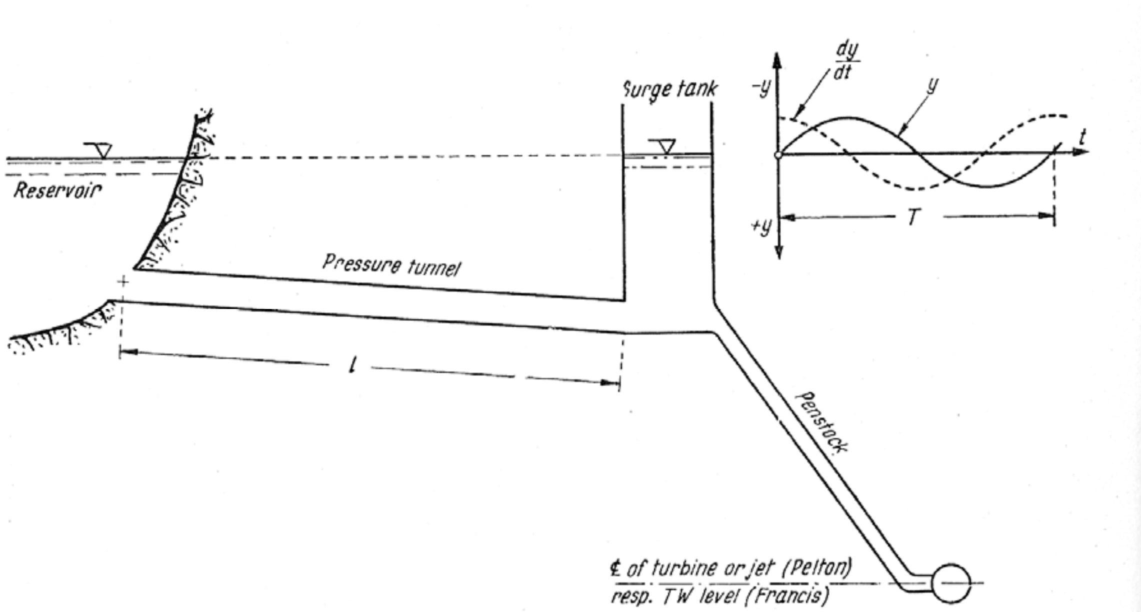


Figure. Undamped oscillations in the surge tank if frictionless conditions are assumed in the pressure tunnel

In the pure theoretical case when no friction is assumed to occur in the pressure tunnel, the water level in the surge tank is on the same elevation as the reservoir whatever the discharge of the system is. Therefore, *hydrostatic and hydrodynamic levels are identical*. The axis of the undamped oscillation is the hydrostatic (and at the same time hydrodynamic) equilibrium level. The penstock is supplied through a surge tank from the frictionless pressure tunnel. The reservoir level may be considered unchanged.

F = Surge tank cross-sectional area,
f = Pressure tunnel cross-sectional area,
l = Pressure tunnel length.

It will be assumed that the time of opening or closure turbine valves is zero (instantaneous). With the above fundamental assumptions, the expressions for the four basic cases are given without derivation.

1. Instantaneous total closure from the maximum discharge of Q_0 (so-called total load rejection).

It is evident that the total closure at maximum turbine discharge results in the greatest possible surges. This highest value of the y_{max} surges occurring in the tank upon rejection of different loads will be distinguished by the notation Y_{max}. The flow velocity in the pressure tunnel for the discharge Q_0 is $V_0 = \frac{Q_0}{f}$.

The absolute value of the widest amplitude in case of the undamped mass oscillation, i.e. the so-called *maximum surge* is,

$$Y_{max} = V_0 \sqrt{\frac{lf}{gF}} \quad \text{(m)}$$

The departure of the water level from its initial position at any arbitrary time t (considering the downward branch of the axis y as positive);

$$y = -Y_{max} Sin \frac{2\pi}{T} t \quad \text{(m)}$$

The varying velocity of water flowing in the pressure tunnel at any time t is,

$$V = V_0 Cos \frac{2\pi}{T} t \quad \text{(m/sec)}$$

At the time t = T/4 (quarter period), the velocity in the tunnel is V = 0, the direction of the flow in the tunnel changes.

The velocity of the water level in the surge tank is,

$$\frac{dy}{dt} = -\frac{f}{F} V_0 Cos \frac{2\pi}{T} t \quad \text{(m/sec)}$$

The time of the total cycle, i.e. the period of the mass oscillation is,

$$T = 2\pi\sqrt{\frac{lF}{gf}} \quad \text{(sec)}$$

Example: The pressure tunnel length is l = 10 km with a cross-sectional area of f = 10 m^2 and steady flow velocity V_0 = 2 m/sec at a hydroelectric power plant. Cylindrical surge tank cross-sectional area is F = 100 m^2. In case of instantaneous closure, compute the maximum surge height and the period of the oscillation assuming the ideal fluid (frictionless).

Solution: Maximum surge height,

$$Y_{max} = V_0\sqrt{\frac{lf}{gF}} = 2\times\sqrt{\frac{10000\times 10}{9.81\times 100}} = 20.20m$$

The period of mass oscillation,

$$T = 2\pi\sqrt{\frac{lF}{gf}} = 2\pi\sqrt{\frac{10000\times 100}{9.81\times 10}} \cong 640\sec$$

Velocities at the maximum surge height in the tunnel and the surge tank are,

$$t = \frac{T}{4} = \frac{640}{4} = 160\sec$$

$$V = V_0 Cos\frac{2\pi}{T}t = 2\times Cos\left(\frac{360}{640}\times 160\right) = 0$$

$$U = \frac{dy}{dt} = -\frac{10}{100}\times 2\times Cos\left(\frac{360}{640}\times 160\right) = 0$$

The water will stop at t = T/4 time for the maximum surge case and will begin to drop in the tank.

