Performance Analysis of Source Transmit Antenna Selection In Space Shift Keying With Cooperative Amplify-and-Forward Relaying

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Abstract—In this paper, we propose a cooperative relaying scheme combining space shift keying (SSK) and transmit antenna selection (TAS), and investigate its error performance. In this scheme, source TAS is performed based on the channel fading coefficients corresponding to source-destination, source-relay and relay-destination links. SSK is applied by using the selected antennas. Moreover, a single-antenna relay is used to amplify the transmitted signal from the source and forward it to the destination. Considerably tight upper-bound and lower-bound expressions for the bit error probability of the proposed SSK system are derived when the number of selected antennas is two. It is demonstrated that the proposed SSK scheme outperforms conventional cooperative SSK scheme without source TAS and also the cooperative SSK scheme in which a single-antenna decode-and-forward relay is used and source-destination channel-based TAS is adopted.

Index Terms—Index modulation, space shift keying, transmit antenna selection, amplify-and-forward.

I. INTRODUCTION

The idea of carrying information by the indices of a transmit antenna has attracted many researchers in recent years [1]–[5]. Space modulation techniques, namely spatial modulation (SM) [3] and space shift keying (SSK) [4], are proposed on the basis of this idea. In such techniques, a new spatial constellation diagram including the indices of transmit antennas is used. There is a one-to-one mapping between the incoming information bits and transmit antenna indices and therefore, typically only one transmit antenna is activated during a transmission interval in conventional SM and SSK. In this context, SM and SSK transmitters require only one radio-frequency (RF) chain. Such property carries space modulation techniques one step further than multiple-input multiple-output (MIMO) techniques, which require multiple transmit RF chains, in terms of system complexity and implementation cost. Moreover, chronic problems of conventional MIMO systems such as high inter-channel interference and the requirement for inter-antenna synchronization are entirely avoided by SM and SSK. Compared to SM, SSK further decreases the transceiver complexity without a degradation in error performance, since the mapping of the information bits to \(M\)-ary symbols is not performed [1]–[4].

On the other hand, conventional SM and SSK cannot provide transmit diversity gain. In this context, studies in recent years on the SM/SSK schemes indicate that cooperative relaying [6]–[10] and/or transmit antenna selection (TAS) [11]–[15] techniques provide diversity gain and improve the bit error probability (BEP) performance. In [6], cooperative decode-and-forward (DF) and amplify-and-forward (AF) relaying schemes with SSK have been investigated. In [7], cooperative DF and AF relaying schemes with SM, where all nodes are equipped with multiple transmit and/or receive antennas, have been studied. The authors of [8] have combined the SSK modulation with a cooperative AF relaying scheme, in which the best and partial relay selection techniques are applied. Novel cooperative spectrum sharing protocols that employ SM and space-time block coded SM at the secondary users have been proposed in [9] and [10], respectively. Furthermore, the authors of [11] and [12] have proposed low complexity TAS techniques for SM. In [13] and [14], novel TAS criteria have been proposed for SSK modulation and the error performance of the corresponding schemes have been investigated. In [15], the performance results of source transmit antenna selection (STAS) for SSK modulation with multiple DF relays has been reported.

In this paper, we propose a novel cooperative AF-SSK scheme that applies STAS. Our contributions are summarized as follows. We extend the scheme of [6], which is a conventional SSK scheme with cooperative AF relays, to a new AF-SSK scheme with STAS. We also generalize the end-to-end antenna selection criterion proposed in [13] to a cooperative relaying scenario in which the source-destination, source-relay and relay-destination links are considered. Our system model differs from the DF-SSK scheme [15] in the following aspects: (i) AF relaying is considered instead of DF relaying; (ii) STAS is performed considering both direct and relaying links unlike DF-SSK scheme in [15], which performs STAS considering only the direct link. We also show that the proposed AF-
SSK scheme outperforms the DF-SSK scheme [15] in terms of BEP. We derive upper-bound and lower-bound expressions for average BEP of the proposed system. Moreover, it is shown via computer simulations that our analytical results are consistent with the simulation results.

II. SYSTEM MODEL AND ANTENNA SELECTION CRITERION

A. System Model

We consider an AF-SSK system model with a source (S) equipped with \( N_t = 2m \) (\( m \geq 1 \) is a positive integer) transmit antennas, a single-antenna AF relay (R) and a single-antenna (\( N_r = 1 \)) destination as illustrated in Fig. 1. At S, \( N_s = 2 \) antennas are selected among \( N_t \) transmit antennas and SSK technique is applied by using the selected antennas. We assume that the direct link transmission between S and D exists and perfect channel state information (CSI) is available at D. Furthermore, there is an error-free feedback channel, which is used to convey the selected transmit antenna subset information from D to S.

