# Chapter 6

# Gradually-Varied Flow in Open Channels

## 6.1. Introduction

A steady non-uniform flow in a prismatic channel with gradual changes in its water-surface elevation is named as *gradually-varied flow (GVF)*. The backwater produced by a dam or weir across a river and drawdown produced at a sudden drop in a channel are few typical examples of GVF. In a GVF, the velocity varies along the channel and consequently the bed slope, water surface slope, and energy line slope will all differ from each other. Regions of high curvature are excluded in the analysis of this flow.

The two basic assumptions involved in the analysis of GVF are:

1. The pressure distribution at any section is assumed to be hydrostatic. This follows from the definition of the flow to have a gradually varied water surface. As gradual changes in the surface curvature give rise to negligible normal accelerations, the departure from the hydrostatic pressure distribution is negligible.
2. The resistance to flow at any depth is assumed to be given by the corresponding uniform flow equation, such as the Manning equation, with the condition that the slope term to be used in the equation is the energy line slope, $S_e$ and not the bed slope, $S_0$. Thus, if in a GVF the depth of flow at any section is y, the energy line slope $S_e$ is given by,

$$S_e = \frac{n^2 V^2}{R^{4/3}} \qquad (6.1)$$

where R = hydraulic radius of the section at depth y.

## 6.2. Basic Differential Equation for the Gradually-Varied Flow Water Surface Profile

Since,

$S_0$ = Channel bed slope for uniform flow depth $y_0$,

dy = Water depth variation for the dx canal reach,

d $(V^2/2g)$ = Velocity head variation for the dx reach.

Writing the energy equation between the cross-sections 1 and 2 (Fig. 6.1.),

$$S_0 dx + y + \frac{V^2}{2g} = (y + dy) + \frac{V^2}{2g} + d\left(\frac{V^2}{2g}\right) + S_e dx$$

$$S_0 dx - S_e dx = dy + d\left(\frac{V^2}{2g}\right)$$

$$S_0 - S_e = \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$

$$S_0 - S_e = \frac{dy}{dx}\left[1 + \frac{d}{dy}\left(\frac{V^2}{2g}\right)\right]$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 + \frac{d}{dy}\left(\frac{V^2}{2g}\right)} \qquad (6.2)$$

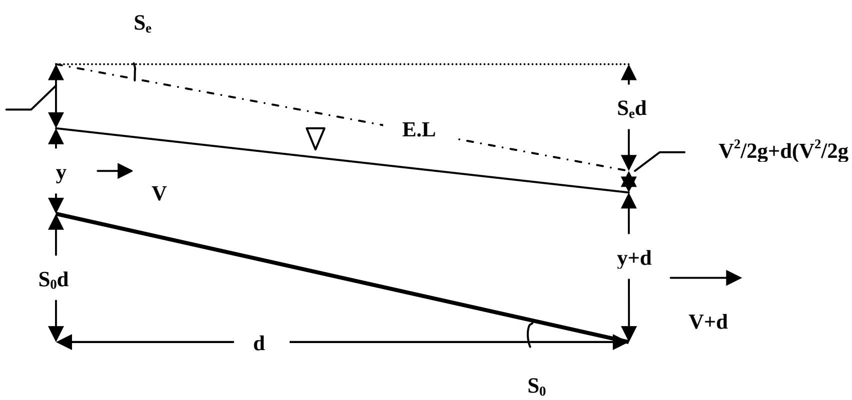



**Figure 6. 1.**

$$\frac{V^2}{2g} = \frac{Q^2}{2gA(y)^2} \rightarrow dA = T(y)dy$$

$$\frac{d}{dy}\left[\frac{Q^2}{2gA(y)^2}\right] = -\frac{Q^2 2A\frac{dA}{dy}}{2gA(y)^4} = -\frac{Q^2 T(y)}{gA(y)^3}$$

$$= -\frac{V^2}{g\frac{A(y)}{T(y)}} = -F_r^2$$

Substituting this to Equ. (6.2),

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} \qquad (6.3)$$

Equ. (6.3) is the *general differential equation of the water surface profile for the gradually varied flows*. dy/dx gives the variation of water depth along the channel in the flow direction.

## 6.3. Classification of Flow Surface Profiles

For a given channel with a known Q = Discharge, n = Manning coefficient, and $S_0$ = Channel bed slope, $y_c$ = critical water depth and $y_0$ = Uniform flow depth can be computed. There are three possible relations between $y_0$ and $y_c$ as 1) $y_0 > y_c$ , 2) $y_0 < y_c$ , 3) $y_0 = y_c$ .

For horizontal ($S_0 = 0$), and adverse slope ( $S_0 < 0$) channels,

$$Q = A\frac{1}{n}R^{2/3}S_0^{1/2}$$

Horizontal channel, $S_0 = 0 \rightarrow Q = 0$,

Adverse channel , $S_0 < 0$ , Q cannot be computed,

*For horizontal and adverse slope channels, uniform flow depth $y_0$ does not exist.*

Based on the information given above, the channels are classified into five categories as indicated in Table (6.1).

**Table 6.1.** Classification of channels

| Number | Channel category | Symbol | Characteristic condition | Remark |
|---|---|---|---|---|
| 1 | Mild slope | M | $y_0 > y_c$ | Subcritical flow at normal depth |
| 2 | Steep slope | S | $y_c > y_0$ | Supercritical flow at normal depth |
| 3 | Critical slope | C | $y_c = y_0$ | Critical flow at normal depth |
| 4 | Horizontal bed | H | $S_0 = 0$ | Cannot sustain uniform flow |
| 5 | Adverse slope | A | $S_0 < 0$ | Cannot sustain uniform flow |

For each of the five categories of channels, lines representing the critical depth ($y_c$ ) and normal depth ($y_0$ ) (if it exists) can be drawn in the longitudinal section. These would divide the whole flow space into three regions as:

Region 1: Space above the topmost line,
Region 2: Space between top line and the next lower line,
Region 3: Space between the second line and the bed.

Figure (6.2) shows these regions in the various categories of channels.

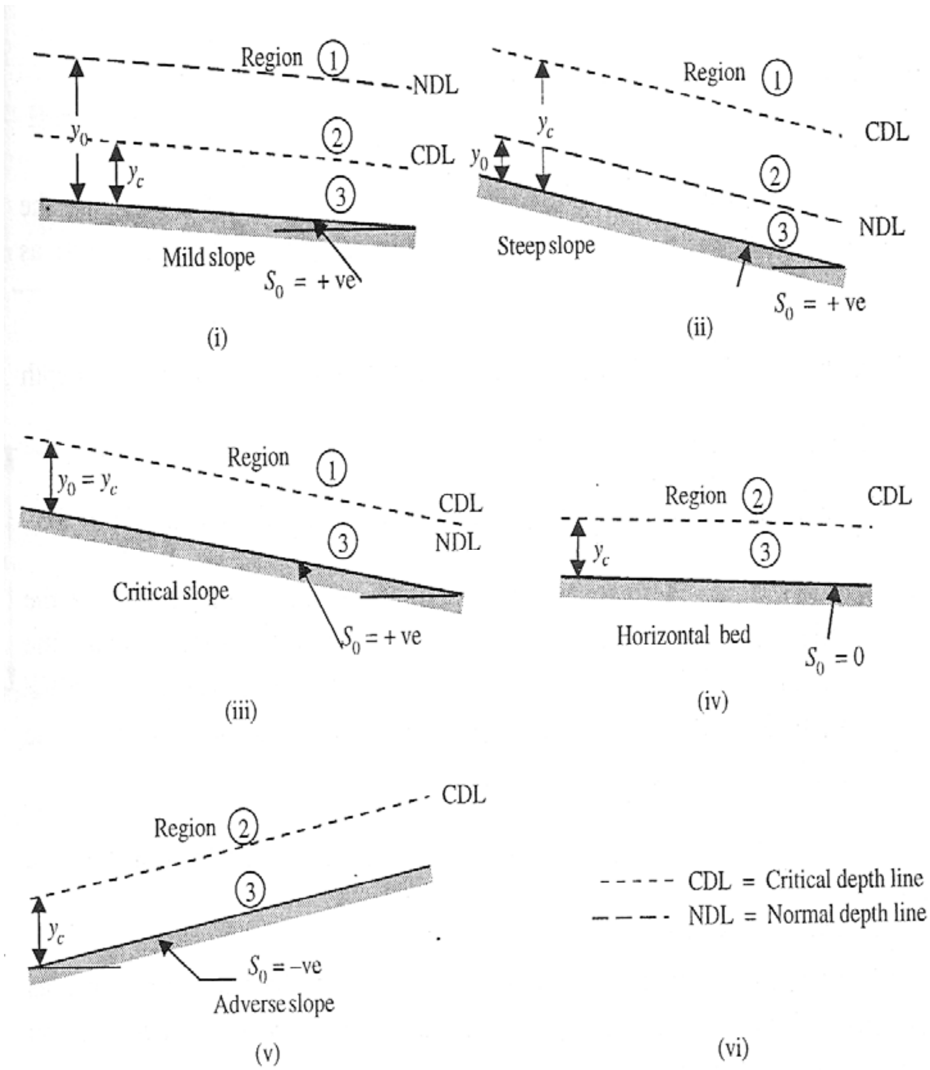



**Figure 6.2.** Regions of flow profiles

Depending upon the channel category and region of flow, the water surface profiles will have characteristics shapes. Whether a given GVF profile will have an increasing or decreasing water depth in the direction of flow will depend upon the term dy/dx in Equ. (6.3) being positive or negative.

