LECTURE NOTES - I

« FLUID MECHANICS »

Prof. Dr. Atıl BULU

Istanbul Technical University
College of Civil Engineering
Civil Engineering Department
Hydraulics Division

CHAPTER 1

FUNDAMENTALS

1.1. INTRODUCTION

Man's desire for knowledge of fluid phenomena began with his problems of water supply, irrigation, navigation, and waterpower.

Matter exists in two states; the solid and the fluid, the fluid state being commonly divided into the liquid and gaseous states. Solids differ from liquids and liquids from gases in the spacing and latitude of motion of their molecules, these variables being large in a gas, smaller in a liquid, and extremely small in a solid. Thus it follows that intermolecular cohesive forces are large in a solid, smaller in a liquid, and extremely small in a gas.

1.2. DIMENSIONS AND UNITS

Dimension = A dimension is the measure by which a physical variable is expressed quantitatively.

Unit = A unit is a particular way of attaching a number to the quantitative dimension.

Thus length is a dimension associated with such variables as distance, displacement, width, deflection, and height, while centimeters or meters are both numerical units for expressing length.

In fluid mechanics, there are only four primary dimensions from which all the dimensions can be derived: mass, length, time, and force. The brackets around a symbol like [M] mean "the dimension" of mass. All other variables in fluid mechanics can be expressed in terms of [M], [L], [T], and [F]. For example, acceleration has the dimensions [LT^{-2}]. Force [F] is directly related to mass, length, and time by Newton's second law,

$$\begin{aligned} F &= ma \\ Force &= Mass \times Acceleration \end{aligned} \qquad (1.1)$$

From this we see that, dimensionally, [F] = [MLT^{-2}].

1 kg-force = 9.81 Newton of force = 9.81 N

Primary Dimensions in SI and MKS Systems

Primary Dimension	MKS Units	SI Units
Force [F]	Kilogram (kg)	Newton ($N=kg.m/s^2$)
Mass [M]	$M=G/g = (kgsec^2/m)$	Kilogram
Length [L]	Meter (m)	Meter (m)
Time [T]	Second (sec)	Second (sec)

Secondary Dimensions in Fluid Mechanics

Secondary Dimension	MKS Units	SI Units
Area $[L^2]$	m^2	m^2
Volume $[L^3]$	m^3	m^3
Velocity $[LT^{-1}]$	m/sec	m/sec
Acceleration $[LT^{-2}]$	m/sec^2	m/sec^2
Pressure or stress $[FL^{-2}] = [ML^{-1}T^{-2}]$	kg/m^2	Pa= N/m^2(Pascal)
Angular Velocity $[T^{-1}]$	sec^{-1}	sec^{-1}
Energy, work $[FL] = [ML^2T^{-2}]$	kg.m	J = Nm (Joule)
Power $[FLT^{-1}] = [ML^2T^{-3}]$	kg.m/sec	W = J/sec (Watt)
Specific mass (ρ) $[ML^{-3}] = [FT^2L^{-4}]$	$kg.sec^2/m^4$	kg/m^3
Specific weight (γ) $[FL^{-3}] = [ML^{-2}T^{-2}]$	Kg/m^3	N/m^3

Specific mass = ρ = The mass, the amount of matter, contained in a volume. This will be expressed in mass-length-time dimensions, and will have the dimensions of mass [M] per unit volume $[L^3]$. Thus,

$$SpecificMass = \frac{Mass}{Volume}$$

$$[\rho]=\left[\frac{M}{L^3}\right]=\left[\frac{FT^2}{L^4}\right],\left(kg\sec^2/m^4\right)$$

Specific weight= γ = will be expressed in force-length-time dimensions and will have dimensions of force [F] per unit volume [L^3].

$$Specific weight=\frac{Weight}{Volume}$$

$$[\gamma]=\left[\frac{F}{L^3}\right]=\left[\frac{M}{L^2T^2}\right],\left(kg/m^3\right)$$

Because the weight (a force), W, related to its mass, M, by Newton's second law of motion in the form

$$W=Mg$$

In which g is the acceleration due to the local force of gravity, specific weight and specific mass will be related by a similar equation,

$$\gamma=\rho g \tag{1.2}$$

EXAMPLE 1.1: Specific weight of the water at 4°C temperature is γ = 1000 kg/m^3. What is its the specific mass?

SOLUTION:

$$\gamma=\rho g=1000\left(kg/m^3\right)$$

$$\rho=\frac{1000}{9.81}=101.94\left(kg\sec^2/m^4\right)$$

EXAMPLE 1.2: A body weighs 1000 kg when exposed to a standard earth gravity g = 9.81 m/sec^2. a) What is its mass? b) What will be the weight of the body be in Newton if it is exposed to the Moon's standard acceleration g_{moon} = 1.62 m/sec^2? c) How fast will the body accelerate if a net force of 100 kg is applied to it on the Moon or on the Earth?