2. **The surge amplitude in case of *partial instantaneous closure*, from the maximum discharge Q_0 to an arbitrary Q_1 value is,**

$$Y = (V_0 - V_1)\sqrt{\frac{lf}{gF}} \quad \text{(m)}$$

Where, $V_1 = Q_1/f$ is the velocity for the reduced discharge. The position of the water level at any time t is given by the expression,

$$y = -YSin\frac{2\pi}{T}t$$

Velocity in the pressure tunnel is,

$$V = V_1 + (V_0 - V_1)Cos\frac{2\pi}{T}t$$

Velocity in the surge tank is,

$$U = \frac{dy}{dt} = -\frac{f}{F}(V_0 - V_1)Cos\frac{2\pi}{T}t$$

The period of oscillation is also,

$$T = 2\pi\sqrt{\frac{lF}{gf}} \qquad \text{(sec)}$$

Example: Using the values given in preceding example, compute the maximum surge for the closure from maximum discharge Q_0 to $0.5Q_0$.

Solution: The discharge of the full load,

$$Q_0 = V_0 f = 2 \times 10 = 20 m^3/\text{sec}$$
$$Q_1 = 0.5Q_0 = 10 m^3/\text{sec}$$
$$V_1 = \frac{10}{10} = 1 m/\text{sec}$$
$$Y = (V_0 - V_1)\sqrt{\frac{lf}{gF}} = (2-1)\times\sqrt{\frac{10000\times 10}{9.81\times 100}} = 10.10 m$$

The period of oscillation will not change.

3. **Oscillations for the *instantaneous partial opening* from some discharge Q_1 to the maximum Q_0 (partial load demand) are given by,**

$$Y = (V_0 - V_1)\sqrt{\frac{lf}{gF}}$$

The momentary position of the water leveling the surge tank is given by the function,

$$y = YSin\frac{2\pi}{T}t$$

Velocities can be computed from the following relations,

$$V = V_0 - (V_0 - V_1) Cos \frac{2\pi}{T} t$$

$$U = \frac{dy}{dt} = \frac{f}{F}(V_0 - V_1) Cos \frac{2\pi}{T} t$$

The oscillation period equation is the same.

4. **The *instantaneous total opening* from the rest (Q = 0) to the maximum discharging capacity of the turbines Q_0 (total load demand) can be characterized by the following relations.**

The maximum surge is equal to the value obtained for total closure,

$$Y_{max} = V_0 \sqrt{\frac{lf}{gF}}$$

The movement of the water surface is,

$$y = Y_{max} Sin \frac{2\pi}{T} t$$

Velocities are obtained as,

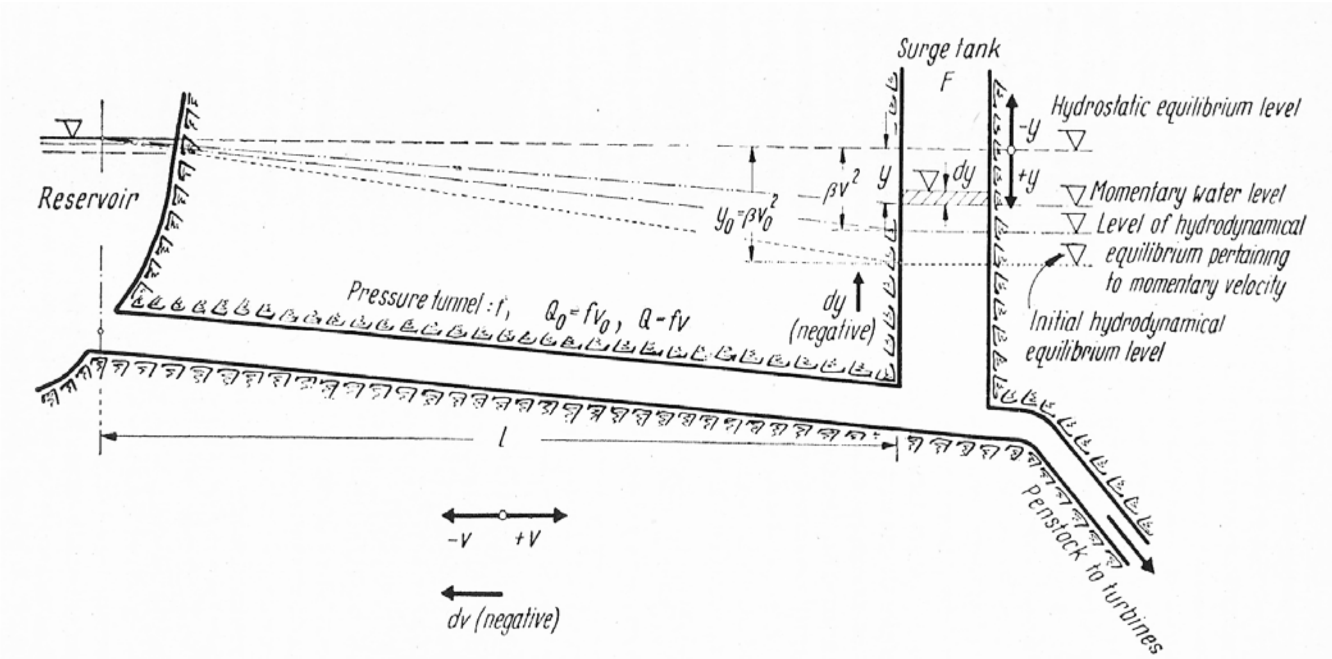
$$V = V_0 - V_0 Cos \frac{2\pi}{T} t$$

$$U = \frac{dy}{dt} = \frac{f}{F} V_0 Cos \frac{2\pi}{T} t$$

Water Surface Oscillations in the Surge Tank by Taking Headloss in the Pressure Tunnel (Damped Oscillations)

The *frictional resistance developing* along the tunnel will be taken into account and its damping effect yielding *damped oscillations* will be dealt with. The only case of damped mass oscillations for which an exact mathematical solution can be found is the total closure. For other circumstances only approximate mathematical and graphical methods are available.