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} \qquad (6.3)$$

For a given Q, n, and $S_0$ at a channel,

$y_0$ = Uniform flow depth,
$y_c$ = Critical flow depth,
y = Non-uniform flow depth.
The depth y is measured vertically from the channel bottom, the slope of the water surface dy/dx is relative to this channel bottom. Fig. (6.3) is basic to the prediction of surface profiles from analysis of Equ. (6.3).

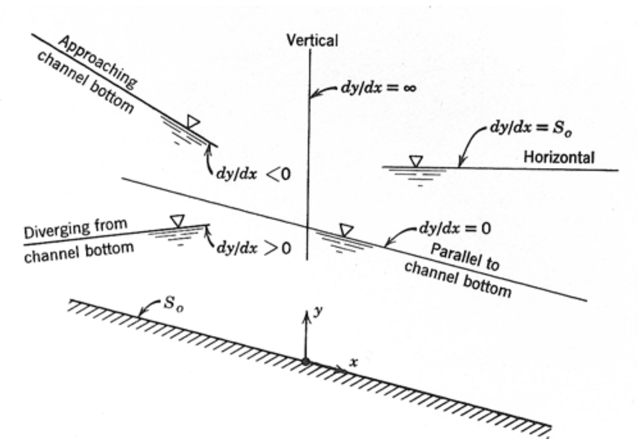



**Figure 6.3**

To assist in the determination of flow profiles in various regions, the behavior of dy/dx at certain key depths is noted by studying Equ. (6.3) as follows:

$$y > y_c \rightarrow F_r < 1$$
$$y = y_c \rightarrow F_r = 1$$
$$y < y_c \rightarrow F_r > 1$$

And also,

$$y > y_0 \rightarrow S_e < S_0$$
$$y = y_0 \rightarrow S_e = S_0$$
$$y < y_0 \rightarrow S_e > S_0$$

1. As $y \rightarrow y_0$ , $V \rightarrow V_0$ , $S_e = S_0$

$$\lim_{y \to y_0} \frac{dy}{dx} = \frac{S_0 - S_0}{1 - F_r^2} = \frac{0}{cons} = 0$$

The water surface approaches the normal depth asymptotically.

2. As $y \to y_c$, $F_r^2 = 1$ , $1 - F_r^2 = 0$ ,

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{S_0 - S_e}{0} = \infty$$

The water surface meets the critical depth line vertically.

3. As $y \to \infty$, $V = 0 \to F_r = 0 \to S_e \to 0$

$$\lim_{y \to \infty} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{S_0}{1} = S_0$$

The water surface meets a very large depth as a horizontal asymptote.

Based on this information, the various possible gradually varied flow profiles are grouped into twelve types (Table 6.2).

**Table 6.2.** Gradually Varied Flow profiles

| **Channel** | **Region** | **Condition** | **Type** |
|---|---|---|---|
| Mild slope | 1<br>2<br>3 | $y > y_0 > y_c$<br>$y_0 > y > y_c$<br>$y_0 > y_c > y$ | $M_1$<br>$M_2$<br>$M_3$ |
| Steep slope | 1<br>2<br>3 | $y > y_c > y_0$<br>$y_c > y > y_0$<br>$y_c > y_0 > y$ | $S_1$<br>$S_2$<br>$S_3$ |
| Critical slope | 1<br><br>3 | $y > y_0 = y_2$<br><br>$y < y_0 = y_c$ | $C_1$<br><br>$C_3$ |
| Horizontal bed | 2<br>3 | $y > y_c$<br>$y < y_c$ | $H_2$<br>$H_3$ |
| Adverse slope | 2<br>3 | $y > y_c$<br>$y < y_c$ | $A_2$<br>$A_3$ |

### 6.4. Water Surface Profiles

### 6.4.1. M – Curves

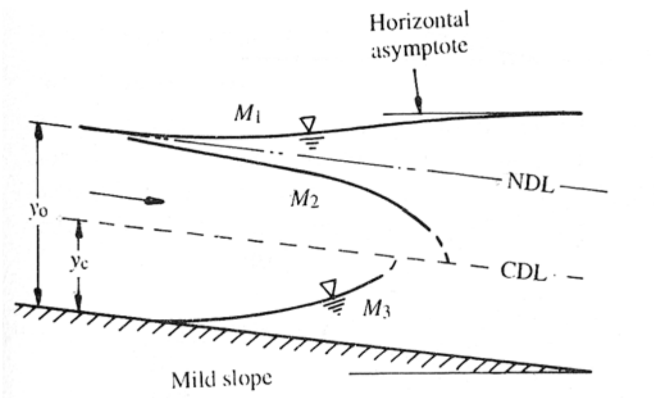



**Figure 6.4**

General shapes of M curves are given in Fig. (6.4). Asymptotic behaviors of each curve will be examined mathematically.

**a) $M_1$ – Curve**

Water surface will be in Region 1 for a mild slope channel and the flow is obviously subcritical.

$$S_e < S_0 \quad \rightarrow \text{Mild slope channel}$$

$$y_0 > y_c \quad \rightarrow \text{Subcritical flow}$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2}$$

$$F_r < 1 \ \rightarrow \ \text{Subcritical flow} \rightarrow (1 - F_r^2) > 0$$

$$y > y_0 \ \rightarrow \ S_e < S_0$$

$$\frac{dy}{dx} = \frac{+}{+} > 0 \quad \text{(Water depth will increase in the flow direction)}$$

Asymptotic behavior of the water surface is;

Water depth can be between ($\infty > y > y_0$) for Region 1. The asymptotic behaviors of the water surface for the limit values ($\infty$ , $y_0$) are;

a) $y \rightarrow \infty$ , $V \rightarrow 0$, $F_r \rightarrow 0$, $(1 - F_r^2) = 1$

$y \rightarrow \infty$ , $V \rightarrow 0$, $S_e \rightarrow 0$

$$\lim_{y \to \infty} \frac{dy}{dx} = \frac{S_0 - 0}{1} = S_0$$

The water surface meets a very large depth as a horizontal asymptote.

b) $y \rightarrow y_0$ , $V \rightarrow V_0$ , $S_e \rightarrow S_0$

$$\lim_{y \to y_0} \frac{dy}{dx} = \frac{S_0 - S_0}{1 - F_r^2} = 0$$

The water surface approaches the normal depth asymptotically.

The most common of all GVF profiles is the $M_1$ type, which is a subcritical flow condition. Obstructions to flow, such as weirs, dams, control structures and natural features, such as bends, produce $M_1$ backwater curves (Fig. 6.5). These extend to several kilometers upstream before merging with the normal depth.

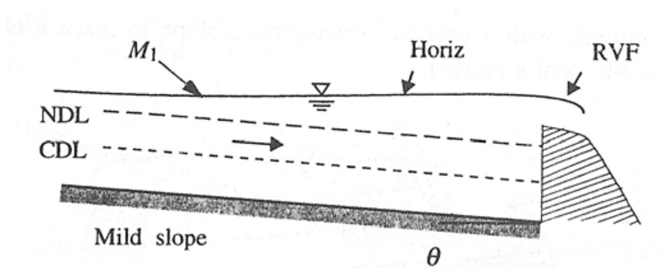



**Figure 6.5.** $M_1$ Profile

**b) $M_2$ – Curve**

Water surface will be in Region 2 for a mild slope channel and the flow is obviously subcritical. (Fig. 6.4).

$$y_0 > y > y_c$$

$$y < y_0 \;\rightarrow\; V > V_0 \;\rightarrow\; S_e > S_0 \;\rightarrow\; (S_0 - S_e) < 0$$

$$y > y_c \;\rightarrow\; F_r < 1 \;\rightarrow\; (1 - F_r^2) > 0$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{+}{-} = -$$ (Water depth decrease in the flow direction)

Asymptotic behavior of the water surface is;

$$y_0 > y > y_c$$

$$y \rightarrow y_0, \quad S_e \rightarrow S_0\ , \ (S_0 - S_e) = 0$$

$$\lim_{y \to y_0} \frac{dy}{dx} = \frac{S_0 - S_0}{1 - F_r^2} = 0$$

The water surface approaches the normal depth asymptotically.

$$y \rightarrow y_c\ , \quad F_r = 1\ , \ (1 - F_r^2) = 0$$

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{0} = \infty$$

The water surface meets the critical depth line vertically.