SOLUTION:

a) Since,
$$W=mg=1000(kg)$$

$$M=\frac{W}{g}=\frac{1000}{9.81}=101.94\left(kg\sec^2/m^4\right)$$

b) The mass of the body remains 101.94 kgsec^2/m regardless of its location. Then,

$$W = mg = 101.94 \times 1.62 = 165.14(kg)$$

In Newtons,

$$165.14 \times 9.81 = 1620(Newton)$$

c) If we apply Newton's second law of motion,

$$F = ma = 100(kg)$$

$$a = \frac{100}{101.94} = 0.98\left(m/\sec^2\right)$$

This acceleration would be the same on the moon or earth or anywhere.

All theoretical equations in mechanics (and in other physical sciences) are *dimensionally homogeneous*, i.e.; each additive term in the equation has the same dimensions.

EXAMPLE 1.3: A useful theoretical equation for computing the relation between the pressure, velocity, and altitude in a steady flow of a nearly inviscid, nearly incompressible fluid is the Bernoulli relation, named after Daniel Bernoulli.

$$p_0 = p + \frac{1}{2}\rho V^2 + \rho g z$$

Where

p_0 = Stagnation pressure
p = Pressure in moving fluid
V = Velocity
ρ = Specific mass
z = Altitude
g = Gravitational acceleration

a) Show that the above equation satisfies the principle of dimensional homogeneity, which states that all additive terms in a physical equation must have the same dimensions. b) Show that consistent units result in MKS units.

SOLUTION:

a) We can express Bernoulli equation dimensionally using brackets by entering the dimensions of each term.

$$[p_0] = [p] + \frac{1}{2}\left[\rho V^2\right] + [\rho g z]$$

The factor ½ is a pure (dimensionless) number, and the exponent 2 is also dimensionless.

$$\left[FL^{-2}\right]=\left[FL^{-2}\right]+\left[FT^{2}L^{-4}\right]\left[L^{2}T^{-2}\right]+\left[FT^{2}L^{-4}\right]\left[LT^{-2}\right]\left[L\right]$$

$$\left[FL^{-2}\right]=\left[FL^{-2}\right]$$

For all terms.

b) If we enter MKS units for each quantity:

$$\left(kg/m^{2}\right)=\left(kg/m^{2}\right)+\left(kg\sec^{2}/m^{4}\right)\left(m^{2}/\sec^{2}\right)+\left(kg\sec^{2}/m^{4}\right)\left(m/\sec^{2}\right)\left(m\right)$$

$$=\left(kg/m^{2}\right)$$

Thus all terms in Bernoulli’s equation have units in kilograms per square meter when MKS units are used.

Many empirical formulas in the engineering literature, arising primarily from correlation of data, are dimensional inconsistent. Dimensionally inconsistent equations, though they abound in engineering practice, are misleading and vague and even dangerous, in the sense that they are often misused outside their range of applicability.

EXAMPLE 1.4: In 1890 Robert Manning proposed the following empirical formula for the average velocity V in uniform flow due to gravity down an open channel.

$$V=\frac{1}{n}R^{2/3}S^{1/2}$$

Where

R = Hydraulics radius of channel
S= Channel slope (tangent of angle that bottom makes with horizontal)
n = Manning’s roughness factor

And n is a constant for a given surface condition for the walls and bottom of the channel. Is Manning’s formula dimensionally consistent?

SOLUTION: Introduce dimensions for each term. The slope S, being a tangent or ratio, is dimensionless, denoted by unity or [$F^0L^0T^0$]. The above equation in dimensional form

$$\left[\frac{L}{T}\right]=\left[\frac{1}{n}\right]\left[L^{2/3}\right]\left[F^{0}L^{0}T^{0}\right]$$

This formula cannot be consistent unless [L/T] = [$L^{1/3}$/T]. In fact, Manning’s formula is inconsistent both dimensionally and physically and does not properly account for channel-roughness effects except in a narrow range of parameters, for water only.

Engineering results often are too small or too large for the common units, with too many zeros one way or the other. For example, to write F = 114000000 ton is long and awkward. Using the prefix "M" to mean 10^6, we convert this to a concise F = 114 Mton (megatons). Similarly, t = 0.000003 sec is a proofreader's nightmare compared to the equivalent t = 3 μsec (microseconds)

TABLE 1.1
CONVENIENT PREFIXES FOR ENGINEERING UNITS

Multiplicative Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1.3 MOLECULAR STRUCTURE OF MATERIALS

Solids, liquids and gases are all composed of molecules in continuous motion. However, the arrangement of these molecules, and the spaces between them, differ, giving rise to the characteristics properties of the three states of matter. In solids, the molecules are densely and regularly packed and movement is slight, each molecule being strained by its neighbors. In liquids, the structure is loser; individual molecules have greater freedom of movement and, although restrained to some degree by the surrounding molecules, can break away from the restraint, causing a change of structure. In gases, there is no formal structure, the spaces between molecules are large and the molecules can move freely.