For the examination of *instantaneous closure* consider the Figure below and notations used therein. The reservoir is connected with a surge tank of cross-sectional area F, by a pressure tunnel of cross-sectional area f, and length l, followed by a penstock conveying a discharge Q_0. The *hydrodynamic-equilibrium water level* in the surge tank for this operating condition is below the *hydrostatic level* by,



Figure

$$y_0 = V_0^2 \frac{\ln^2}{R^{4/3}} = \beta V_0^2$$

Where the static level is equal to that in the reservoir, and $V_0 = Q_0/ f$. Hence y_0 is the hydraulic resistance of the tunnel at a flow velocity V_0. This is the headloss due to the friction in the tunnel computed by the Manning equation as,

$$V_0 = \frac{1}{n} R^{2/3} S_0^{1/2} = \frac{1}{n} R^{2/3} \left(\frac{\Delta h}{l} \right)^{1/2}$$

Whence the friction headloss is,

$$\Delta h = \frac{V_0^2 n^2 l}{R^{4/3}}$$

The *resistance factor of the tunnel* is,

$$\beta = \frac{l \cdot n^2}{R^{4/3}}$$

In case of *instantaneous opening* of turbine valves, the discharge for the turbine cannot be supplied by the pressure tunnel because of the velocity differences among the pressure tunnel and penstock. This water volume difference will be supplied by the surge tank initially so that the water level in the surge tank will drop. Air entrance to the penstock should not be permitted in order not to cause bursting of the penstock. There should be minimum water height of 1.50 m over the top of penstock in the surge tank for the minimum water level which is the case of instantaneous opening of turbines for full load. In order to be on the safe side, manning roughness coefficient n should be selected high for concrete lining as n = 0.015 to obtain a higher β resistance factor.

Vogt Dimensionless Variables

Tables have been prepared to compute surge amplitudes and periods for the surge tanks using dimensionless variables.

$$\varepsilon = \frac{l}{g} \cdot \frac{f}{F} \cdot \frac{V_0^2}{(\Delta h_0)^2}$$

$$z = \frac{V}{V_0}$$

$$x = \frac{y}{\Delta h_0}$$

Δh_0 = Head loss for the steady flow case (will get negative values since y values are taken positive for upward direction).

a) Instantaneous full closure case

Forchhmeir has given for the first maximum surge height for steady flow of Q_0 discharge with Δh_0 headloss,

$$\left(1 + \frac{2}{\varepsilon} x_{max}\right) - Ln\left(1 + \frac{2}{\varepsilon} x_{max}\right) = 1 + \frac{2}{\varepsilon}$$

For m = Damping factor,

$$m = \frac{2}{\varepsilon \Delta h_0} = \frac{2gF\Delta h_0}{lfV_0^2}$$

The equation takes the form of,

$$(1 + y_{max}) - Ln(1 + y_{max}) = 1 + m\Delta h_0$$

m dimensional variables are always negative since ε dimensionless variables are positive and y direction is taken positive for upward direction with Δh negative values. Forchhmeir equation is solved by using the Table.

$$m=\frac{2gF\beta V_0^2}{\ell F V_0^2}, \quad \varepsilon=\frac{\ell F V_0^2}{gF(\beta V_0^2)^2}, \quad \Delta h_0=\beta V_0^2$$

$m\Delta h_0$	$m\Delta y_{max}$	$m\Delta h_0$	$m\Delta y_{max}$	$m\,h_0$	$m\Delta y_{max}$	$m\,h_0$	$m\Delta y_{max}$
$\frac{2}{\varepsilon}$	$\frac{2}{\varepsilon}x_{max}$	$\frac{2}{\varepsilon}$	$\frac{2}{\varepsilon}x_{max}$	$\frac{2}{\varepsilon}$	$\frac{2}{\varepsilon}x_{max}$	$\frac{2}{\varepsilon}$	$\frac{2}{\varepsilon}x_{max}$
0,00005	−0,0100	0,026	−0,211	0,30	−0,589	0,92	−0,825
0,0001	−0,0145	0,028	−0,218	0,31	−0,596	0,94	−0,830
0,0002	−0,0200	0,030	−0,225	0,32	−0,602	0,96	−0,834
0,0003	−0,0241	0,035	−0,242	0,33	−0,609	0,98	−0,837
0,0004	−0,0280	0,040	−0,257	0,34	−0,615	1,00	−0,841
0,0005	−0,0312	0,045	−0,271	0,35	−0,621	1,05	−0,850
0,0006	−0,0342	0,050	−0,284	0,36	−0,627	1,10	−0,859
0,0007	−0,0370	0,055	−0,296	0,37	−0,633	1,15	−0,867
0,0008	−0,0396	0,060	−0,308	0,38	−0,639	1,20	−0,874
0,0009	−0,0419	0,065	−0,318	0,39	−0,644	1,25	−0,882
0,0010	−0,0439	0,070	−0,329	0,40	−0,650	1,30	−0,888
0,0015	−0,0535	0,075	−0,339	0,42	−0,661	1,35	−0,894
0,0020	−0,0615	0,080	−0,348	0,44	−0,671	1,40	−0,900
0,0025	−0,0686	0,085	−0,358	0,46	−0,680	1,45	−0,905
0,0030	−0,0750	0,090	−0,366	0,48	−0,689	1,50	−0,910
0,0035	−0,0809	0,095	−0,375	0,50	−0,698	1,60	−0,920
0,0040	−0,0864	0,10	−0,383	0,52	−0,707	1,70	−0,928
0,0045	−0,0915	0,11	−0,399	0,54	−0,715	1,80	−0,935
0,0050	−0,0962	0,12	−0,413	0,56	−0,723	1,90	−0,942
0,0060	−0,105	0,13	−0,427	0,58	−0,730	2,00	−0,948
0,0070	−0,113	0,14	−0,440	0,60	−0,737	2,10	−0,953
0,0080	−0,121	0,15	−0,453	0,62	−0,744	2,20	−0,957
0,0090	−0,128	0,16	−0,465	0,64	−0,751	2,30	−0,962
0,010	−0,134	0,17	−0,476	0,66	−0,758	2,40	−0,965
0,011	−0,141	0,18	−0,486	0,68	−0,764	2,50	−0,969
0,012	−0,147	0,19	−0,497	0,70	−0,770	2,60	−0,972
0,013	−0,153	0,20	−0,507	0,72	−0,776	2,70	−0,975
0,014	−0,158	0,21	−0,516	0,74	−0,782	2,80	−0,977
0,015	−0,163	0,22	−0,525	0,76	−0,787	2,90	−0,979
0,016	−0,168	0,23	−0,534	0,78	−0,792	3,00	−0,981
0,017	−0,173	0,24	−0,543	0,80	−0,798	3,50	−0,989
0,018	−0,178	0,25	−0,551	0,82	−0,803	4,00	−0,993
0,019	−0,182	0,26	−0,559	0,84	−0,807	4,50	−0,996
0,020	−0,187	0,27	−0,567	0,86	−0,812	5,00	−0,998
0,022	−0,196	0,28	−0,574	0,88	−0,817		
0,024	−0,204	0,29	−0,582	0,90	−0,821		