The $M_2$ profiles occur at a sudden drop of the channel, at constriction type of transitions and at the canal outlet into pools (Fig. 6.6).

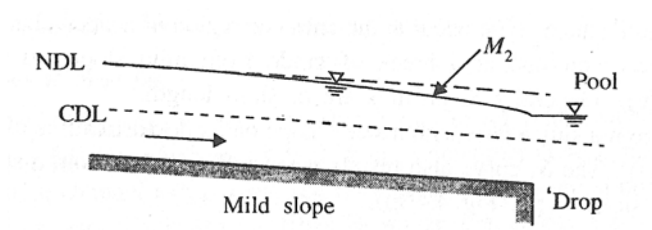



**Figure 6.6**

### c) $M_3$ – Curve

Water surface will be in Region 3 for a mild slope channel and the flow is obviously subcritical. (Fig. 6.4).

$$y_0 > y_c > 0$$

$$y < y_0 \ \rightarrow \ V > V_0 \ \rightarrow \ S_e > S_0 \rightarrow (S_0 - S_e) < 0$$

$$y < y_c \ \rightarrow \text{Supercritical flow} \ \rightarrow \ F_r > 1 \ \rightarrow (1 - F_r^2) < 0$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{-}{-} = + \quad \text{(Water depth will increase in the flow direction)}$$

Asymptotic behavior of the water surface is;

$$y \rightarrow y_c \;\rightarrow F_r = 1 \rightarrow \;(1 - F_r^2) = 0$$

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{0} = \infty$$

The water surface meets the critical depth line vertically.

$$y \rightarrow 0,\;\; S_e \rightarrow \infty,\;\; (S_0 - S_e) = \infty$$

$$y \rightarrow 0,\;\; F_r = \frac{V}{\sqrt{gy}} = \infty$$

$$\lim_{y \to 0} \frac{dy}{dx} = \frac{\infty}{\infty} \quad \text{(Unknown)}$$

The angle of the water surface with the channel bed may be taken as $S_0 \left( \frac{y_0}{y_c} \right)^3$.

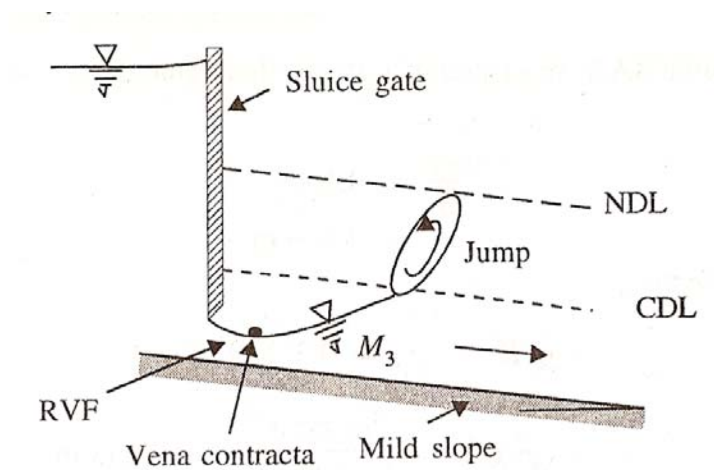



**Figure 6.7.** $M_3$ Profile

Where a supercritical stream enters a mild slope channel, $M_3$ type of profile occurs. The flow leading from a spillway or a sluice gate to a mild slope forms a typical example (Fig. 6.7). The beginning of the $M_3$ curve is usually followed by a small stretch of rapidly varied flow and the downstream is generally terminated by a hydraulic jump. Compared to $M_1$ and $M_2$ profiles, $M_3$ curves are of relatively short length.

**Example 6.1 :** A rectangular channel with a bottom width of 4.0 m and a bottom slope of 0.0008 has a discharge of 1.50 $m^3$/sec. In a gradually varied flow in this channel, the depth at a certain location is found to be 0.30 m. assuming n = 0.016, determine the type of GVF profile.

**Solution:**

a) *To find the normal depth $y_0$,*

$$A = By_0 = 4y_0$$
$$P = B + 2y_0 = 4 + 2y_0$$
$$R = \frac{A}{P} = \frac{4y_0}{4 + 2y_0}$$

$$Q = VA = A\frac{1}{n}\left(\frac{A}{P}\right)^{2/3} S_0^{1/2}$$
$$1.50 = \frac{1}{0.016} \times \frac{(4y_0)^{5/3}}{(4 + 2y_0)^{2/3}} \times 0.0008^{0.5}$$

$$0.0842 = \frac{y_0^{5/3}}{(4 + 2y_0)^{2/3}}$$

By trial and error, $y_0$ = 0.43 m.

b) *Critical depth $y_c$,*

$$q = \frac{Q}{B} = \frac{1.50}{4.0} = 0.375\, m^2/\text{sec}$$
$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.375^2}{9.81}\right)^{1/3} = 0.24m$$

c) *Type of profile,*

$y_0$ = 0.43 m > $y_c$ = 0.24 m (Mild slope channel, M profile)

y = 0.30 m

$y_0 > y > y_c$ (Region 2)

Water surface profile is of the $M_2$ type.

### 6.4.2. S – Curves

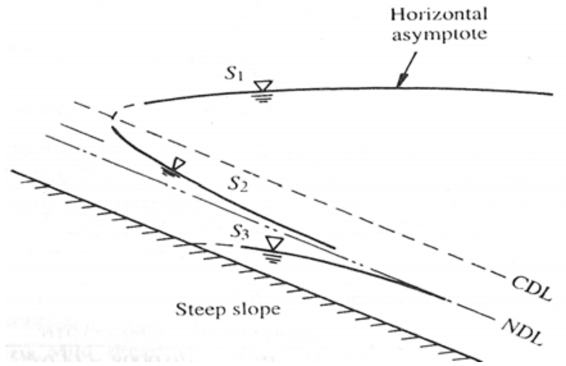



**Figure 6.8**

General shapes of S curves are given in Fig. (6.8). Asymptotic behaviors of each curve will be examined mathematically.

**a) $S_1$ – Curve**

Water surface will be in Region 1 for a steep slope channel and the flow is obviously supercritical.

$$y > y_c > y_0$$

$$y > y_0 \rightarrow \quad V < V_0 \rightarrow S_e < S_0 \rightarrow (S_0 - S_e) > 0$$

$$y > y_c \rightarrow F_r < 1 \rightarrow (1 - F_r^2) > 0$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{+}{+} = +$$

(Water depth will increase in the flow direction)

Asymptotic behavior of the water surface is;

$$\infty > y > y_c$$

$$y \rightarrow \infty \ , \ V \rightarrow 0 \ , F_r = 0 \ , \ (1- F_r^2) = 1$$

$$y \rightarrow \infty \ , \ V \rightarrow 0 \ , S_e = 0$$

$$\lim_{y \to \infty} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{S_0 - 0}{1 - 0} = S_0$$

The water surface meets a very large depth as a horizontal asymptote.

$$y \rightarrow y_c, \quad F_r = 1, (1 - F_r^2) = 0$$

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{S_0 - S_e}{0} = \infty$$

The water surface meets the critical depth line vertically.

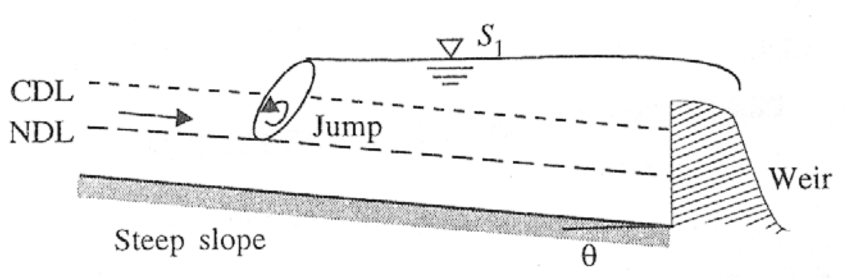



**Figure 6.9.** $S_1$ Profile

The $S_1$ profile is produced when the flow from a steep channel is terminated by a deep pool created by an obstruction, such as a weir or dam (Fig. 6.9). At the beginning of the curve, the flow changes from the normal depth (supercritical flow) to subcritical flow through a hydraulic jump. The profiles extend downstream with a positive water slope to reach a horizontal asymptote at the pool elevation.

