

Multi-hop Decode-and-Forward STBC Spatial Modulation

Mohammadreza Babaei, Ertugrul Basar and Ümit Aygözü

Department of Electronics and Communications Eng.

Istanbul Technical University

34469, Maslak, Istanbul, Turkey

Emails: {babaei,basarer,aygolu}@itu.edu.tr

Abstract—In this paper, a multi-hop relaying system involving space-time block coded spatial modulation (STBC-SM) is proposed for MIMO networks. STBC-SM is applied at each node in which information bits are conveyed by both modulated symbols and antenna combinations. In this scheme, each relay node is equipped by N_t^R transmit and N_r^R receive antennas while the source and destination are equipped by N_t^S transmit and N_r^D receive antennas, respectively. Each node decodes the received STBC-SM signals from the previous node and forwards to the next node until the source's signals reach the destination. An upper bound on the bit error probability (BEP) for the multi-hop STBC-SM system is analytically derived and supported via computer simulations. Theoretical and simulation results show that the proposed system significantly improves the BEP performance compared to the equivalent reference schemes given in the literature.

Index Terms—spatial modulation, space-time coding, multi-hop relaying.

I. INTRODUCTION

The increasing demand for high data rates, improved reliability and capacity have led to the application multiple-input multiple-output (MIMO) transmission techniques in wireless systems. Spatial multiplexing and space-time block coding (STBC) have been proposed as two main techniques to improve the spectral efficiency and error performance of MIMO transmission, respectively. Spatial multiplexing systems such as vertical-Bell Laboratories layered space-time (V-BLAST) [1], divides the input stream into several sub-streams and all sub-streams are transmitted simultaneously, each one via a different antenna. This transmission technique increases the capacity, however, it causes a high level of inter-channel interference (ICI) at the receive antennas and considerably increases the decoding complexity. On the other hand, STBC, which provides diversity and coding gains, is also favorable due to its low decoding complexity and implementation simplicity in comparison to V-BLAST [2]. Both methods experience several limitations such as ICI and inter antenna synchronization (IAS) due to simultaneous data transmission.

Spatial modulation (SM) is a relatively new modulation technique proposed for MIMO communication systems [3], [4]. SM extends two dimensional signal constellations to the third dimension, which is the spatial dimension. Therefore,

information bits are conveyed by both modulated symbols and antenna indices. In this technique, ICI is removed since only one antenna among the available transmit antennas is active during each signaling interval. As a result, SM helps to totally overcome IAS, while only one radio frequency (RF) chain is required at the transmitter. Space shift keying (SSK) modulation introduced in [5] is a special case of SM for which the information bits are conveyed only by antenna indices. This simplifies the system design and decreases the decoding complexity. Similar to SM, ICI is completely eliminated since again only one antenna is active during each signaling interval. However, neither SM nor SSK do not have the potential to exploit transmit diversity. A transmission scheme, which combines SM and STBC for MIMO systems, is proposed in [6] as space-time block coded spatial modulation (STBC-SM) by taking the advantages of both STBC and SM while avoiding their drawbacks. In STBC-SM, information bits are conveyed in addition to the modulated symbols by antenna combinations in which only two antennas are activated during each signaling interval. STBC-SM provides transmit diversity with low complexity due to single symbol decoding property of Alamouti's STBC and significantly improves the system bit error probability (BEP) performance.

Relays are commonly considered in the practical setups to improve the performance of wireless communication systems. On one hand, for non-cooperative relaying systems, relay can be considered as a solution to shorten the distance between source (S) and destination (D). For example, a system that utilizes a relay between S and D can be assumed as a satellite where the relay forwards the uplink carrier to D. Furthermore, in multihop relaying, relays can be used along the transmission route to forward the source signal to D by enabling greater coverage for the wireless system without increasing the source's transmission power [7]–[14]. On the other hand, for cooperative communication systems, D receives signals from both the direct link between S and D and from the relay. As a result, this approach provides diversity and improves the system reliability. As in classical cooperative systems, amplify-and-forward (AF) and decode-and-forward (DF) are two most common techniques used in multihop relaying schemes. In the former, each intermediate node forwards the amplified noisy version of the signal received from the previous node, to the next relay until it reaches D. In the latter, relays forward to

the next node, the decoded version of the signal received from the previous node until it reaches D.