Table 17. Values for the decrease of the discharge from Q_0 to nQ (Frank, 1957)

			$1/\sqrt{\varepsilon}$										
		n	0,00	0,05	0,10	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50
1. Max.	x	0,0	z=Z	-19,20	-9,30	-5,99	-4,35	-3,36	-2,70	-2,24	-1,88	-1,61	-1,396
	x	0,5	z=Z	-9,00	-4,20	-2,53	-1,70	-1,216	-0,89	-0,653	-0,481	-0,346	-0,240
	x	1,0	z=0	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	0,25	0,255	0,261	0,267	0,273	0,279	0,286	0,293	0,300	0,307	0,315
	τ	0,5	0,25	0,257	0,264	0,272	0,280	0,289	0,298	0,308	0,318	0,329	0,341
	τ	1,0	0,25	0,258	0,267	0,277	0,288	0,299	0,312	0,326	0,340	0,357	0,374
1. Inflec.	x	0,0	z=0	-0,85	-0,76	-0,68	-0,62	-0,55	-0,49	-0,45	-0,41	-0,37	-0,34
	x	0,5	z=0	+0,004	+0,006	+0,013	+0,021	+0,032	+0,044	+0,060	+0,072	+0,086	+0,10
	x	1,0	z=0	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	0,50	0,501	0,502	0,504	0,507	0,511	0,515	0,520	0,525	0,531	0,537
	τ	0,5	0,50	0,505	0,510	0,516	0,521	0,527	0,534	0,541	0,548	0,556	0,564
	τ	1,0	0,50	0,501	0,502	0,505	0,510	0,515	0,522	0,530	0,540	0,550	0,563
1. Minimum	x	0,0	z=-Z	+17,60	+8,10	+5,02	+3,51	+2,61	+2,02	+1,63	+1,34	+1,11	+0,95
	x	0,5	z=-Z	+8,80	+4,20	+2,665	+1,85	+1,36	+1,055	+0,833	+0,687	+0,582	+0,50
	x	1,0	z=0	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	0,75	0,756	0,761	0,768	0,775	0,782	0,789	0,797	0,805	0,813	0,822
	τ	0,5	0,75	0,757	0,765	0,772	0,782	0,791	0,800	0,812	0,824	0,837	0,855
	τ	1,0	0,75	0,757	0,769	0,783	0,797	0,815	0,834	0,856	0,879	0,907	0,937
2. Inflec.	x	0,0	z=0	+0,71	+0,58	+0,49	+0,42	+0,35	+0,30	+0,256	+0,218	+0,193	+0,172
	x	0,5	z=0	+0,765	+0,740	+0,661	+0,585	+0,519	+0,476	+0,429	+0,394	+0,366	+0,343
	x	1,0	z=0	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	1,0	1,001	1,003	1,007	1,012	1,017	1,022	1,030	1,037	1,045	1,052
	τ	0,5	1,0	1,001	1,003	1,007	1,012	1,017	1,023	1,032	1,041	1,053	1,068
	τ	1,0	1,0	1,001	1,005	1,012	1,020	1,031	1,046	1,062	1,080	1,102	1,126
2. Max.	x	0,0	z=Z	-16,00	-7,00	-4,30	-2,92	-2,13	-1,62	-1,274	-1,025	-0,850	-0,716
	x	0,5	z=Z	-6,80	-2,90	-1,51	-0,825	-0,432	-0,211	-0,057	-0,038	-0,109	-0,148
	x	1,0	z=0	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	1,25	1,255	1,262	1,270	1,277	1,283	1,292	1,300	1,309	1,317	1,325
	τ	0,5	1,25	1,258	1,266	1,274	1,284	1,299	1,309	1,321	1,337	1,352	1,372
	τ	1,0	1,25	1,260	1,271	1,288	1,309	1,330	1,357	1,387	1,420	1,458	1,500

In order to calculate the other extreme surge values after calculation the first y_{max} value, *Braun* equations are used.

$$(1-my_1)-Ln(1-my_1)=(1-my_{max})-Ln(1-my_{max})$$
$$(1+my_2)-Ln(1+my_2)=(1+my_1)-Ln(1+my_1)$$
$$(1-my_3)-Ln(1-my_3)=(1-my_2)-Ln(1-my_2)$$
$$(1+my_4)-Ln(1+my_4)=(1+my_3)-Ln(1+my_3)$$

The following steps are taken for the solution of the aforementioned equations,

1) ε dimensionless variable is calculated,
2) y_{max} value is computed by *Forchhmeir* equation by using the Table for ($m\Delta h_0$) to get (my_{max}) value,
3) After calculation y_{max}, the other y surge values are calculated by using above giving equations and the Table.

b) Partial Closure of the Turbine Valve

Q_0 full load discharge may be reduced to nQ_0 for ($n < 1$). It will be instantaneous full closure if ($n = 0$). *Frank`s* Table can be used to calculate the surge values for partial closure. The values in the Table can be defined as,

$$\varepsilon=\frac{lV_0^2 f}{gF(\Delta h_0)^2}$$

$$x_{max}=\frac{y_{max}}{\Delta h_0}$$

$$\tau=\frac{t}{T}=\frac{t}{2\pi\sqrt{\frac{lF}{gf}}}$$

The Table has been prepared for circular simple surge tanks.

