LECTURE NOTES - V

« FLUID MECHANICS »

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CHAPTER 5

FLOW OF A REAL FLUID

5.1 INTRODUCTION

The flow of a real fluid is more complex than that of an ideal fluid, owing to the phenomena caused by the existence of viscosity. Viscosity introduces resistance to motion by causing shear and friction forces between fluid particles and boundary walls. For flow to take place, work must be done against resistance forces, and in the process energy is converted to heat. The inclusion of viscosity also allows the possibility of two physically different flow regimes. The effects of viscosity on the velocity profile also render invalid the assumption of uniform velocity distribution. Although the Euler equations may be altered to include shear stresses of a real fluid, the result is a set of partial differential equations to which no general solution is known.

5.2 REYNOLD'S EXPERIMENTS

The effects of viscosity cause the flow of a real fluid to occur under two different conditions or regimes: that of *laminar* flow and that of *turbulent* flow. The characteristics of these regimes were first demonstrated by Reynolds, with an apparatus similar to that of Fig. 5.1.

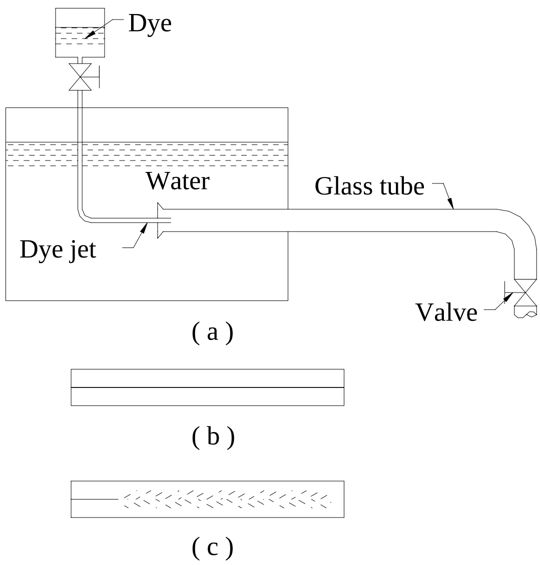

Fig. 5.1

Water flows from a tank through a bell-mouthed glass pipe, the flow being controlled by the valve. A thin tube leading from a reservoir of dye has its opening within the entrance of the glass pipe.

Reynolds discovered that, for low velocities of flow in the glass pipe, a thin filament of dye issuing from the tube did not diffuse but was maintained intact throughout the pipe, forming a thin straight line parallel to the axis of the pipe (Fig.5.1b). As the valve was opened, however, and greater velocities were attained, the dye filament wavered and broke, eventually diffusing through the flowing water in glass pipe (Fig.5.1c).

Since mixing of fluid particles during flow would cause diffusion of the dye filament, Reynolds deduced from his experiments that at low velocities this mixing was absent and that the fluid particles moved in parallel layers, or laminae, sliding past adjacent laminae but not mixing with them, this is the regime of *laminar flow*. Since at higher velocities the dye filament diffused through the pipe, it was apparent that mixing of fluid particles was occurring, or, in other words, the flow was *turbulent*. Laminar flow broke down into the turbulent flow at some *critical velocity* above that at which turbulent flow was restored to the laminar condition.

Reynolds was able to generalize his conclusions from his dye stream experiments by the introduction of a dimensionless term Re, later called the *Reynolds Number*, which was defined by

$$\mathrm{Re} = \frac{Vd\rho}{\mu} = \frac{Vd}{\nu} \qquad (5.1)$$

In which V is the mean velocity of the fluid in pipe, d is the diameter of the pipe, and ρ, μ and ν are the specific mass, dynamic viscosity and kinematic viscosity of the fluid flowing therein.

The upper limit of laminar flow to be defined by $2100<\mathrm{Re}_{cr}<4000$. The lower limit of turbulent flow, defined by the lower critical Reynolds number, is of greater engineering importance; it defines a condition below which all turbulence entering the flow from any source will eventually be damped out by viscosity. This lower critical Reynolds number thus sets a limit below which laminar flow will always occur; many experiments have indicated the lower critical Reynolds number to have a value of approximately 2100. Between Reynolds number 2100 and 4000 a region of uncertainty exits.

The concept of a critical Reynolds number to the flow of any fluid in cylindrical pipes, one may predict that the flow will be laminar if Re<2000 and turbulent if Re>4000. However, critical Reynolds number is very much a function of boundary geometry.

Flow	**d length**	**Re_{cr}**
Flow in cylindrical pipes	pipe diameter	2000
Flow between parallel walls	Spacing between the walls	1000
Flow in a wide open channel	Water depth	500
Flow about a sphere	Sphere diameter	1

EXAMPLE 5.1: Water of kinematic viscosity 1.15×10^{-6} m^2/sec flows in a cylindrical pipe of 30 mm diameter. Calculate the largest discharge for which laminar flow can be expected. What is the equivalent discharge for air? $\nu_{air} = 1.37\times10^{-5}$ m^2/sec.