**b) $S_2$ – Curve**

Water surface will be in Region 2 for a steep slope channel and the flow is supercritical.

$$y_0 < y < y_c$$

$$y > y_0 \rightarrow S_e < S_0 \rightarrow (S_0 - S_e) > 0$$

$$y < y_c \rightarrow \text{Supercritical flow} \rightarrow F_r > 1 \rightarrow (1 - F_r^2) < 0$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{+}{-} = -$$

(Water depth will decrease in the flow direction)

Asymptotic behavior of the water surface is;

$$y_0 < y < y_c$$

$$y \rightarrow y_c \;,\; F_r \rightarrow 1 \;,\; (1 - F_r^2) = 0$$

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{S_0 - S_e}{0} = \infty$$

The water surface meets the critical depth line vertically.

$$y \rightarrow y_0 \;,\; V \rightarrow V_0 \;,\; S_e \rightarrow S_0 \;,\; (S_0 - S_e) = 0$$

$$\lim_{y \to y_0} \frac{dy}{dx} = \frac{0}{1 - F_r^2} = 0$$

The water surface approaches the normal depth asymptotically.

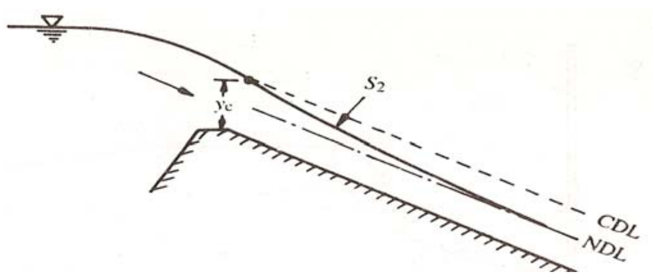



**Figure 6.10.** $S_2$ Profile

Profiles of the $S_2$ type occur at the entrance region of a steep channel leading from a reservoir and at a brake of grade from mild slopes to steep slope (Fig. 6.10). Generally $S_2$ profiles are short of length.

### c) $S_3$ – Curve

Water surface will be in Region 3 for a steep slope channel and the flow is supercritical.

$$0 < y < y_0$$

$$y < y_0 \rightarrow S_e > S_0 \rightarrow (S_0 - S_e) < 0$$

$$y < y_c \rightarrow \text{Supercritical flow} \rightarrow F_r > 1 \rightarrow (1 - F_r^2) < 0$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{-}{-} = +$$

(Water depth will increase in the flow direction)

Asymptotic behavior of the water surface is;

$$0 < y < y_0$$

$$y \rightarrow 0,\ S_e \rightarrow \infty\ ,\ (S_0 - S_e) = -\infty$$

$$y \rightarrow 0\ ,\ \ F_r = \frac{V}{\sqrt{gy}} = \infty$$

$$\lim_{y \to 0} \frac{dy}{dx} = \frac{-\infty}{\infty} \quad \text{(Unknown)}$$

The angle of the water surface with the channel bed may be taken as $S_0 \left( \frac{y_0}{y_c} \right)^3$.

$$y \rightarrow y_0\ ,\ \ S_e \rightarrow S_0\ ,\ \ (S_0 - S_e) = 0$$

$$\lim_{y \to y_0} \frac{dy}{dx} = \frac{0}{1 - F_r^2} = 0$$

The water surface approaches the normal depth $y_0$ asymptotically.

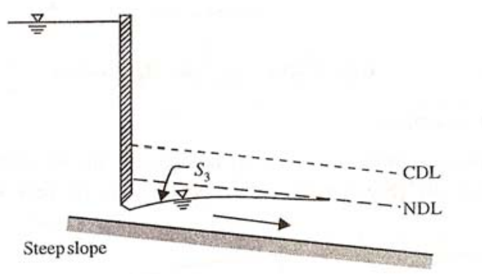



**Figure 6.11.** $S_3$ Profile

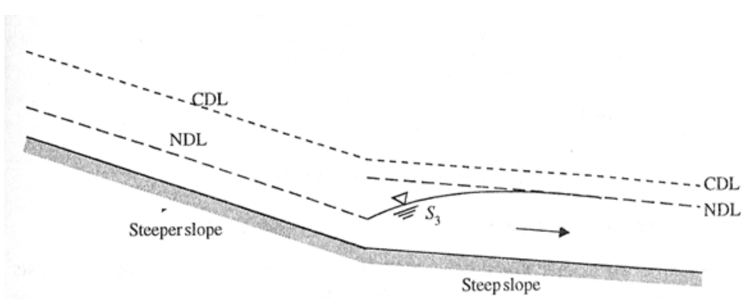



**Figure 6.12.** $S_3$ Profile

Free flow from a sluice gate with a steep slope on its downstream is of the $S_3$ type (Fig. 6.11). The $S_3$ curve also results when a flow exists from a steeper slope to a less steep slope (Fig. 6.12).

### 6.4.3. C – Curves

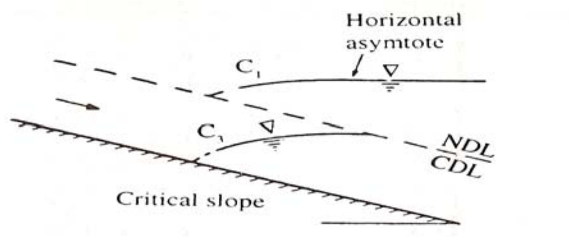



**Figure 6.13**

General shapes of C curves are given in Fig. (6.13). Asymptotic behaviors of each curve will be examined mathematically. Since the flow is at critical stage, $y_0 = y_c$, there is no Region 2.

**a) $C_1$ – Curve**

Water surface will be in Region 1 for a critical slope channel.

$$y_0 = y_c < y < \infty$$

$$y > y_c \rightarrow S_e < S_0 \rightarrow (S_0 - S_e) > 0$$

$$y > y_c \rightarrow \text{Subcritical flow} \rightarrow F_r < 1 \rightarrow (1 - F_r^2) > 0$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{+}{+} = +$$

(Water depth will increase in the flow direction)

Asymptotic behavior of the water surface is;

$$y \rightarrow \infty \;,\; S_e \rightarrow 0 \;,\; (S_0 - S_e) = S_0$$

$$y \rightarrow \infty, \quad V = 0 \rightarrow F_r = 0 \rightarrow (1 - F_r^2) = 1$$

$$\lim_{y \to \infty} \frac{dy}{dx} = \frac{S_0 - 0}{1 - 0} = S_0$$

The water surface meets a very large depth as a horizontal asymptote.

$$y \rightarrow y_c \;,\; F_r = 1 \rightarrow (1 - F_r^2) = 0$$

$$y \rightarrow y_c = y_0 \;,\; S_e = S_0 = S_c \rightarrow (S_0 - S_e) = 0$$

$$\lim_{y \to y_c = y_0} \frac{dy}{dx} = \frac{0}{0} \quad \text{(Unknown)}$$

$C_1$ and $C_3$ profiles are very rare and highly unstable

## 6.4.4. H – Curves

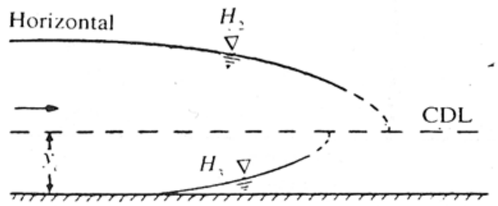



**Figure 6.14**

General shapes of H curves are given in Fig. (6.14). For horizontal slope channels, uniform flow depth $y_0$ does not exist. Critical water depth can be computed for a given discharge Q and therefore critical water depth line can be drawn. Since there is no uniform water depth $y_0$, Region 1 does not exist.

### a) $H_2$ – Curve

Water surface will be in Region 2 for a horizontal slope channel.

$$\infty > y > y_c$$

$$y > y_c \rightarrow S_e < S_0 = 0 \rightarrow (S_0 - S_e) < 0$$

$$y > y_c \rightarrow \text{subcritical flow} \rightarrow F_r < 1 \rightarrow (1 - F_r^2) > 0$$

$$\frac{dy}{dx} = \frac{-}{+} = - \quad \text{(Water depth will decrease in the flow direction)}$$

Asymptotic behavior of the water surface is;

$$y \rightarrow \infty \; , S_e \rightarrow 0 \; , \; (S_0 - S_e) = S_0$$

$$y \rightarrow \infty, \quad V = 0 \rightarrow F_r = 0 \rightarrow (1 - F_r^2) = 1$$

$$\lim_{y \to \infty} \frac{dy}{dx} = \frac{S_0 - 0}{1 - 0} = S_0$$

The water surface meets a very large depth as a horizontal asymptote.

$$y \rightarrow y_c \; , \; F_r = 1 \rightarrow (1 - F_r^2) = 0$$

$$y \rightarrow y_c \; , \; S_e = S_c \rightarrow S_0 = 0, \; (S_0 - S_e) = -S_e$$

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{0 - S_e}{0} = \infty$$

The water surface meets the critical depth line vertically.