Example: An hydroelectric power plant with a design discharge Q = 30 m^3/sec is fed by a pressure tunnel with a diameter D = 4 m, length l = 5000 m, and Manning coefficient n = 0.014. Compute the extreme surge heights for instantaneous full turbine closure in the surge tank with cross-sectional area F = 150 m^2,

a) By using *Forchhmeir* method,
b) By the help of *Frank*`s table.

Solution:

a) Physical characteristics of the plant are,

$$f=\frac{\pi D^2}{4}=\frac{\pi\times 4^2}{4}=12.57m^2$$

$$R = \frac{D}{4} = 1m$$

$$V_0 = \frac{Q}{f} = \frac{30.0}{12.57} = 2.39\, m/\sec$$

$$\Delta h_0 = \frac{V_0^2 n^2 l}{R^{4/3}} = \frac{2.39^2 \times 12.57 \times 5000}{1^{4/3}} = 5.60m$$

$$\varepsilon = \frac{lfV_0^2}{gF(\Delta h_0)^2} = \frac{5000 \times 12.57 \times 2.39^2}{9.81 \times 150 \times 5.60^2} = 7.78$$

$$m = \frac{2}{\varepsilon \Delta h_0} = \frac{2}{7.78 \times (-5.60)} = -0.046$$

$$m\Delta h_0 = (-0.046) \times (-5.60) = 0.257$$

From the *Forchhmeir Table*,

$$m\Delta h_0 = 0.25 \quad \rightarrow \quad my_{max} = -0.551$$

$$m\Delta h_0 = 0.26 \quad \rightarrow \quad my_{max} = -0.559$$

$$m\Delta h_0 = 0.257 \quad \rightarrow \quad my_{max} = -0.557$$

$$y_{max} = \frac{-0.557}{-0.046} = 12.11m$$

The first minimum level,

$$(1 - my_1) - Ln(1 - my_1) = (1 - my_{max}) - Ln(1 - my_{max})$$

$$(1 - my_1) - Ln(1 - my_1) = (1 + 0.557) - Ln(1 + 0.557)$$

$$(1 - my_1) - Ln(1 - my_1) = 1.557 - 0.443 = 1.114$$

$$1 + m\Delta h_0 = 1.114$$

$$m\Delta h_0 = 0.114$$

From the *Frank`s Table*,

$$m\Delta h_0 = 0.11 \quad \rightarrow \quad my_{max} = -0.399$$

$$m\Delta h_0 = 0.12 \quad \rightarrow \quad my_{max} = -0.413$$

$$m\Delta h_0 = 0.114 \quad \rightarrow \quad my_{max} = -0.405$$

$$y_1 = \frac{-0.405}{0.046} = -8.80m$$

Second maximum level,

$$\left(1+my_2\right)-Ln\left(1+my_2\right)=\left(1+my_1\right)-Ln\left(1+my_1\right)$$
$$1+m\Delta h_0=\left(1-0.405\right)-Ln\left(1-0.405\right)$$
$$1+m\Delta h_0=0.595+0.519=1.114$$
$$m\Delta h_0=0.114$$

The same $m\Delta h_0$ value has been obtained coincidentally.

$$m\Delta h_0 = 0.114 \quad \rightarrow \quad my_2 = -0.405$$

$$y_2=\frac{0.405}{0.046}=8.80m$$

Second minimum level,

$$\left(1-my_3\right)-Ln\left(1-my_3\right)=\left(1-my_2\right)-Ln\left(1-my_2\right)$$
$$1+m\Delta h_0=\left(1+0.405\right)-Ln\left(1+0.405\right)$$
$$1+m\Delta h_0=1.065$$

$$m\Delta h_0 = 0.065 \quad \rightarrow \quad my_3 = -0.318$$
$$y_3=\frac{-0.318}{0.046}=-6.91m$$

Third maximum level,

$$\left(1+my_4\right)-Ln\left(1+my_4\right)=\left(1-0.318\right)-Ln\left(1-0.318\right)$$
$$1+m\Delta h_0=0.682+0.383=1.065$$

$$m\Delta h_0 = 0.065 \quad \rightarrow \quad my_4 = -0.318$$

$$y_4=\frac{0.318}{0.046}=6.91m$$

b) *Franks Table* will be used for surge calculations for instantaneous closure, n = 0.

$$\varepsilon=7.78\rightarrow\frac{1}{\sqrt{\varepsilon}}=\frac{1}{\sqrt{7.78}}=0.36$$

The first maximum level for n = 0,

$$\frac{1}{\sqrt{\varepsilon}}=0.35 \quad \rightarrow \quad x = -2.24 \quad , \quad \tau = 0.293$$
$$\frac{1}{\sqrt{\varepsilon}}=0.40 \quad \rightarrow \quad x = 1.88 \quad , \quad \tau = 0.300$$
$$\frac{1}{\sqrt{\varepsilon}}=0.36 \quad \rightarrow \quad x = -2.17 \quad , \quad \tau = 0.294$$

Oscillation period,

$$T = 2\pi\sqrt{\frac{lF}{gf}} = 2\pi\sqrt{\frac{5000\times150}{9.81\times12.57}} = 490\sec$$

$$\Delta h_0 = -5.60\text{m}$$

$$y_{max} = x_{max}\times\Delta h_0 = (-2.17)\times(-5.60) = 12.15m$$
$$t_{max} = \tau\times T = 0.294\times490 = 144\sec$$