The end-to-end BEP performance and outage analysis of dual-hop DF (DH-DF) transmission system are performed in [7] where it is shown that DF relaying technique outperforms AF relaying technique. In [8], the performances of conventional and cooperative multi-hop transmissions are compared where each node applies DF or AF relaying to the received signal and transmits it to the next node until it reaches D. A multi-branch multi-hop scheme with DF where each relay forwards only when it correctly decodes, is analyzed in [9]. In [10], a cooperative multi-branch multi-hop scheme is considered in which S and D communicate through direct link and also by relays. Each relay node sends the signal to the next node only if it is correctly decoded.

In relaying systems, spectral efficiency is reduced by the number of transmission phases. For example, in the non-cooperative DH case, data rate is halved due to the fact that two time slots are required for the signal to reach D from S by the intervention of a relay. Different protocols for SM and SSK are considered in [11]–[13] to partially mitigate the decrease in spectral efficiency of the multi-hop systems thanks to the extra data transmission by antenna indices. Consequently, a lower modulation order can be used by applying SM or STBC-SM which convey information bits by antenna indices or antenna combinations in addition to the modulated symbols. Therefore, SM or STBC-SM can be used in multi-hop MIMO networks to increase the data rate and decrease the multi-hop burden. In [11], a DH-SSK scheme with AF relaying is proposed and the BER performance is investigated for correlated and uncorrelated channels with no direct link between S and D. A hybrid SM for DH scheme is studied in [12] for the case of imperfect channel knowledge at the transmitter. In [13], two different SSK-DF multi-hop MIMO network structures are proposed. In the first and second one, multi-hop diversity and multi-branch multi-hop with direct link between S and D are considered, respectively. In both schemes, each relay decodes the received signal and forwards when it is correctly decoded. In [14], DH-SM with DF relaying is introduced where SM is applied to transmit the signal in each node. Significant BER performance improvements and an increase in the system rate are provided for the proposed protocol.

In this paper, the application of STBC-SM as a possible relaying technique to multi-hop wireless networks, is proposed. STBC-SM provides diversity and increases the data rate as well as it avoids the interference at receive antennas due to the single symbol decoding property of Alamouti's STBC [2]. In the considered system, it is assumed that each relay node broadcasts the STBC-SM signal to the next node and each node only receives, the signal from the previous one. All intermediate nodes including D apply the maximum-likelihood (ML) decoding to estimate STBC-SM symbols. An upper bound on the BEP of the proposed multi-hop STBC-SM system is analytically derived for DF relaying technique and supported via computer simulations. Theoretical and simulation results show that the proposed system significantly

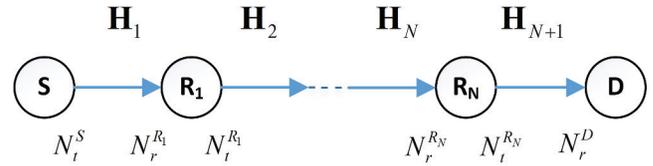


Fig. 1. Considered multi-hop relaying scheme.

improves the BEP performance compared to the reference schemes given in the literature.

The rest of the paper is organized as follows. The system model is presented in Section II and the BEP analysis is performed in Section III. Section IV deals with the performance results and comparisons. Finally, Section V concludes the paper.