**b) $H_3$ – Curve**

$$0 > y > y_c$$

$$y < y_c \rightarrow S_e > S_0 = 0 \rightarrow (S_0 - S_e) = -S_e$$

$$y \rightarrow y_c \text{ , supercritical flow , } F_r > 1 \rightarrow (1 - F_r^2) < 0$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{-}{-} = +$$

(Water depth will increase in the flow direction)

Asymptotic behavior of the water surface is;

$$y \rightarrow 0, \; S_e \rightarrow \infty \; , (S_0 - S_e) = -\infty$$

$$y \rightarrow 0 \, , \quad F_r = \frac{V}{\sqrt{gy}} = \infty$$

$$\lim_{y \to 0} \frac{dy}{dx} = \frac{-\infty}{\infty} \quad \text{(Unknown)}$$

$$y \rightarrow y_c \rightarrow F_r = 1 \rightarrow (1 - F_r^2) = 0$$

$$y \rightarrow y_c \; , \; S_e = S_c \rightarrow S_0 = 0, \; (S_0 - S_e) = -S_e$$

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{0 - S_e}{0} = \infty$$

The water surface meets the critical depth line vertically.

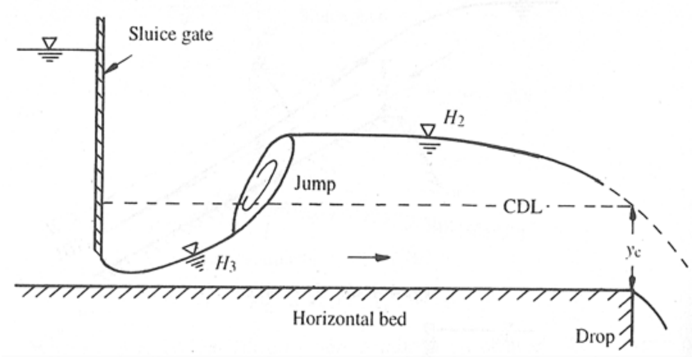



**Figure 6.15**

A horizontal channel can be considered as the lower limit reached by a mild slope as its bed slope becomes flatter. The $H_2$ and $H_3$ profiles are similar to $M_2$ and $M_3$ profiles respectively (Fig. 6.15). However, the $H_2$ curve has a horizontal asymptote.

### 6.4.5. A– Curves

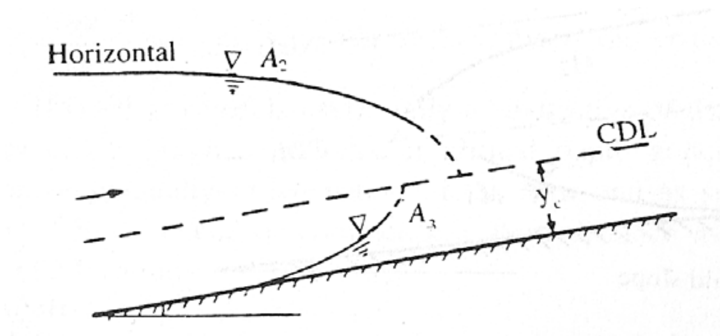



**Figure 6.16**

General shapes of A curves are given in Fig. (6.16). For adverse slope channels, uniform flow depth $y_0$ does not exist. Critical water depth can be computed for a given discharge Q and therefore critical water depth line can be drawn. Since there is no uniform water depth $y_0$, Region 1 does not exist as well as in A curves. $A_2$ and $A_3$ curves are similar to $H_2$ and $H_3$ curves respectively.

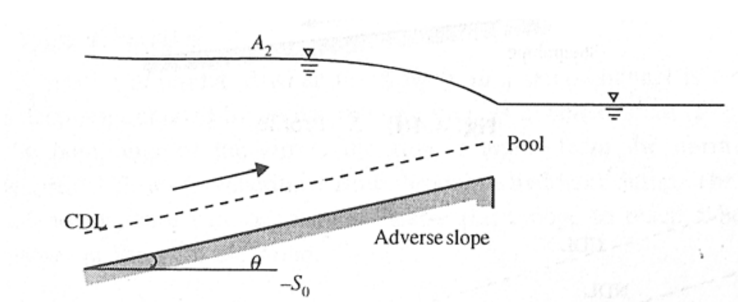
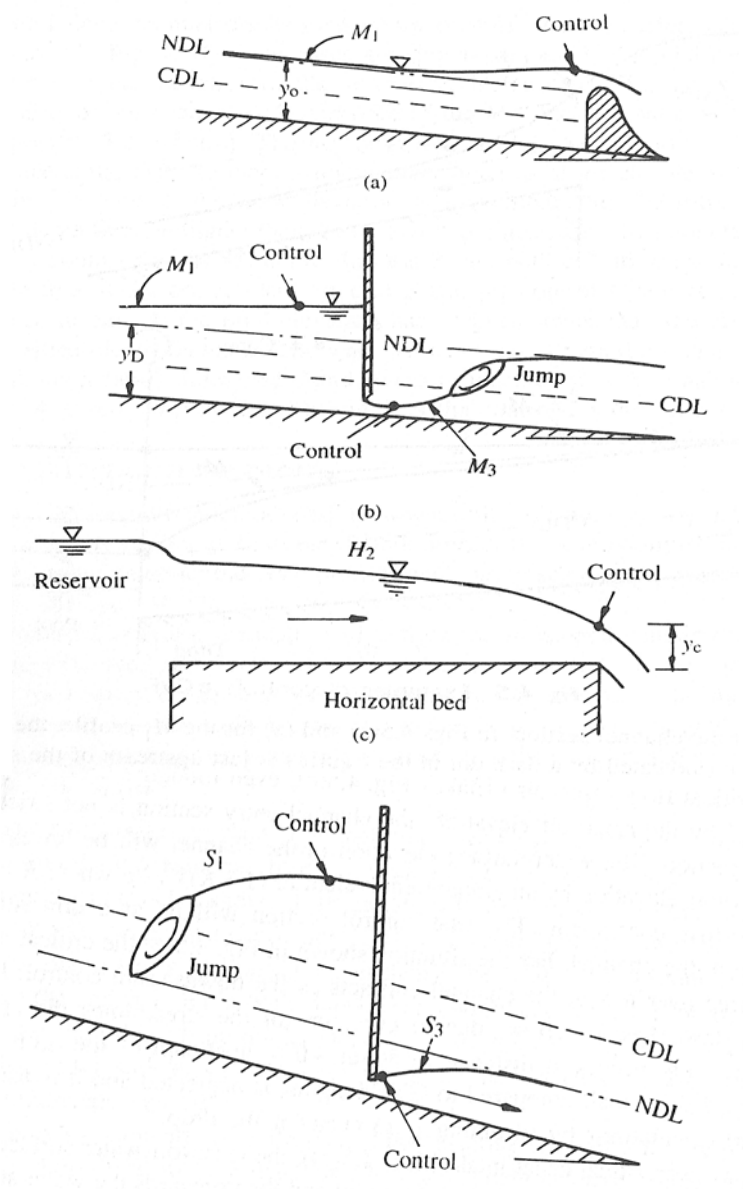



**Figure 6.17**

Adverse slopes are rather rare (Fig. 6.17). These profiles are of very short length.

### 6.5. Control Sections

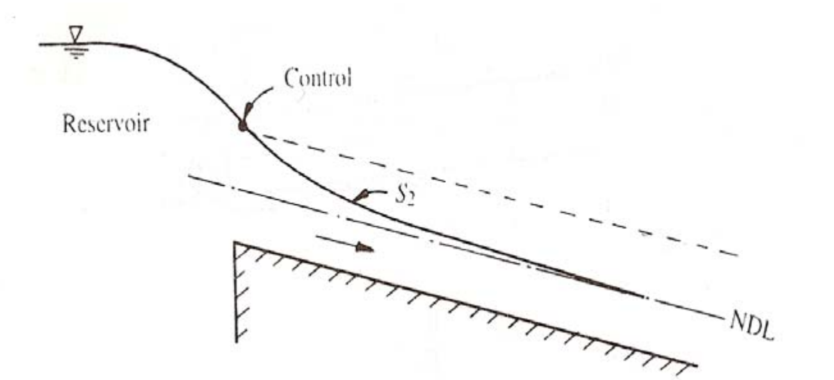
A *control section* is defined as a section in which a fixed relationship exists between the discharge and depth of flow. Weirs, spillways, sluice gates are some typical examples of structures which give rise to control sections. *The critical depth is also a control point.* However, it is effective in a flow profile which changes from subcritical to supercritical flow. In the reverse case of transition from supercritical flow to subcritical flow, a hydraulic jump is usually formed bypassing the critical depth as a control point. Any GVF profile will have at least one control section.