In this book, fluids will be assumed to be continuous substances, and, when the behavior of a small element or particle of fluid is studied, it will be assumed that it contains so many molecules that it can be treated as part of this continuum. Quantities such as velocity and pressure can be considered to be constant at any point, and changes due to molecular motion may be ignored. Variations in such quantities can also be assumed to take place smoothly, from point to point.

1.4. COMPRESSIBILITY: BEHAVIOR OF FLUIDS AGAINST PRESSURE

For most purposes a liquid may be considered as incompressible. The compressibility of a liquid is expressed by its *bulk modulus of elasticity*. The mechanics of compression of a fluid may be demonstrated by imagining the cylinder and piston of Fig.1.1 to be perfectly rigid (inelastic) and to contain a volume of fluid V. Application of a force, F, to piston will increase the pressure, p, in the fluid and cause the volume decrease –dV. The bulk modulus of elasticity, E, for the volume V of a liquid

$$E = -\frac{dp}{dV/V} \tag{1.3}$$

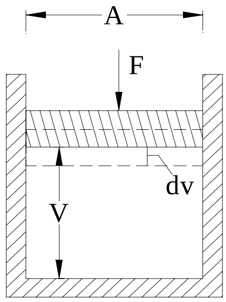

Fig. 1.1

Since dV/V is dimensionless, E is expressed in the units of pressure, p. For water at ordinary temperatures and pressures, $E = 2\times10^4$ kg/cm^2.

For liquids, the changes in pressure occurring in many fluid mechanics problems are not sufficiently great to cause appreciable changes in specific mass. It is, therefore, usual to ignore such changes and to treat liquids as incompressible.

ρ = Constant

1.5. VISCOSITY: BEHAVIOR OF FLUIDS AGAINST SHEAR STRESS

When real fluid motions are observed carefully, two basic types of motion are seen. The first is a smooth motion in which fluid elements or particles appear to slide over each other in layers or laminae; this motion is called *laminar* flow. The second distinct motion that occurs is characterized by a random or chaotic motion of individual particles; this motion is called *turbulent* flow.

Now consider the laminar motion of a real fluid along a solid boundary as in Fig. 1.2. Observations show that, while the fluid has a finite velocity, u, at any finite distance from the boundary, there is no velocity at the boundary. Thus, the velocity increases with increasing distance from the boundary. These facts are summarized on the *velocity profile*, which indicates relative motion between adjacent layers. Two such layers are showing having thickness dy, the lower layer moving with velocity u, the upper with velocity u+du. Two particles 1 and 2, starting on the same vertical line, move different distances d_1 = udt and d_2 = (u+du) dt in an infinitesimal time dt.

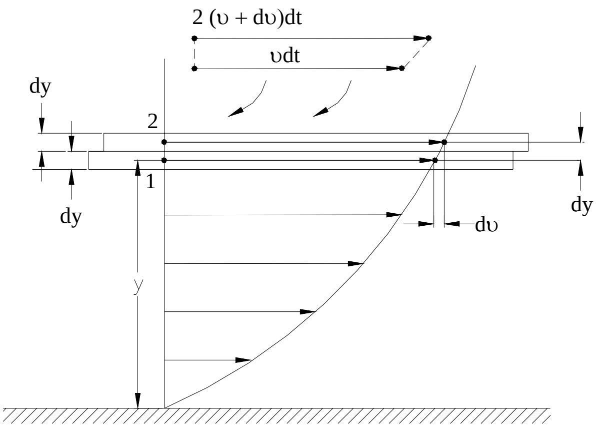

Fig. 1.2

It is evident that a frictional or shearing force must exist between the fluid layers; it may be expressed as a *shearing* or *frictional stress* per unit of contact area. This stress, designated by τ, has been found for laminar (nonturbulent) motion to be proportional to the velocity gradient, du/dy, with a constant of proportionality, μ, defined as *coefficient of viscosity* or *dynamic viscosity*. Thus,

$$\tau = \mu \frac{du}{dy} \tag{1.4}$$

All real fluids possess viscosity and therefore exhibit certain frictional phenomena when motion occurs. Viscosity results fundamentally from cohesion and molecular momentum exchange between fluid layers and, as flow occurs, these effects appear as tangential or shearing stresses between the moving layers. This equation is called as *Newton's law of viscosity*.