First inflection point,

$$\frac{1}{\sqrt{\varepsilon}} = 0.35 \quad\rightarrow\quad x = -0.45 \quad,\quad \tau = 0.520$$
$$\frac{1}{\sqrt{\varepsilon}} = 0.40 \quad\rightarrow\quad x = -0.41 \quad,\quad \tau = 0.525$$
$$\frac{1}{\sqrt{\varepsilon}} = 0.36 \quad\rightarrow\quad x = 0.44 \quad,\quad \tau = 0.521$$

$$y_{\inf_1} = (-0.44)\times(-5.60) = 2.46m$$
$$t_{\inf_1} = 490\times0.521 = 255\sec$$

First minimum level,

$$\frac{1}{\sqrt{\varepsilon}} = 0.35 \quad\rightarrow\quad x = +1.63 \quad,\quad \tau = 0.797$$
$$\frac{1}{\sqrt{\varepsilon}} = 0.40 \quad\rightarrow\quad x = +1.34 \quad,\quad \tau = 0.805$$
$$\frac{1}{\sqrt{\varepsilon}} = 0.36 \quad\rightarrow\quad x = +1.57 \quad,\quad \tau = 0.799$$

$$y_{\min_1} = 1.57\times(-5.60) = -8.79m$$
$$t_{\min_1} = 490\times0.799 = 392\sec$$

Second inflection point,

$$\frac{1}{\sqrt{\varepsilon}} = 0.35 \quad\rightarrow\quad x = 0.256 \quad,\quad \tau = 1.030$$
$$\frac{1}{\sqrt{\varepsilon}} = 0.40 \quad\rightarrow\quad x = 0.218 \quad,\quad \tau = 1.037$$
$$\frac{1}{\sqrt{\varepsilon}} = 0.36 \quad\rightarrow\quad x = 0.248 \quad,\quad \tau = 1.031$$

$$y_{\inf_2} = 0.256 \times (-5.60) = 1.43m$$
$$t_{\inf_2} = 490 \times 1.031 = 505\,\text{sec}$$

Second maximum level,

$$\frac{1}{\sqrt{\varepsilon}} = 0.35 \quad \rightarrow \quad x = -1.274 \quad , \quad \tau = 1.300$$
$$\frac{1}{\sqrt{\varepsilon}} = 0.40 \quad \rightarrow \quad x = -1.025 \quad , \quad \tau = 1.309$$
$$\frac{1}{\sqrt{\varepsilon}} = 0.36 \quad \rightarrow \quad x = -1.224 \quad , \quad \tau = 1.302$$

$$y_{\max_2} = (-1.274) \times (-5.60) = 7.13m$$
$$t_{\max_2} = 1.302 \times 490 = 638\,\text{sec}$$

Placing the surge values to the table,

y	**Forchheimer**	**Frank**
y_{max}	12.11	12.15
y_1	-8.80	-8.79

The values are close for the both methods.

Table 18. Discharge increase from nQ_0 to Q_0

			$1/\sqrt{\varepsilon}$										
		n	0,00	0,05	0,10	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50
1. Minimum	x	0,0	z=-Z	+20,00	+10,10	+6,75	+5,10	+4,11	+3,44	+2,96	+2,61	+2,34	+2,11
		0,5	z=-Z	+10,60	+5,40	+3,75	+2,90	+2,41	+2,06	+1,83	+1,66	+1,52	+1,41
		1,0	z=+1	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	0,25	0,252	0,255	0,258	0,262	0,265	0,268	0,272	0,276	0,280	0,284
		0,5	0,25	0,255	0,261	0,267	0,274	0,281	0,288	0,297	0,306	0,314	0,325
		1,0	0,25	0,258	0,267	0,277	0,288	0,299	0,312	0,326	0,340	0,357	0,374
1. Inflect.	x	0,0	z=0	+3,46	+3,06	+2,75	+2,50	+2,30	+2,14	+2,00	+1,864	+1,755	+1,66
		0,5	z=0	+2,02	+1,88	+1,76	+1,64	+1,54	+1,46	+1,39	+1,33	+1,28	+1,23
		1,0	z=0	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	0,50	0,490	0,484	0,479	0,475	0,472	0,469	0,466	0,465	0,463	0,462
		0,5	0,50	0,495	0,493	0,492	0,491	0,493	0,495	0,498	0,501	0,504	0,510
		1,0	0,50	0,501	0,502	0,505	0,510	0,515	0,522	0,530	0,540	0,550	0,563
1. Maximum	x	0,0	z=Z	-14,40	-5,00	-2,13	-0,83	-0,14	+0,27	+0,52	+0,68	+0,80	+0,87
		0,5	z=Z	-6,40	-2,00	-0,58	+0,08	+0,43	+0,64	+0,78	+0,85	+0,90	+0,94
		1,0	z=0	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	0,75	0,753	0,760	0,768	0,779	0,791	0,804	0,817	0,833	0,850	0,870
		0,5	0,75	0,756	0,765	0,775	0,787	0,801	0,815	0,834	0,853	0,873	0,895
		1,0	0,75	0,757	0,769	0,783	0,797	0,815	0,834	0,856	0,879	0,907	0,937
2. Inflec.	x	0,0	z=0	+0,07	+0,21	+0,37	+0,51	+0,63	+0,72	+0,80	+0,85	+0,90	+0,93
		0,5	z=0	+0,41	+0,55	+0,64	+0,73	+0,80	+0,86	+0,90	+0,93	+0,95	+0,97
		1,0	z=0	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	1,00	1,000	1,001	1,002	1,006	1,012	1,018	1,026	1,035	1,047	1,060
		0,5	1,00	1,000	1,002	1,007	1,012	1,020	1,030	1,040	1,053	1,068	1,086
		1,0	1,00	1,001	1,005	1,012	1,020	1,031	1,046	1,062	1,080	1,102	1,126
2. Mini.	x	0,0	z=-Z	+15,20	+5,80	+3,15	+2,05	+1,54	+1,29	+1,16	+1,09	+1,05	+1,02
		0,5	z=-Z	+8,00	+3,30	+2,09	+1,52	+1,28	+1,14	+1,08	+1,04	+1,02	+1,01
		1,0	z=0	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00	+1,00
	τ	0,0	1,25	1,253	1,261	1,272	1,286	1,302	1,322	1,345	1,370	1,399	1,430
		0,5	1,25	1,256	1,267	1,280	1,296	1,316	1,339	1,364	1,393	1,422	1,458
		1,0	1,25	1,260	1,271	1,288	1,309	1,330	1,357	1,387	1,420	1,458	1,500

c) Instantaneous Opening of the Turbines

The discharge increase to the turbines by instantaneous opening is done from nQ_0 to Q_0 ($n < 1$). If the turbines are not running, there will be no discharge feeding the penstock which is ($n = 0$) case. Instantaneous partial opening case may be computed to find out the surge heights by using *Frank`s* Table 18.