Error probability of the SSK scheme depends only on the Euclidean distances among channel fading coefficients and the antenna subsets, which maximize the minimum Euclidean distances among the channel coefficients, are selected for TAS to improve the error performance. Since the channel fading coefficients in different Euclidean distances are common, these distances are statistically dependent for both SM and SSK. This important property complicates the mathematical tractability of error probability analysis belonging to TAS in SM and SSK. In SM, when the constellation size grows, the number of statistically dependent random variables increases dramatically compared to SSK. Therefore, the use of SSK instead of SM simplifies the mathematical analysis. Also, SM receiver decodes both the transmitted symbol and the transmit antenna index to detect the information bits. However, SSK receiver decodes only the transmit antenna index to detect the information bits. Hence, the use of SSK instead of SM decreases the transceiver complexity. Besides these advantages, SSK provides almost identical error performance with SM for the same data rates [4]. Due to the all of above, SSK modulation is applied at S.

Antenna selection is performed by considering the channel fading coefficients corresponding to the direct (S-D) and relaying links (S-R and R-D links) in order to maximize the error performance of the system. Moreover, to decrease the complexity as well as to simplify the mathematical analysis, we modify the antenna selection criterion proposed in [13] for a cooperative AF network. In this selection criterion, channel fading coefficients corresponding to available \( N_t \) transmit antennas are separated into \((N_t/N_s)\) disjoint subsets where we assume \( N_s = 2 \). D determines one of the subsets according to the Euclidean distances between the channel fading coefficients corresponding to the direct (S-D) and relaying links (S-R and R-D links). A detailed description of antenna selection criterion will be the subject of the next subsection.

The transmission occurs in two phases. In the first phase, SSK technique is applied at S by using the selected \( N_s = 2 \) transmit antennas, where an SSK signal is transmitted from S to R and D with an average energy of \( E_S \). Let \( \nu = 1, \ldots, N_t/2 \), \( \nu \) and \( l \in \{1, 2\} \) denote the subset indices, the selected subset and the active antenna index in the selected subset, respectively, the received signals at R and D can be expressed, respectively, as

\[
y^{SR} = \sqrt{E_S} h^{SR}_{\nu,l} + n^{SR},
\]

\[
y^{SD} = \sqrt{E_S} h^{SD}_{\nu,l} + n^{SD}
\]

where \( h^{SR}_{\nu,l} \) and \( h^{SD}_{\nu,l} \) denote the selected S-R and S-D channel fading coefficients, respectively, which belong to the \( \nu \)th transmit antenna in the selected subset and they are distributed with \( \mathcal{CN}(0,1) \). \( n^{SR} \) and \( n^{SD} \) are additive white Gaussian noise (AWGN) samples at R and D, respectively, which are distributed with \( \mathcal{CN}(0,N_0) \).

In the second phase, S remains silent and R forwards its received signal to D by following the AF protocol. The received signal at D can be given as

\[
y^{RD} = \sqrt{E_R} E_R A h^{RD}_{\nu,l} + \sqrt{E_R} A h^{RD}_{\nu,l} n^{RD} + n^{RD}
\]

where \( E_R \) is the transmitted energy of R and \( h^{RD}_{\nu,l} \) is the R-D channel fading coefficient which is distributed with \( \mathcal{CN}(0,1) \). Moreover, \( A = 1/\sqrt{E_S + N_0} \) is the fixed-gain amplification factor and \( n^{RD} \) is the AWGN sample which has the same distribution as \( n^{SD} \) and \( n^{SR} \). After the noise normalization, (3) can be rewritten as

\[
y^{RD} = \frac{y^{RD}}{\sqrt{\Omega}} = G h^{RD} h^{SR}_{\nu,l} + \tilde{n}^{RD}
\]

where \( \Omega = \frac{1}{E_R A^2 |h^{RD}_{\nu,l}|^2 + 1} \), \( G = \sqrt{E_R A^2 |h^{RD}_{\nu,l}|^2 + 1} \) and \( \tilde{n}^{RD} \) is the AWGN sample, which is distributed as \( \mathcal{CN}(0,N_0) \). Finally, D uses the maximum likelihood principle to estimate the active source transmit antenna index and using (2) and (4), the estimated antenna index can be expressed as

\[
i = \arg \min_{\nu \in \{1, 2\}, l \in \{1, 2\}} \left\{ \left| y^{SD} - \sqrt{E_S} h^{SD}_{\nu,l} \right|^2 + \left| y^{RD} - G h^{RD} h^{SR}_{\nu,l} \right|^2 \right\}.
\]
B. Antenna Selection Criterion