(d)

**Figure 6.18**

In the synthesis of GVF profiles occurring in a serially connected channel elements, the control sections provide a key to the identification of proper profile shapes. A few typical control sections are shown in Fig. (6.18, a-e). It may be noted that subcritical flows have controls in the downstream end, while supercritical flows are governed by control sections existing at the upstream end of the channel section. In Figs. (6.18 a and b) for the $M_1$ profile, the control section (indicated by a dark dot in the figures) is just at the upstream of the spillway and sluice gate respectively. In Figs. (6.18 b and d) for $M_3$ and $S_3$ profiles respectively, the control point is at the vena contracta of the sluice gate flow. In subcritical flow reservoir offtakes (Fig. 6.18 c), even though the discharge is governed by the reservoir elevation, the channel entry section is not strictly a control section. The water surface elevation in the channel, will be lower than the reservoir elevation by a headloss amount equivalent to $(1 + \zeta)V^2/2g$ where $\zeta$ is the entrance loss coefficient. The true control section will be at a downstream location in the channel. For the situation shown in Fig. (6.18 c) the critical depth at the free overflow at the channel end acts as the downstream control. For a sudden drop (free overflow) due to curvature of the streamlines the critical depth usually occurs at distance of about $4y_c$ upstream of the drop. This distance, being small compared to GVF lengths, is neglected and it is usual to perform calculations by assuming $y_c$ to occur at the drop.




(e)

For a supercritical canal intake (Fig. 6.18 e), the reservoir water surface falls to the critical depth at the head of the canal and then onwards the water surface follows the $S_2$ curve. The critical depth occurring at the upstream end of the channel is the control for this flow.

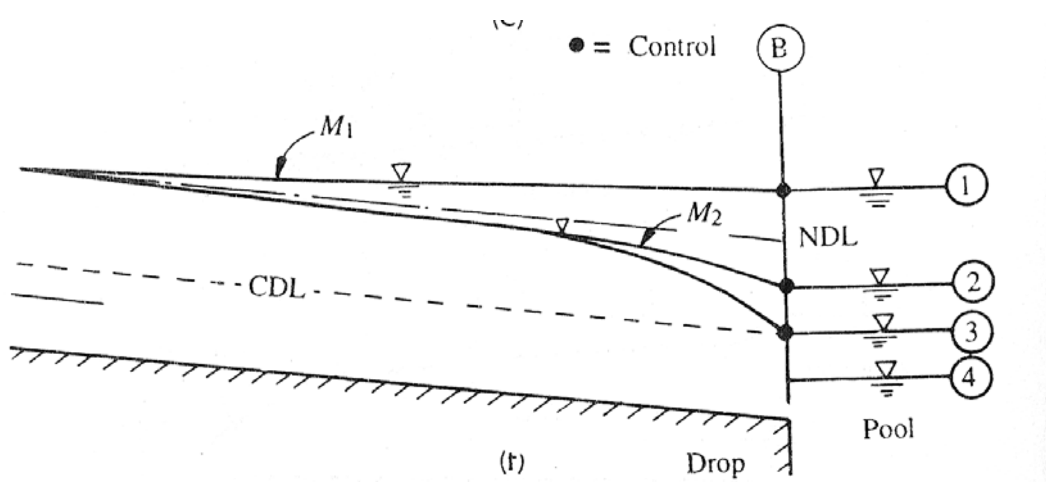



**Figure 6.19**

### 6.5.1. Feeding a Pool for Subcritical Flow

A mild slope channel discharging into a pool of variable surface elevation is indicated in Fig. (6.19). Four cases are shown.

1) The pool elevation is higher than the elevation of the normal (uniform) water depth line at B. This gives rise to a drowning of the channel end. A profile of the $M_1$ type is produced with the pool level at B as control.

2) The pool elevation is lower than the elevation of the normal depth line but higher than the critical depth line at B. The pool elevation acts as a control for the $M_2$ curve.

3) The pool elevation has dropped down to that of the critical depth line at B and the control is still at the pool elevation.

4) The pool elevation has dropped lower than the elevation of the critical depth line at B. The water surface cannot pass through a critical depth at any location other than B and hence a sudden drop in the water surface at B is observed. The critical depth at B is the control of this flow.

### 6.5.2. Feeding a Pool for Supercritical Flow

When the slope of the channel is steep, that is greater than the critical slope, the flow in the channel becomes supercritical. In practical applications, steep channels are usually short, such as raft and log chutes that are used as spillways.

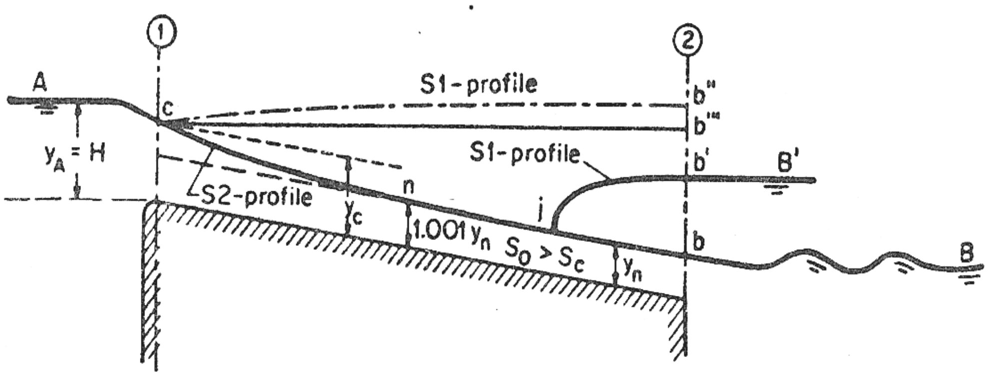



**Figure 6.20**

As the control section in a channel of supercritical flow is at the upstream end, the delivery of the channel is fully governed by the critical discharge at section 1.

1) When the tailwater level B is less than the outlet depth (normal depth) at section 2, the flow in the canal is unaffected by the tailwater. The flow profile passes through the critical water depth near C and approaches the normal depth by means of a smooth drawdown curve of the $S_2$ type.

2) When the tailwater level B is greater than the outlet depth, the tailwater will raise the water level in the upstream portion of the canal to form an $S_1$ profile between j and b`, producing a hydraulic jump at the end j of the profile. However, the flow upstream from the jump will not be affected by the tailwater.

   Such a hydraulic jump is objectionable and dangerous, particularly when the canal is raft chute or some other structure intended to transport a floating raft from the upstream reservoir to a downstream pool. A *neutralizing reach* may be suggested for the solution of this problem. (Chow, 1959). (Fig. 6.21)

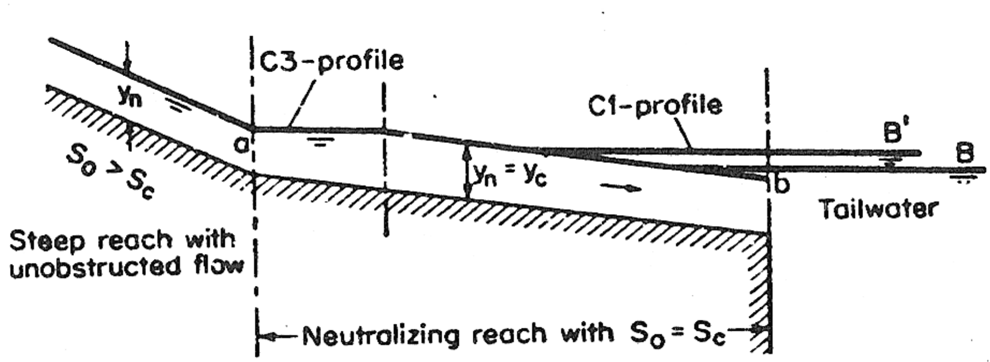



**Figure 6.21.** Elimination of hydraulic jump by neutralizing reach.

   In this reach the bottom slope of the channel is made equal to the critical slope. According to a corresponding case of $C_1$ profile in Fig. (6.21), the tailwater levels will be approximately horizontal lines which intersect the surface of flow in the canal without causing any disturbance. At the point of intersection, theoretically, there is a jump of zero height.

3) As the tailwater rises further, the jump will move upstream, maintaining its height and form in the uniform flow zone nb, until it reaches point n. The height of the jump becomes zero when it reaches the critical depth at c. In the meantime, the flow profile reaches its theoretical limit cb`` of the $S_1$ profile. Beyond this limit the incoming flow will be directly affected by the tailwater. In practical applications, the horizontal line cb``` may be taken as the practical limit of the tailwater stage.