Because Equ. (1.4) is basic to all problems of fluid resistance, its implications and restrictions are to be emphasized:

1) The nonappearance of pressure in the equation shows that both τ and μ are independent of pressure, and that therefore fluid friction is different from that between moving solids, where plays a large part,
2) Any shear stress τ, however small, will cause flow because applied tangential forces must produce a velocity gradient, that is, relative motion between adjacent fluid layers,
3) Where du/dy = 0, τ = 0, regardless of the magnitude of μ, the shearing stress in viscous fluids at rest will be zero,
4) The velocity profile cannot be tangent to a solid boundary because this would require an infinite velocity gradient and infinite shearing stress between fluids and solids,
5) The equation is limited to nonturbulent (laminar) fluid motion, in which viscous action is strong.
6) The velocity at a solid boundary is zero, that is, there is no slip between fluid and solid for all fluids that can be treated as a continuum.

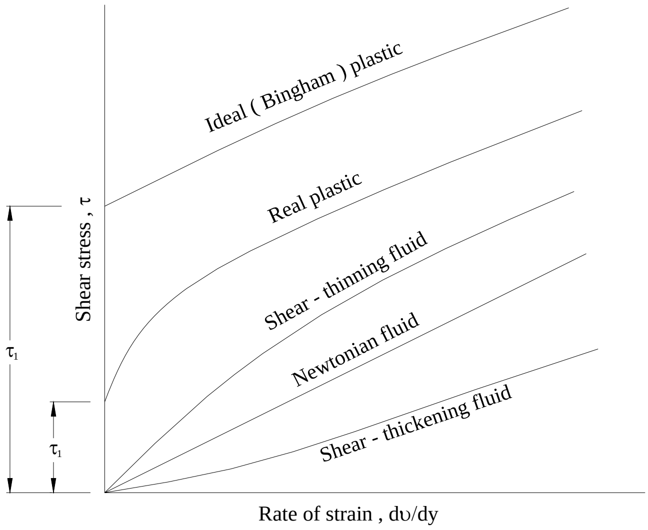

Fig. 1.3

Equ. (1.4) may be usefully visualized on the plot of Fig.1.3 on which μ is the slope of a straight line passing through the origin, here du will be considered as displacement per unit time and the velocity gradient du/dy as time of strain. Fluids that follow Newton's viscosity law are commonly known as *Newtonian* fluids. It is these fluids with which this book is concerned. Other fluids are classed as *non-Newtonian* fluids. The science of Rheology, which broadly is the study of the deformation and flow of matter, is concerned with plastics, blood, suspensions, paints, and foods, which flow but whose resistance is not characterized by Equ. (1.4).

The dimensions of the (dynamic) viscosity μ may be determined from dimensional homogeneity as follows:

$$[\mu] = \frac{[\tau]}{[du/dy]} = \frac{[FL^{-2}]}{[LT^{-1}L^{-1}]} = [FL^{-2}T] \quad , \ (kg.\sec/m^2)$$

In SI units, (Pa×sec). These combination times 10^{-1} is given the special name *poises*.

Viscosity varies widely with temperature. The shear stress and thus the viscosity of gases will increase with temperature. Liquid viscosities decrease as temperature rises.

Owing to the appearance of the ratio μ/ρ in many of the equations of the fluid flow, this term has been defined by,

$$\vartheta = \frac{\mu}{\rho} \tag{1.5}$$

in which υ is called the *kinematic viscosity*. Dimensional considerations of Equ. (1.5) shows the units of υ to be square meters per second, a combination of kinematic terms, which explains the name kinematic viscosity. The dimensional combination times 10^{-4} is known as *stokes*.

1.6.VAPOR PRESSURE AND CAVITATION

Vapor pressure is the pressure at which a liquid boils and is in equilibrium with its own vapor. For example, the vapor pressure of water at 10^0C is 0.125 t/m^2, and at 40^0C is 0.75 t/m^2. If the liquid pressure is greater than the vapor pressure, the only exchange between liquid and vapor is evaporation at the interface. If, however, the liquid pressure falls below the vapor pressure, vapor bubbles begin to appear in the liquid. When the liquid pressure is dropped below the vapor pressure due to the flow phenomenon, we call the process *cavitation*. Cavitation can cause serious problems, since the flow of liquid can sweep this cloud of bubbles on into an area of higher pressure where the bubbles will collapse suddenly. If this should occur in contact with a solid surface, very serious damage can result due to the very large force with which the liquid hits the surface. Cavitation can affect the performance of hydraulic machinery such as pumps, turbines and propellers, and the impact of collapsing bubbles can cause local erosion of metal surfaces.