Pairwise error probability (PEP), $P(l \rightarrow \hat{l})$, is the probability of the error event at $D$, which corresponds to deciding $\hat{l}$ instead of $l$ as the index of active transmit antenna of $S$. PEP of AF-SSK systems can be expressed as $[6, 8]$,

$$ P(l \rightarrow \hat{l}) = E \left[ Q \left( \sqrt{\gamma^{\text{Eq}}} \right) \right] $$  \hspace{1cm} (6)

where $\gamma^{\text{Eq}} = \gamma^{SD}_t + \gamma^{SD}_r$, $\gamma^{SD}_t = \frac{E_s}{\frac{1}{2} N_0 |\hat{h}_t|^2 + \gamma}, \gamma^{SD}_r = \frac{E_s}{\frac{1}{2} N_0 |\hat{h}_r|^2 + \gamma}$, $\gamma^{RD}_s = \frac{E_s \gamma^{SD}_s}{N_0}$, $\gamma^{RD}_r = \frac{E_s \gamma^{SD}_r}{N_0}$, and $C = \frac{1}{N_0}$. Moreover, $Q(l) = \int_{l}^{\infty} (1/\sqrt{2\pi}) e^{-z^2/2} dz$ is the Gaussian $Q$ function. Here, the index $\nu$ stands for the any two-element ($N_s = 2$) subset of $N_t$ transmit antennas. Note that the PEP is equivalent to average bit error probability (BEP) when $N_s = 2$ for SSK systems. Hence, we use average BEP ($\bar{P}_b$) instead of PEP ($P(l \rightarrow \hat{l})$) in the rest of the paper.

Obviously, the optimal antenna selection scheme minimizes the PEP given in (6). However, such selection strategy searches for all $\binom{N_t}{2}$ antenna combinations and therefore, the complexity of the exhaustive antenna selection becomes prohibitively high for large values of $N_t$ and $N_s$. On the other hand, the antenna elements in each subset for the optimal antenna selection are common. Consequently, the squared Euclidean distances, which are the metrics of the antenna selection, become statistically dependent. It is considerably hard to provide exact expressions for the dependent random variables. As a result, in order to avoid high complexity and statistical dependency problems of the optimal antenna selection, we use the antenna selection criterion proposed in [13], which is used for the end-to-end SSK systems, and we generalize this selection strategy for the AF-SSK systems. In our selection scheme, available $N_t$ transmit antennas are divided into $N_t/2$ disjoint subsets as in [13]. The selected subset is the one that minimizes the PEP of the AF-SSK system, i.e.,

$$ \hat{\nu} = \arg \min_{\nu=1,\ldots,N_t/2} P(l \rightarrow \hat{l}) = \arg \max_{\nu=1,\ldots,N_t/2} \{ \gamma^{\text{Eq}} \}. $$  \hspace{1cm} (7)

Note that since we denote the selected subset as $\hat{\nu}$, the corresponding random variable is denoted by $\gamma^{\text{Eq}}$.

III. PERFORMANCE ANALYSIS

In this section, upper-bound and lower-bound expressions for the proposed AF-SSK system with STAS are derived. Since the complexity of an exact BEP expression for the proposed AF-SSK system becomes excessively high for the higher number of transmit antennas, even the numerical evaluation of such an expression is not easily solvable for the mathematical softwares. Hence, we propose upper-bound and lower-bound expressions in order to avoid the excessive complexity of the exact BEP expression.

A. Upper-Bound Expression For Bit Error Probability

In this subsection, we derive a tight upper-bound on the BEP performance of the AF-SSK by using the following inequality

$$ \gamma^{\text{Eq}} > \gamma^{lb} $$  \hspace{1cm} (8)

where $\gamma^{lb} = \max_{\nu=1,\ldots,N_t/2} \{ \gamma^{SD,SRD} \}$ and $\gamma^{SD,SRD} = \max \{ \gamma^{SD}, \gamma^{SRD} \}$. Note that although $\gamma^{lb}$ is a lower-bound for $\gamma^{\text{Eq}}$, the error probability corresponding to $\gamma^{lb}$ is an upper-bound for the error probability of the system due to the characteristic of the Gaussian $Q$ function in (6).