## 6.6. Analysis of Flow Profile

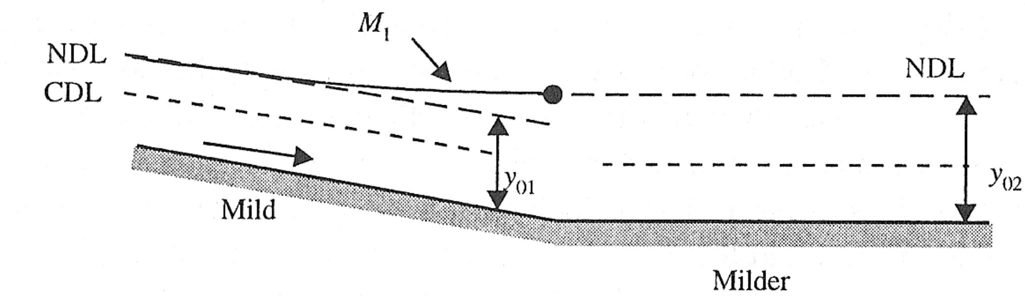
A channel carrying a gradually varied flow can in general contain different prismoidal channel cross-sections of varying hydraulic properties. There can be a number of control sections at various locations. To determine the resulting water surface profile in a given case, one should be in a position to analyze the effects of various channel sections and controls connected in series. Simple cases are illustrated to provide information and experience to handle more complex cases.

### 6.6.1. Break in Grades

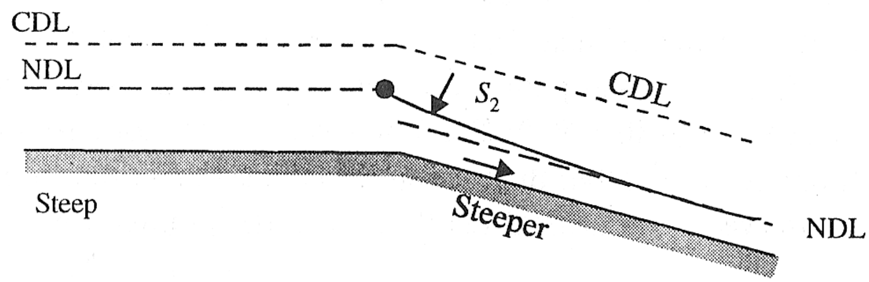
Simple situations of a series combination of two channel sections with differing bed slopes are considered. In Fig. (6.22.a), a break in grade from a mild channel to a milder channel is shown. It is necessary to first draw the critical-depth line (CDL) and the normal-depth line (NDL) for both slopes. Since $y_c$ does not depend upon the slope for a taken Q = discharge, the CDL is at a constant height above the channel bed in both slopes. The normal depth $y_{01}$ for the mild slope is lower than that of the of the milder slope ($y_{02}$). In this case, $y_{02}$ acts as a control, similar to the weir or spillway case and an $M_1$ backwater curve is produced in the mild slope channel.

Various combinations of slopes and the resulting GVF profiles are presented in Fig. ( 6.22, a-h). It may be noted that in some situations there can be more than one possible profiles. For example, in Fig. (6.22 e), a jump and $S_1$ profile or an $M_3$ profile and a jump possible. The particular curve in this case depends on the channel and its flow properties.

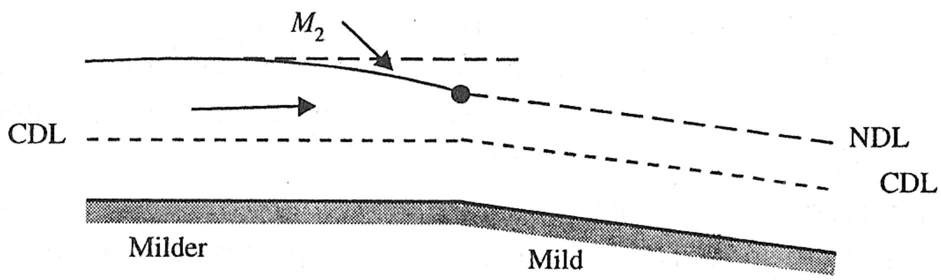
In the examples indicated in Fig. (6.22), the section where the grade changes acts a control section and this can be classified as a natural control. It should be noted that even though the bed slope is considered as the only variable in the above examples, the same type of analysis would hold good for channel sections in which there is a marked change in the roughness characteristics with or without change in the bed slope. A long reach of unlined canal followed by a line reach serves as a typical example for the same. A change in the channel geometry (the bed width or side slope) beyond a section while retaining the prismoidal nature in each reach also leads to a natural control section.


M1
NDL
CDL
y01
Mild
NDL
y02
Milder

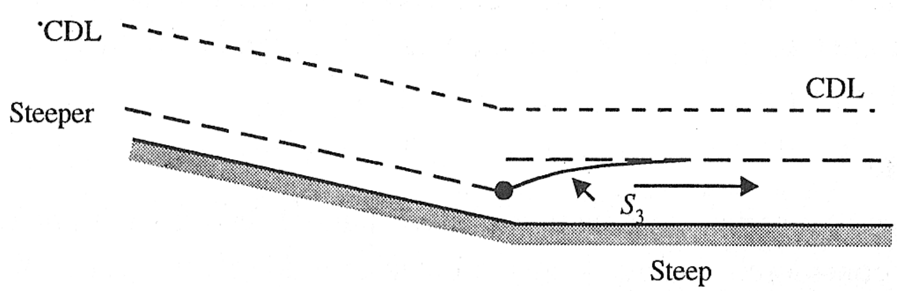
(a)


CDL
NDL
S2
CDL
Steep
Steeper
NDL

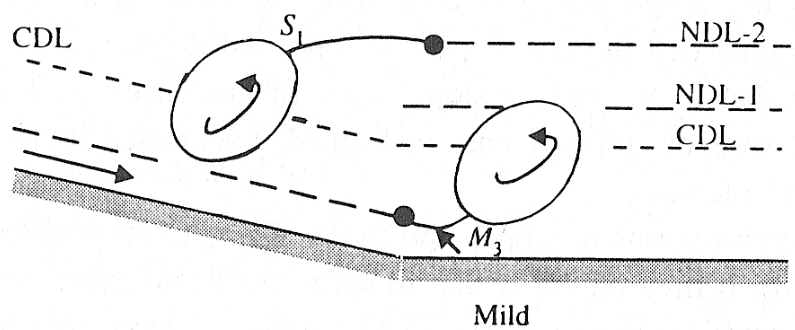
(b)


M2
CDL
NDL
CDL
Milder
Mild

(c)

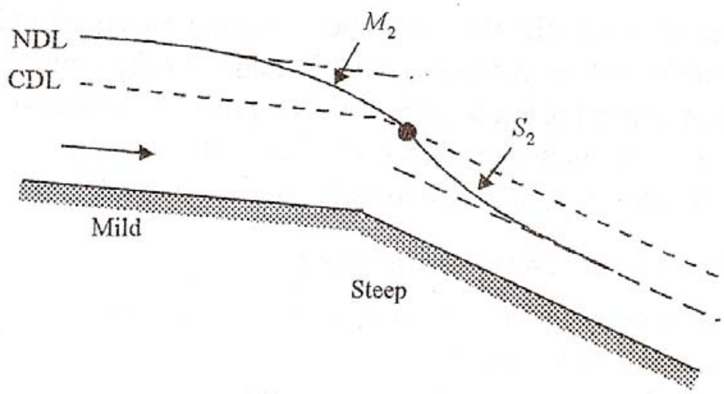

CDL
CDL
Steeper
S3
Steep

(d)



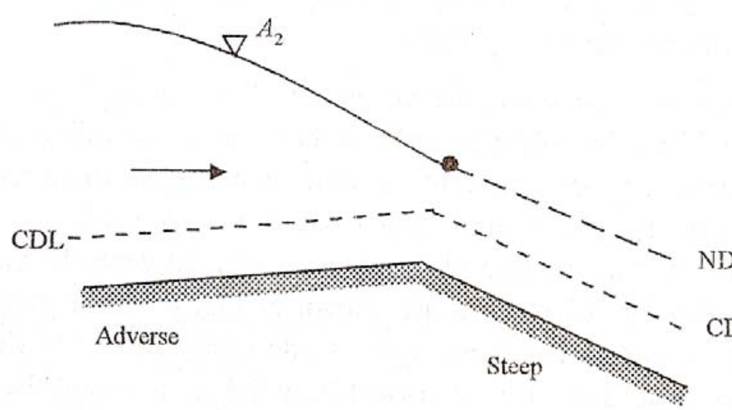
CDL
$S_1$
NDL-2
NDL-1
CDL
$M_3$
Mild


(e)



NDL
$M_2$
CDL
$S_2$
Mild
Steep


(f)



$A_2$
CDL
NDL
CDL
Adverse
Steep


(g)

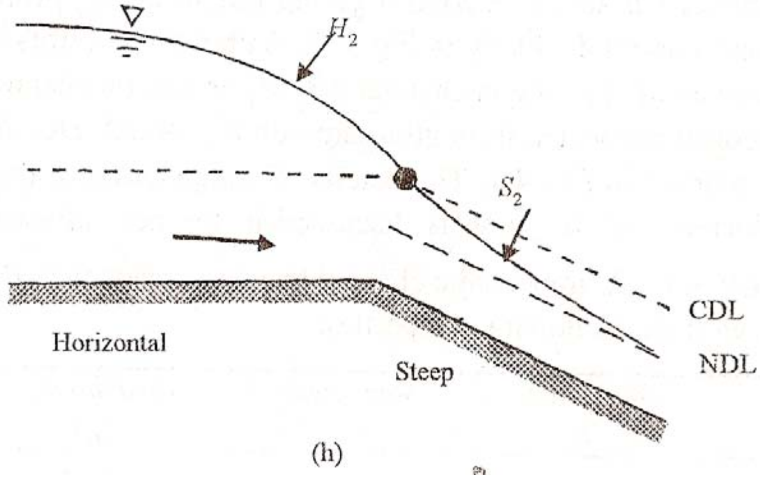



**Figure 6.22**

### 6.6.2. Serial Combination of Channel Sections

To analyze a general problem of many channel sections and controls, the following steps are to be applied.