II. SYSTEM MODEL AND PROTOCOL DESCRIPTION

The considered multi-hop relaying system is shown in Fig. 1, which consists of a single S, a single D and intermediate relay nodes R_i where $i = 1, 2, \dots, N$ and N being the number of relays. S and D are linked to each other with N relays and communicate through $N + 1$ hops in the lack of a direct link between them. S and all relays have N_t^S and N_t^R transmit antennas, and all relays and D have N_r^R and N_r^D receive antennas, respectively. In Fig. 1, \mathbf{H}_j for $j = 1, 2, \dots, N + 1$, stand for the channel fading coefficient matrices for each hop whose entries are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables denoted by $\mathcal{CN}(0, d^{-v})$, where d is the link distance and v is the path-loss coefficient. Furthermore, it is assumed that the channel state information (CSI) is perfectly known at all receivers where noise components are assumed to be the i.i.d. samples of additive white Gaussian noise (AWGN) process obeying $\mathcal{CN}(0, N_0)$, where $N_0/2$ is the variance per dimension. In this paper, bold capital and lower case letters denote matrices and vectors, respectively. $\|\cdot\|$, $(\cdot)^*$ and $(\cdot)^H$ represent Frobenius norm, complex conjugate, and Hermitian transpose operations, respectively. Transmit power taken equal for each node is denoted by ρ and M denotes the modulation order.

In this paper, the STBC-SM concept is applied to a multi-hop relaying system. STBC-SM uses the advantages of both STBC and SM to provide diversity and to increase the spectral efficiency. At each node, only two antennas are active during each signaling interval where Alamouti's STBC [2] is applied to broadcast the SM signal pairs as given by

$$\mathbf{X} = (\mathbf{c}_1 \quad \mathbf{c}_2) = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \quad (1)$$

where x_1 and x_2 represent complex transmitted symbols drawn from M -PSK or M -QAM signal constellations. In (1), columns and rows represent transmit antennas and time slots, respectively. As an example, based on [6], if we assume that $N_t^S = N_t^R = 4$ then, each node transmits the estimated data

from the previous node by one of the following four antenna combinations given as

$$\begin{aligned} \{\mathbf{X}_{11}, \mathbf{X}_{12}\} &= \left\{ \begin{pmatrix} \mathbf{X} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \mathbf{X} \end{pmatrix}, \begin{pmatrix} \mathbf{0}_{2 \times 2} & \mathbf{X} \\ \mathbf{X} & \mathbf{0}_{2 \times 2} \end{pmatrix} \right\}, \\ \{\mathbf{X}_{21}, \mathbf{X}_{22}\} &= \left\{ \begin{pmatrix} \mathbf{0}_{2 \times 1} & \mathbf{X} & \mathbf{0}_{2 \times 1} \\ \mathbf{c}_2 & \mathbf{0}_{2 \times 2} & \mathbf{c}_1 \end{pmatrix} \right\} \varphi \quad (2) \end{aligned}$$

where $\varphi = e^{j\theta}$, $\mathbf{X}_{i,j}$ for $i, j = 1, 2$ are the corresponding codewords for the data blocks 00, 01, 10, 11 assigned to antenna combinations and $\mathbf{0}_{a \times b}$ is the $a \times b$ all-zero matrix. Minimum coding gains distance (CGD) is calculated as a function of θ to maximize the diversity and coding gain [6, Eqs. (5),(9)]. Optimum θ values for BPSK and 4-QAM are found as equal to 1.57 and 0.61 radians in [6], respectively.

Transmission over each link consists of two time slots and the receiver of each link tries to decode and forward STBC-SM signal pairs. The received signal matrix at node i is given as

$$\mathbf{Y}_i = \sqrt{\frac{\rho}{\mu}} \mathbf{X}_x \mathbf{H}_i + \mathbf{N}_i \quad (3)$$

where \mathbf{Y}_i stands for $2 \times N_r^R$ received signal matrix, \mathbf{H}_i and \mathbf{N}_i denote the $N_t^R \times N_r^R$ channel and $2 \times N_r^R$ AWGN noise matrices for the i th link, respectively. \mathbf{X}_x is the transmitted codeword from the transmitting node of the i th link, taken from (2) and $\mu = 2$ is the normalization factor of the transmit antenna's power. Using Alamouti concept, the decoder at each node can extract the embedded information from (3) and estimate the received signals where more details are given in [6].