The column with $\frac{1}{\sqrt{\varepsilon}} = 0$ value corresponds to $\varepsilon \rightarrow \infty$. Since l, f, V_0^2, F are physical magnitudes, this $\varepsilon \rightarrow \infty$ corresponds to $\Delta h_0 = S_0 l \rightarrow 0$ which is the ideal fluid case. The x and τ values of this column can only be used for ideal fluids which is no friction losses would occur in the plant.

Example: An hydroelectric power plant with a pressure tunnel of the length l = 5000 m, diameter D = 4 m, and Manning coefficient n = 0.014 is feeding the turbines. The cross-sectional area of the cylindrical surge tank is F = 150 m^2. Calculate the extreme surge levels by using *Frank* tables for,

a) Instantaneous discharge increase from 0 m^3/sec to 10 m^3/sec,
b) Instantaneous discharge increase from 10 m^3/sec to 30 m^3/sec.

Solution:

a) $Q_0 = 10$ m^3/sec,

$$f = \frac{\pi D^2}{4} = \frac{\pi \times 4^2}{4} = 12.57 m^2$$

$$R = \frac{D}{4} = \frac{4}{4} = 1m$$

$$V_0 = \frac{Q}{f} = \frac{10}{12.57} = 0.80\, m/\sec$$

$$\Delta h_0 = \frac{V_0^2 n^2 l}{R^{4/3}} = \frac{0.80^2 \times 0.014^2 \times 5000}{1^{4/3}} = 0.63m$$

$$\varepsilon = \frac{l}{g} \cdot \frac{f}{F} \cdot \frac{V_0^2}{(\Delta h_0)^2} = \frac{5000 \times 12.57 \times 0.80^2}{9.81 \times 150 \times 0.63^2} = 68.87$$

$$\frac{1}{\sqrt{\varepsilon}} = \frac{1}{\sqrt{68.87}} = 0.12$$

$$T = 2\pi \sqrt{\frac{lF}{gf}} = 2\pi \sqrt{\frac{5000 \times 150}{9.81 \times 12.57}} = 490 \sec$$

First minimum surge tank level for n = 0 by using Table 18,

$$\frac{1}{\sqrt{\varepsilon}} = 0.10 \quad \rightarrow \quad x = 10.10 \quad , \quad \tau = 0.255$$

$$\frac{1}{\sqrt{\varepsilon}} = 0.15 \quad \rightarrow \quad x = 6.75 \quad , \quad \tau = 0.258$$

$$\frac{1}{\sqrt{\varepsilon}} = 0.12 \quad \rightarrow \quad x = 8.76 \quad , \quad \tau = 0.256$$

$$y_{\min_1} = 8.76 \times (-0.63) = -5.52m$$

$$\tau = 490 \times 0.256 = 125 \sec$$

First maximum level,

$$\frac{1}{\sqrt{\varepsilon}} = 0.10 \quad \rightarrow \quad x = -5.00 \quad , \quad \tau = 0.760$$

$$\frac{1}{\sqrt{\varepsilon}} = 0.15 \quad \rightarrow \quad x = -2.13 \quad , \quad \tau = 0.768$$

$$\frac{1}{\sqrt{\varepsilon}} = 0.12 \quad \rightarrow \quad x = -3.85 \quad , \quad \tau = 0.763$$

$$y_{\max_1} = (-3.85) \times (-0.63) = 2.43m$$

$$\tau = 490 \times 0.763 = 374 \sec$$

Second minimum level,

$$\frac{1}{\sqrt{\varepsilon}} = 0.10 \quad \rightarrow \quad x = 5.80 \quad , \quad \tau = 1.261$$

$$\frac{1}{\sqrt{\varepsilon}} = 0.15 \quad \rightarrow \quad x = 3.15 \quad , \quad \tau = 1.272$$

$$\frac{1}{\sqrt{\varepsilon}} = 0.12 \quad \rightarrow \quad x = 4.74 \quad , \quad \tau = 1.265$$

$$y_{\min_2} = 4.74 \times (-0.63) \cong -3.00m$$

$$\tau = 490 \times 1.265 = 620 \sec$$

b) Instantaneous discharge increase from Q = 10 m^3/sec to Q_0 = 30 m^3/sec.

Q_0 = 30 m^3/sec , f = 12.57 m^2 , R = 1 m.

$$V_0 = \frac{30}{12.57} = 2.39\, m/\sec$$

$$\Delta h_0 = \frac{V_0^2 n^2 l}{R^{4/3}} = \frac{2.39^2 \times 0.014^2 \times 5000}{1^{4/3}} = 5.60m$$

$$\varepsilon = \frac{l}{g} \cdot \frac{f}{F} \cdot \frac{V_0^2}{(\Delta h_0)^2} = \frac{5000 \times 12.57 \times 2.39^2}{9.81 \times 150 \times 5.60^2} = 7.78m$$

$$\frac{1}{\sqrt{\varepsilon}} = \frac{1}{\sqrt{7.78}} = 0.36 \quad , \quad n = \frac{10}{30} = 0.333$$

Interpolation will be done for the required $\frac{1}{\sqrt{\varepsilon}}$, and n values using Table 18.