1. Draw the longitudinal section of the system.
2. Calculate the critical depth and normal depths of various reaches and draw the CDL and NDL in all reaches.
3. Mark all the controls, both the imposed as well as natural controls.
4. Identify the possible profiles.

**Example 6. 2:** Identify and sketch the GVF profiles in three mild slopes which could be described as mild, steeper mild and milder. The three slopes are in series. The last slope has a sluice gate in the middle of the reach and the downstream end of the channel has a free overfall.

**Solution:** The longitudinal profile of the channel, critical depth line and normal depth lines for the various reaches are shown in the Fig. (6.23). The free overfall at E is obviously a control. The vena contracta downstream of the sluice gate at D is another control. Since for subcritical flow the control is at the downstream end of the channel, the higher of the two normal depths at C acts as a control for the reach CB, giving rise to an $M_1$ profile over CB. At B, the normal depth of the channel CB acts as a control giving rise to an $M_2$ profile over AB. The controls are marked distinctly in Fig. (6.23). With these controls the possible flow profiles are: an $M_2$ profile on channel AB, $M_1$ profile on channel BC, $M_3$ profile and $M_2$ profile through a jump on the stretch DE.

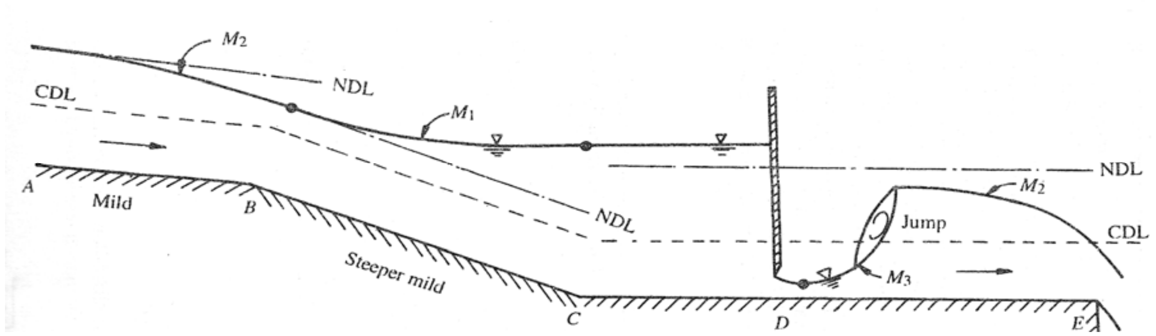


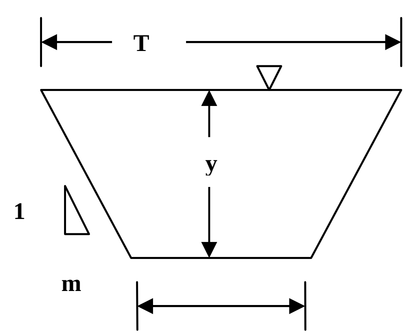

**Figure 6.23**

**Example 6.3:** A trapezoidal channel has three reaches A, B, and C connected in series with the following physical characteristics.

| Reach | Bed width (B) | Side slope (m) | Bed slope $S_0$ | n |
|---|---|---|---|---|
| 1 | 4.0 m | 1.0 | 0.0004 | 0.015 |
| 2 | 4.0 m | 1.0 | 0.009 | 0.012 |
| 3 | 4.0 m | 1.0 | 0.004 | 0.015 |

For a discharge Q = 22.5 $m^3$/sec through this channel, sketch the resulting water surface profiles. The length of the reaches can be assumed to be sufficiently long for the GVF profiles to develop fully.

**Solution:** The normal depths and critical water depths in the various reaches are calculated as:




$$A = (B + my)y$$

$$P = B + 2y\sqrt{1 + m^2}$$

$$T = B + 2my$$

$$A = (4 + y)y$$
$$P = 4 + 2y\sqrt{1+1^2} = 4 + 2^{3/2}y$$
$$T = 4 + 2y$$

$$Q = AV = \frac{A}{n}\left(\frac{A}{P}\right)^{2/3} S_0^{0.5}$$
$$22.5 = \frac{\left[(4 + y_0)y_0\right]^{5/3}}{n} \times \frac{S_0^{0.5}}{\left(4 + 2^{3/2} \times y_0\right)^{2/3}}$$

Uniform flow depths for the given data for every reach are calculated by trial and error method;

Reach A: $S_{0A} = 0.0004$ , $n_A = 0.015$ $\rightarrow$ $y_{0A} = 2.26$ m

Reach B: $S_{0B} = 0.009$ , $n_B = 0.012$ $\rightarrow$ $y_{0B} = 0.81$ m

Reach C: $S_{0C} = 0.004$ , $n_C = 0.015$ $\rightarrow$ $y_{0C} = 1.17$ m

Since the channel is prismoidal (the geometry does not change in the reaches), there will be only one critical water depth.

$$\frac{Q^2 T_c}{g A_c^3} = 1$$
$$\frac{22.5^2 \times (4 + 2y_c)}{9.81 \times \left[(4 + y_c)y_c\right]^3} = 1$$

$$y_c = 1.32 \text{ m}$$

Reach A is a mild slope channel as $y_{0A}$ = 2.26 m > $y_{0c}$ = 1.32 m and the flow is subcritical. Reach B and C are steep slope channels as $y_{0B}$, $y_{0C} < y_c$ and the flow is supercritical on both reaches. Reach B is steeper than reach C. The various reaches are schematically shown in Fig. (6.24). The CDL is drawn at a height of 1.32 m above the bed level and NDLs are drawn at the calculated $y_0$ values.

The controls are marked in the figure. Reach A will have an $M_2$ drawdown curve, reach B an $S_2$ drawdown curve and reach C a rising curve as shown in the figure. It may be noted that the resulting profile as above is a serial combination of Fig. (6.22 d and f).

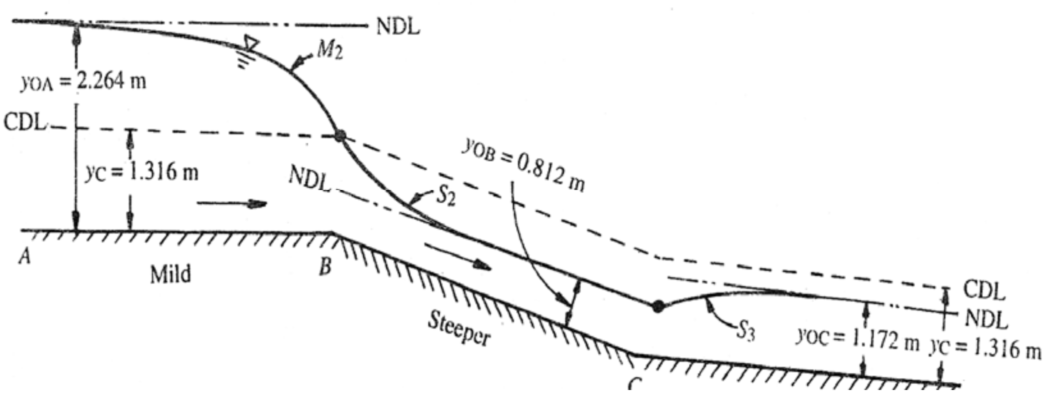

NDL
$M_2$
$y_{OA}$ = 2.264 m
CDL
$y_C$ = 1.316 m
NDL
$S_2$
$y_{OB}$ = 0.812 m
A
Mild
B
Steeper
C
$S_3$
CDL
NDL
$y_{OC}$ = 1.172 m
$y_C$ = 1.316 m

**Figure 6.23**