SOLUTION:

Taking $Re_{cr} = 2100$ as the upper limit for laminar flow,

$$\text{Re}_{cr} = \frac{Vd}{\vartheta}$$

$$2100 = \frac{V\times 0.03}{1.15\times 10^{-6}}$$

$$V = 0.0805 m/\sec$$

$$Q_{water} = 0.0805\times\frac{\pi}{4}\times 0.03^2 = 5.69\times 10^{-5}\ m^3/\sec$$

$$2100 = \frac{V_{air}\times 0.03}{1.37\times 10^{-5}}$$

$$V_{air} = 0.959\, m/\sec$$

$$Q_{air} = 0.959\times\frac{\pi}{4}\times 0.03^2 = 6.78\times 10^{-4}\ m^3/\sec$$

$$Q_{air} \cong 12 Q_{water}$$

5.3. LAMINAR AND TURBULENT FLOW

In laminar flow, fluid particles are constrained to motion in parallel paths by the action of viscosity. The shearing stress between adjacent moving layers is determined in laminar flow by the Newton’s viscosity law.

$$\tau = \mu\frac{du}{dy} \qquad (1.4)$$

The stress is the product of viscosity and velocity gradient (Fig.5.2). If the laminar flow is disturbed by wall roughness or some other obstacle, the disturbances are rapidly damped by viscous action and downstream the flow is smooth again. A laminar flow is stable against such disturbances, but a turbulent flow is not.

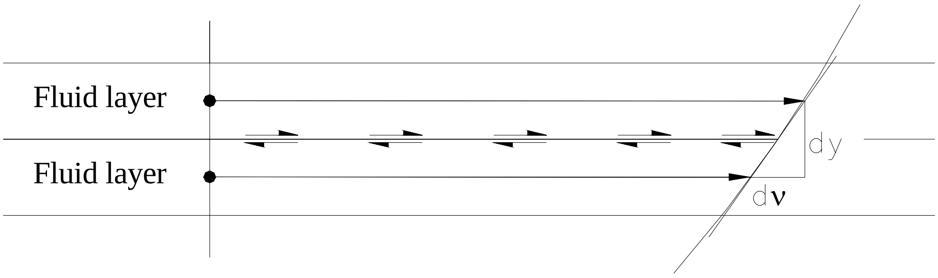

Fig. 5.2

The instability of laminar flow at a high Reynolds number causes disruption of the laminar pattern of fluid motion. With sufficient disturbances the result is known as *turbulence*. Turbulence is characterized by the irregular, chaotic motion of fluid particles. Experiments show that, at any fixed point in a completely developed flow, the *instantaneous velocity* and, consequently, the *instantaneous pressure* fluctuate regularly about a *mean value* with respect to both time and spatial direction. A typical curve showing velocity fluctuations in the x-direction of a turbulent flow is plotted as a function of time in Fig. 5.3.

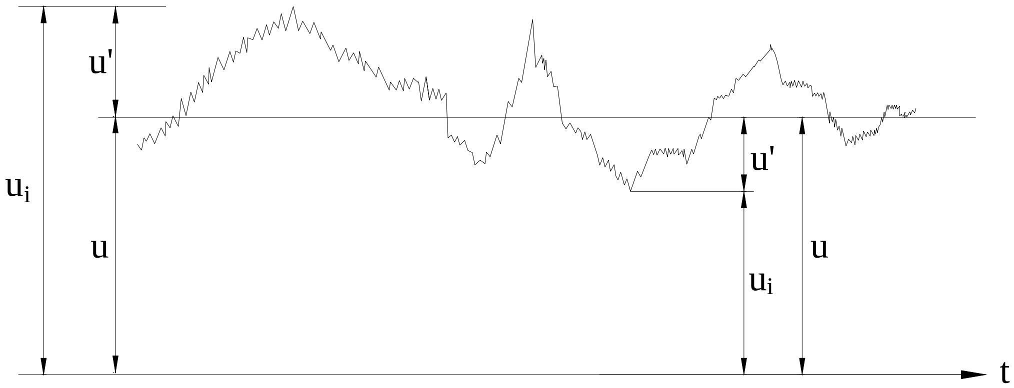

Fig. 5.3

In the theoretical study it is convenient to split the instantaneous velocities, u_i, v_i, and w_i, into their *time average components*, $\overline{u}$, $\overline{v}$, and $\overline{w}$, and *momentary fluctuation components*, u′, v′, and w′. Thus

$$u_i = \overline{u} + u'$$
$$v_i = \overline{v} + v' \qquad (5.2)$$
$$w_i = \overline{w} + w'$$

and, similarly, for pressure:

$$p_i = \overline{p} + p' \qquad (5.3)$$

In accordance with their definition, the following relationships exit:

$$\frac{1}{T}\int_0^T u_i dt = \overline{u}$$
$$\frac{1}{T}\int_0^T v_i dt = \overline{v} = 0$$
$$\frac{1}{T}\int_0^T w_i dt = \overline{w} = 0 \qquad (5.4)$$
$$\frac{1}{T}\int_0^T p_i dt = \overline{p}$$