$\gamma^{SD}$ and $\gamma^{SR}$ are distributed with exponential distribution and their cumulative distribution functions (CDF) are given by

$$ F_{\gamma^{SD}}(x) = F_{\gamma^{SR}}(x) = 1 - e^{-\frac{x}{\bar{D}}} $$  \hspace{1cm} (9)

where $f_{\nu}(z) = \frac{e^{-\frac{z^2}{2\bar{D}}}}{\sqrt{2\pi} \bar{D}}$ is the probability density function of $\gamma^{RD}$ where $\gamma^{RD} = \frac{z}{\bar{D}}$. By substituting $f_{\nu}(z)$ into (10) and using [16, (3.471.9)], the CDF of $\gamma^{SD,SRD}$ ($F_{\gamma^{SD,SRD}}(x)$) can be given as

$$ F_{\gamma^{SD,SRD}}(x) = \int_{0}^{\infty} \left( 1 - e^{-\frac{1}{\sqrt{\bar{D}}}} x \right) \frac{e^{-\frac{x^2}{2\bar{D}}}}{\sqrt{2\pi} \bar{D}} dz $$

$$ = 1 - 2e^{\frac{1}{\sqrt{\bar{D}}}} \sqrt{\bar{D}} K_0 \left( \sqrt{\bar{D}}x \right) $$  \hspace{1cm} (11)

where $D = \frac{C}{N_0 R^{\nu}}$ and $K_0(u)$ is the 0th order modified Bessel function of the second kind [16, (8.432.1)].

Since $\gamma^{SD,SRD}$ is defined as $\gamma^{SD,SRD} = \max \{ \gamma^{SD}, \gamma^{SRD} \}$, the CDF of $\gamma^{SD,SRD}$ can be written with the help of order statistics as

$$ F_{\gamma^{SD,SRD}}(x) = F_{\gamma^{SD}}(x) F_{\gamma^{SRD}}(x) $$

$$ = \left( 1 - e^{-\frac{x}{\bar{D}}} \right) \left( 1 - 2 e^{\frac{1}{\sqrt{\bar{D}}}} \sqrt{\bar{D}} K_0 \left( \sqrt{\bar{D}}x \right) \right) $$  \hspace{1cm} (12)

On the other hand, $\gamma^{lb}$ is the maximum of $N_t/2$ $\gamma^{SD,SRD}$ random variables. Hence, the CDF of $\gamma^{lb}$ can be given as

$$ F_{\gamma^{lb}}(x) = \left[ F_{\gamma^{SD,SRD}}(x) \right]^{N_t/2} $$

$$ = \left( 1 - e^{-\frac{x}{\bar{D}}} \right) \left( 1 - 2 e^{\frac{1}{\sqrt{\bar{D}}}} \sqrt{\bar{D}} K_0 \left( \sqrt{\bar{D}}x \right) \right)^{N_t/2} $$  \hspace{1cm} (13)

The upper-bound on the average BEP of the system can be obtained, by using [17, (32)], as

$$ \bar{P}_b^{ub} = \frac{1}{2} \int_{0}^{\infty} F_{\gamma^{lb}}(x) dx $$

$$ = \frac{1}{2} \int_{0}^{\infty} \frac{1}{\sqrt{\bar{D}}} e^{-\frac{1}{\sqrt{\bar{D}}} x} F_{\gamma^{lb}}(x) dx $$  \hspace{1cm} (14)

By substituting (13) into (14) and evaluating the integrals numerically with the help of common mathematical softwares, the upper-bound expression $\bar{P}_b^{ub}$ can be calculated.
B. Lower-Bound Expression For Bit Error Probability

On the other hand, a lower-bound on the average BEP of the proposed system can be obtained by using the following inequality

\[ \gamma_{ub}^{E_S} \leq \gamma_{ub} \]

(15)

where \( \gamma_{ub}^{E_S} = \gamma_{SD}^{SRD} + \gamma_{SD}^{SRD} \), \( \gamma_{SD}^{SRD} = \max_{\nu = 1, \ldots, N_t/2} \{ \gamma_{SD}^{SRD} \} \) and \( \gamma_{SD}^{SRD} = \bar{x} - \nu \) is an upper-bound for \( \gamma_{ub}^{E_S} \), the error probability corresponding to \( \gamma_{ub}^{E_S} \) is a lower-bound for the error probability of the system.

Using the moment generating function (MGF) approach [18], the lower-bound on the average BEP is obtained as

\[ \bar{F}_b = \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{SD}} \left( \frac{1}{2 \sin^2 \theta} \right) \frac{1}{M_{\gamma_{SD}} \left( \frac{1}{2 \sin^2 \theta} \right)} d\theta \]

(16)

where \( M_{\gamma}(\cdot) \) is the MGF of \( \gamma \).