III. PERFORMANCE ANALYSIS

In this section, theoretical upper bound for the BEP of multi-hop STBC-SM system in which $2m = \log_2 4M^2$ information bits are transmitted by $\omega = 2^{2m}$ different STBC-SM transmission matrices at each link, is derived. An upper bound on the BEP for the i th link (hop) of the proposed STBC-SM system is calculated by the well-known union bound [15] as

$$P_{b,i} \leq \frac{1}{\omega} \sum_{k=1}^{\omega} \sum_{l=1, k \neq l}^{\omega} \frac{P(\mathbf{X}_k \rightarrow \mathbf{X}_l) n_{k,l}}{2m} \quad (4)$$

where $i = 1, 2, \dots, N+1$, $n_{k,l}$ is the number of bit errors between the transmitted matrix \mathbf{X}_k and the erroneously decided matrix \mathbf{X}_l and, $P(\mathbf{X}_k \rightarrow \mathbf{X}_l)$ is the pairwise error probability (PEP) between these two matrices. The conditional PEP (CPEP) is calculated as

$$P(\mathbf{X}_k \rightarrow \mathbf{X}_l | \mathbf{H}) = Q \left(\sqrt{\frac{\rho}{2}} \|\mathbf{X}_k - \mathbf{X}_l\| \mathbf{H} \right) \quad (5)$$

where $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty \exp(-y^2/2) dy$. Using Craig formula from [15], (5) can be rewritten as

$$P(\mathbf{X}_k \rightarrow \mathbf{X}_l | \mathbf{H}) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{\rho\gamma}{4\sin^2\phi}\right) d\phi \quad (6)$$

where $\gamma = \|(\mathbf{X}_k - \mathbf{X}_l)\mathbf{H}\|^2$. Averaging over the pdf of SNR, PEP can be expressed as

$$P(\mathbf{X}_k \rightarrow \mathbf{X}_l) = \frac{1}{\pi} \int_0^{\pi/2} M_\gamma\left(-\frac{\rho}{4\sin^2\phi}\right) d\phi \quad (7)$$

where $M_\gamma(\cdot)$ is the moment generating function (MGF) of γ . Considering the quadratic form of γ as $\gamma = \sum_{k=1}^{N_r^R} \mathbf{h}_k^H \mathbf{Q} \mathbf{h}_k$ with $\mathbf{Q} = (\mathbf{X}_k - \mathbf{X}_l)^H (\mathbf{X}_k - \mathbf{X}_l)$ and $E\{\mathbf{h}_k \mathbf{h}_k^H\} = \mathbf{I}_{N_t^S}$ where \mathbf{h}_k is the k th column of \mathbf{H} and $\mathbf{I}_{N_t^S}$ is the unit matrix of dimension N_t^S , MGF of γ can be expressed and further simplified as [16]

$$M_\gamma(t) = \prod_{i=1}^R (1 - t\lambda_{k,l,i})^{-N_r^R} \quad (8)$$

where $\lambda_{k,l,i}$'s are the eigenvalues of the matrix \mathbf{Q} , $R = \text{rank}(\mathbf{Q})$ with $R = 2$ for the STBC-SM system of [6]. Substituting (8) in (7) PEP between \mathbf{X}_k and \mathbf{X}_l can be calculated as

$$P(\mathbf{X}_k \rightarrow \mathbf{X}_l) = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{1}{1+\kappa} \cdot \frac{1}{1+\beta} \right)^{N_r^R} d\phi \quad (9)$$

where $\kappa = (\rho\lambda_{k,l,1}/4\sin^2\phi)$ and $\beta = (\rho\lambda_{k,l,2}/4\sin^2\phi)$.

Theoretical upper bound on the BEP of the i th hop can be obtained by substituting (9) in (4). Finally, by assuming that an error at any one of the receiving nodes will cause an error at D, the end-to-end BEP upper bound for multi-hop STBC-SM system can be expressed as

$$P_b \leq 1 - \prod_{i=1}^{N+1} (1 - P_{b,i}) \quad (10)$$

where $P_{b,i}$ is given in (4).