Also, the time averages of fluctuation components must be equal to zero:

$$\frac{1}{T}\int_0^T u' dt = \overline{u}' = 0, \overline{v'} = 0, \overline{w'} = 0, \overline{p'} = 0 \qquad (5.5)$$

Let's try to examine the flow in a pipe in (Fig.5.4). dxdz is a cylindrical surface area with the same axis of the pipe. v is the velocity component normal the pipe axis, x. vdxdz is the volume of fluid passing through dxdz in unit time. ρvdxdz is the mass of fluid flowing through this area. (ρvdxdz)u is the momentum of the fluid flowing normal to this area over the x-axis per unit time. Therefore, the momentum increase in the x-axis direction is:

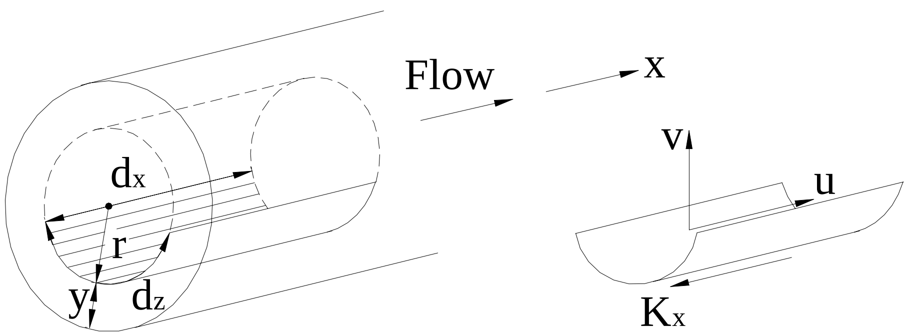

Fig. 5.4

$$\frac{dI_x}{dt} = \rho uvdxdz \qquad (5.6)$$

Here, I_x is the momentum component in the x-axis direction. According to the Newton's second law, the change in the momentum will create a force:

$$-K_x = \frac{dI_x}{dt} \qquad (5.7)$$

K_x is the force created and will act normal to the dxdz surface. Since K_x is taken opposite to the x-axis, there will be a minus sign in front of the force, K_x.

Using Eqs. (5.6) and (5.7),

$$-K_x = \rho uvdxdz \tag{5.8}$$

Shearing stress may be found as for the dxdz area,

$$\frac{K_x}{dxdz} = -\rho uv \tag{5.9}$$

Since u and v changes over their mean value by time, uv term will also change with time. The mean shearing stress may be found by applying Equ. (5.9),

$$\begin{aligned}
\tau &= \lim_{T\to\infty}\frac{1}{T}\int_0^T(-\rho uv)dt \\
&= \lim_{T\to\infty}\frac{1}{T}\int_0^T\left[-\rho(\bar{u}+u')(0+v')\right]dt \\
&= -\rho\bar{u}\lim_{T\to\infty}\frac{1}{T}\int_0^T v'dt - \rho\lim_{T\to\infty}\frac{1}{T}\int_0^T u'v'dt
\end{aligned}$$

Since the first integral of the last equation is the mean of the fluctuation component, then $\overline{v'} = 0$, and the second integral is the mean value of the product of u′ and v′, $\overline{u'v'}$. Then the mean shearing stress may be found as,

$$\tau = -\rho\overline{u'v'} \tag{5.10}$$

Terms of the form $-\rho\overline{u'v'}$ now called *Reynolds stresses*. In general, the shearing stress of the turbulent flow may be written as in the following form:

$$\tau = \tau_l + \tau_t = \mu\frac{du}{dy} + \left(-\rho\overline{u'v'}\right) \tag{5.11}$$

The first term of Equ. (5.11) is the result of viscous effect and the second term is the result of turbulence effect. If the flow is laminar, u′ and v′ velocity fluctuations will be zero, so Equ. (5.11) will take the form of Equ. (1.4).

In turbulent flow the numerical value of Reynolds stress ($-\rho\overline{u'v'}$) is generally several times greater than that of (μdu/dy). Therefore, the viscosity term (μdu/dy) may be neglected in case of turbulent flow.

5.3.1. Turbulence Viscosity

Shearing stress caused by turbulence effect in Equ. (5.10) can be written in the similar form as the viscous effect shearing stress as

$$-\rho\overline{u'v'} = \mu_T \frac{du}{dy} \tag{5.12}$$

Here μ_T is known as *turbulence viscosity*.