Since \( \gamma_{SD}^{SRD} \) is the maximum among \( N_t/2 \) random variables, the CDF of \( \gamma_{SD}^{SRD} \) can be given with the help of order statistics as

\[ F_{\gamma_{SD}}(x) = \left[ F_{\gamma_{SD}} \right]^{N_t/2} \left( 1 - e^{-x} \right)^{N_t/2} \]

(17)

Then, by using the binomial expansion, the MGF of \( \gamma_{SD}^{SRD} \) can be obtained as

\[ M_{\gamma_{SD}}(s) = s \sum_{k=0}^{N_t/2} \binom{N_t/2}{k} \left( -1 \right)^k \frac{1}{s + \frac{x}{2}} \]

(19)

Moreover, the CDF of \( \gamma_{SD}^{SRD} \) can be given with the help of order statistics as

\[ F_{\gamma_{SD}}(x) = \left[ F_{\gamma_{SD}} \right]^{N_t/2} \left( 1 - e^{-x} \right)^{N_t/2} \]

(20)

Then, the MGF of \( \gamma_{SD}^{SRD} \) can be given as

\[ M_{\gamma_{SD}}(s) = s \int_0^\infty e^{-sx} F_{\gamma_{SD}}(x) dx \]

\[ = s \int_0^\infty e^{-sx} \left[ 1 - 2x \tau_2 \sqrt{D_x K_1 \left( \sqrt{D_x} \right)} \right]^{N_t/2} dx \]

(21)

By substituting (19) and (21) into (16) and evaluating the integrals numerically with the help of common mathematical softwares, a lower-bound to \( \bar{F}_b \) can be calculated.

IV. NUMERICAL RESULTS

In this section, analytical upper-bound and lower-bound expressions derived in the previous section are validated through computer simulations. Bit error rate (BER) results are provided for the AF-SSK system with STAS. Moreover, we perform performance comparisons between the proposed SSK and the DF-SSK systems [15] in which STAS is applied considering only S-D link. Our results are plotted as a function of \( E_{tot}/N_0 \) where \( E_{tot} = E_S + E_R \). For simplicity, we assume \( E_S = E_R \).

In Fig. 2, the BER performance of the proposed AF-SSK system is given for \( N_t \in \{2, 4, 8\} \). Here, the case with \( N_t = 2 \) corresponds to the conventional cooperative AF-SSK system without TAS [6]. As seen from the figure, the system performance is improved in the presence of STAS and increasing the number of transmit antennas enhances the system performance remarkably. Furthermore, Fig. 2 clearly indicates that the analytical upper-bound and lower-bound results match with the simulation results and the derived analytical bounds are sufficiently tight for especially high SNR values.

Fig. 3 compares the BER performance of the proposed AF-SSK system with the DF-SSK system [15] for \( N_t \in \{2, 4, 6, 8\} \). In order to make fair comparisons, we assume that the AF-SSK and DF-SSK systems have the same number of available transmit antennas \( N_t \) at S and the number of selected and receive antennas at S and D, respectively, are chosen as \( N_s = 2 \) and \( N_r = 1 \) for both systems. As seen from Fig. 3, DF-SSK system outperforms the AF-SSK system when there is no STAS \( (N_t = 2) \); however, the effectiveness of the proposed AF-SSK system against the DF-SSK system is observed when STAS is applied and the AF-SSK system...
outperforms the DF-SSK system at high SNR values by more than 0.5 dB for $N_t \in \{4, 6, 8\}$. Such a result arises from the antenna selection strategy used in the AF-SSK system, which considers the direct (S-D) and relaying (S-R, R-D) links to select transmit antennas unlike the DF-SSK system, which considers only the direct (S-D) link to select transmit antennas. Hence, the complexity of the antenna selection strategy used in DF-SSK system is lower than the one used in the proposed AF-SSK system. As a result, we observe an interesting trade-off between the error performance and system complexity of the AF-SSK and DF-SSK systems.

V. CONCLUSION

In this paper, we have proposed an AF-SSK system with STAS and derived upper-bound and lower-bound expressions for the proposed system. It has been shown that the proposed SSK system outperforms the existing SSK system with cooperative AF relaying [6] and the DF-SSK system with STAS [15]. Furthermore, we have proven that there is an interesting trade-off between the proposed AF-SSK and DF-SSK [15] systems in terms of error performance and system complexity. Computer simulation results have been provided to verify the analytical results.

REFERENCES