IV. PERFORMANCE EVALUATION

In this section, the theoretical and simulation results for the bit error rate (BER) performance of the proposed multi-hop STBC-SM system are presented and compared for different number of transmit antennas and data rates. Comparisons for the DH case are also provided with the reference systems applying SM [14] (conventional DH-SM), classical Alamouti's STBC (DH-Alamouti) and DF with classical M -QAM (conventional DH-DF) at relay. The conventional SM system uses the optimal decoder given in [4]. In Alamouti's STBC, two transmit antennas are used at S and each relay to transmit an STBC signal pair. In conventional DH-DF, relay employs maximum ratio combining (MRC) to decode the received signals and forward it to D. Moreover, comparisons for 2, 3, 5 and 7 bits/s/Hz values of the spectral efficiency with the reference systems are also provided. BER curves are depicted as a function of SNR. In each figure, $(N_t^S, N_r^R, N_t^R, N_r^D)$ represents the numbers of transmit antennas at S, receive antennas at the relays, transmit antennas at the relays and receive antennas at D, respectively. Especially, results are provided for the case of $N_t^S = N_t^R$ and $N_r^R = N_r^D$ and with $d = 1$ for all links. The path-loss coefficient is taken equal to 4. In addition, all links are assumed equidistant for

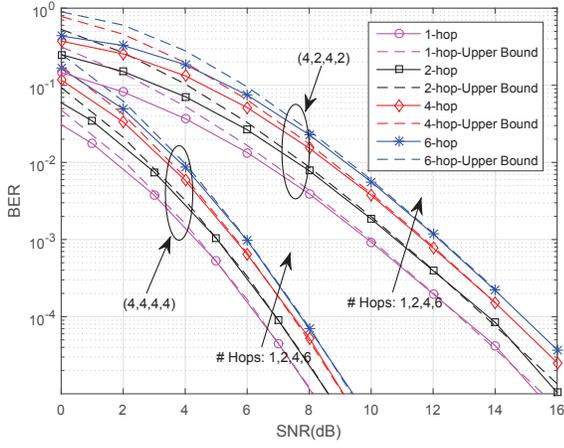


Fig. 2. Multi-hop BER performances of 2 bits/s/Hz STBC-SM.

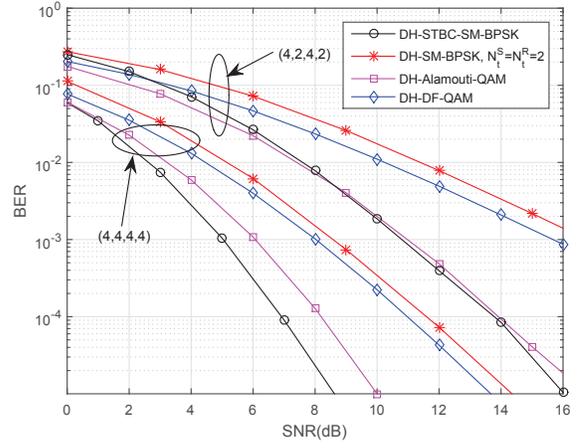


Fig. 4. DH BER performances of 2 bits/s/Hz per link.

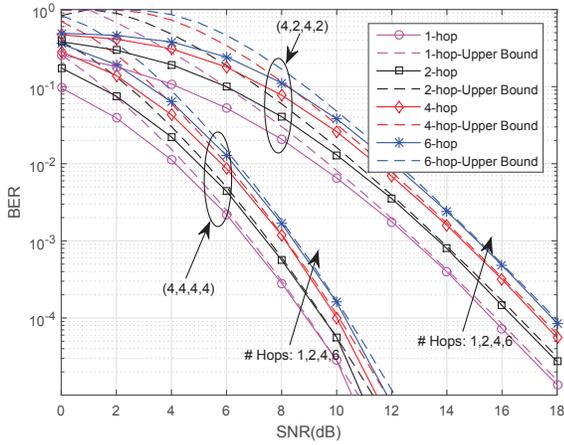


Fig. 3. Multi-hop BER performances of 3 bits/s/Hz STBC-SM.