Minimum surge tank level for n = 0.333,

$$n = 0 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.35 \rightarrow x = 2.96 \quad , \quad \tau = 0.272$$

$$n = 0 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.40 \rightarrow x = 2.61 \quad , \rightarrow \tau = 0.276$$

$$n = 0 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.36 \rightarrow x = 2.89 \quad , \quad \tau = 0.273$$

$$n = 0.5 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.35 \rightarrow x = 1.83 \quad , \quad \tau = 0.297$$

$$n = 0.5 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.40 \rightarrow x = 1.06 \quad , \quad \tau = 0.306$$

$$n = 0.5 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.36 \rightarrow x = 1.80 \quad , \quad \tau = 0.299$$

$$n = 0.333 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.36 \rightarrow x = 2.17 \quad , \quad \tau = 0.290$$

$$y_{\min_1} = 2.17 \times (-5.60) = -12.15m$$
$$t_{\min_1} = 0.29 \times 490 = 142 \sec$$

First maximum surge tank level,

$$n = 0 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.35 \rightarrow x = 0.52 \quad , \quad \tau = 0.817$$

$$n = 0 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.40 \rightarrow x = 0.68 \quad , \quad \tau = 0.833$$

$$n = 0 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.36 \rightarrow x = 0.55 \quad , \quad \tau = 0.820$$

$$n = 0.5 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.35 \rightarrow x = 0.78 \quad , \quad \tau = 0.834$$

$$n = 0.5 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.40 \rightarrow x = 0.85 \quad , \quad \tau = 0.853$$

$$n = 0.5 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.36 \rightarrow x = 0.79 \quad , \quad \tau = 0.838$$

$$n = 0.333 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.36 \rightarrow x = 0.71 \quad , \quad \tau = 0.832$$

$$y_{max_1} = 0.71 \times (-5.60) = -3.98m$$
$$t = 490 \times 0.832 = 408 \sec$$

Second minimum surge tank level,

$$n = 0 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.35 \rightarrow x = 1.16 \quad , \quad \tau = 1.345$$
$$n = 0 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.40 \rightarrow x = 1.09 \quad , \quad \tau = 1.370$$
$$n = 0 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.36 \rightarrow x = 1.15 \quad , \quad \tau = 1.35$$
$$n = 0.5 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.35 \rightarrow x = 1.08, \quad \tau = 1.364$$
$$n = 0.5 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.40 \rightarrow x = 1.04 \quad , \quad \tau = 1.393$$
$$n = 0.333 \rightarrow \frac{1}{\sqrt{\varepsilon}} = 0.36 \rightarrow x = 1.10 \quad , \quad \tau = 1.36$$
$$y_{min_2} = 1.10 \times (-5.60) = -6.16m$$
$$t = 490 \times 1.36 = 666 \sec$$

Stability Conditions of the Surge Tanks

Stability conditions of the surge tanks were first established by *D. Thoma* and *F. Vogt*. They stated that in order to prevent the development of unstable oscillations the cross-section of the surge tank should exceed a critical value.

According to the Thoma equation suggested in *small oscillations*, the limit cross-sectional area of the surge tank is,

$$F > F_{thm} = k \frac{lf}{2g\beta H_0} \quad (m^2)$$

k = Factor of safety,
V_0 = Pressure tunnel velocity for the new dynamic equilibrium level, i.e. to the power output to be succeeded after opening (m/sec),
β = Resistance factor of the pressure tunnel (sec^2/m),
l = Length of the tunnel (m),
$H_0 = H - \beta V_0^2 = H - \Delta h_0$ = net head (by neglecting the headloss in the penstock) (m).

Substituting the damping factor m,

$$m = \frac{2gF\beta}{lf}$$

The minimum value of head succeeding surge stability in case of a given cross-sectional area F of the surge tank is,

$$F = \frac{klf}{2g\beta H_0} = \frac{mlf}{2g\beta}$$

$$H_0 = \frac{k}{m}$$

Assuming local headlosses can be neglected with respect to friction losses, and with the substitution,

$$\beta = \frac{n^2 l}{R^{4/3}}$$

$$F_{thm} = k\frac{lfR^{4/3}}{2gH_0 n^2 l} = k\frac{R^{4/3} f}{2gH_0 n^2}$$

Is obtained, which can be simplified in case of a *circular pressure tunnel cross-section*, R = D/4 as hydraulic radius, f = $\pi D^2/4$ cross sectional are, to the form of,

$$F = k\frac{D^{4/3}\pi D^2}{4^{4/3}\times 4\times 19.62\times H_0 n^2}$$

$$F = k\frac{D^{10/3}}{160 H_0 n^2}$$

A *safety factor k* of 1.5 to 1.8 may be adopted.

As can be seen from the equation, the lower the friction factor β, the larger the cross-sectional area of the surge tank. Limit values of F are thus obtained by simultaneous assumption of the highest safety factor k and lowest Manning coefficient n. Substituting the pairs of values k = 1.5, n = 0.014 as well as k = 1.8, n = 0.0106, we obtain,

$$F_1 = \frac{1.5}{160\times 0.014^2}\times\frac{D^{10/3}}{H_0} \cong 50\frac{D^{10/3}}{H_0}$$

$$F_2 = \frac{1.8}{160\times 0.0106^2}\times\frac{D^{10/3}}{H_0} \cong 100\frac{D^{10/3}}{H_0}$$

$$\frac{F_2}{F_1} = 2$$

In case of a concrete lined pressure tunnel, the deviation depending on the choice of the friction coefficient n, as well as on the safety factor k, is considerable between extreme $F_2/F_1 = 2$. For a lining carried out with steel, the mean value n = 0.0143 – 0.0133 may be applied.

For *great amplitudes the Thoma equation* was modified by *Ch. Jaeger*, demonstrating that the safety factor can n0 longer be considered constant. According to *Jaeger*, the cross-sectional area necessary for stability should not be less than,

$$F = k^* \frac{lf}{2g\beta H_0} = k^* \frac{R^{4/3} f}{2gn^2 H_0}$$

For a circular cross-section,

$$F = k^* \frac{D^{10/3}}{160n^2 H_0}$$

The safety factor is,

$$k^* = 1 + 0.482 \frac{y_{max}}{H_0}$$

y_{max} is the amplitude of the undamped (frictionless) surge.