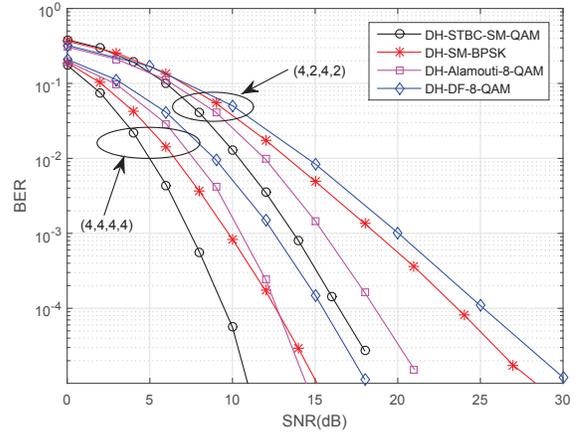


Fig. 5. DH BER performances for 3 bits/s/Hz per link.

the proposed and reference schemes and the average SNR is taken equal for all links. Markers with solid lines in Figs. 2 and 3 represent the simulation results and dashed lines are for the theoretical upper bounds. Gray coding is applied for all considered signal constellations.

A. Multi-Hop Transmission Performance Results

Figs. 2 and 3 show the simulation results and the theoretical upper bounds when BPSK and 4-QAM modulations are employed to achieve at 2 and 3 bits/s/Hz data rates per link, respectively. The simulation results are given for $N_t^S = N_t^R = 4$ and for varying numbers of receive antennas at relay and D, and compared with the analytical results of the previous section. It can be seen that for (4,2,4,2) and (4,4,4,4) cases in each figure, we have an approximately 0.5 dB loss in SNR for each increase in the number of hops between S and D. Moreover, from Figs. 2 and 3 it can be concluded that the simulation curves and theoretical upper bounds are in perfect match for the proposed protocol in the high SNR region for

all cases, which validates the derived BEP upper bound of the proposed multi-hop STBC-SM system.

B. DH comparisons with SM, DF and Alamouti's STBC

In Fig. 4, simulation results for the proposed DH-STBC-SM with BPSK modulation are provided. For 2 bits/s/Hz transmission per link, we compare our system with conventional DH-SM for $N_t^S = N_t^R = 2$ and BPSK modulation, DH-Alamouti and conventional DH-DF with 4-QAM. As seen from Fig. 4, we have 5, 1 and 4 dB SNR gains for (4,4,4,4) case and nearly 5, 0 and 4 dB SNR gains for (4,2,4,2) case compared to the conventional DH-SM, DH-Alamouti and conventional DH-DF reference schemes, respectively.

Fig. 5 compares the BER performance of DH-STBC-SM with 4-QAM, conventional DH-SM with BPSK, DH-Alamouti and conventional DH-DF with 8-QAM for 3 bits/s/Hz per link. The average end-to-end spectral efficiency from S to D is 1.5 bits/s/Hz for all systems. It is observed that we have SNR gains of 4, 3 and 8 dB in (4,4,4,4) case and 9, 3 and 11 dB in (4,2,4,2) case over conventional DH-SM, DH-Alamouti,

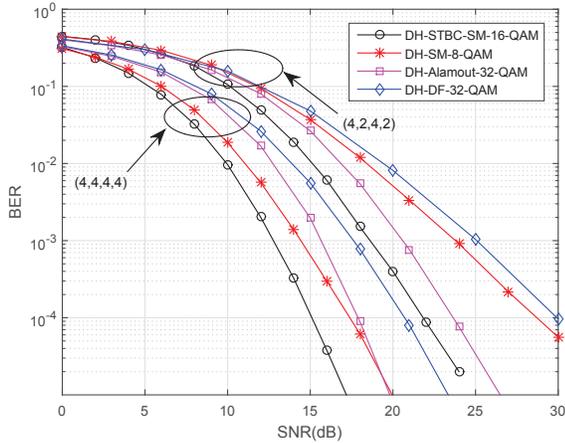


Fig. 6. DH BER performances for 5 bits/s/Hz per link.

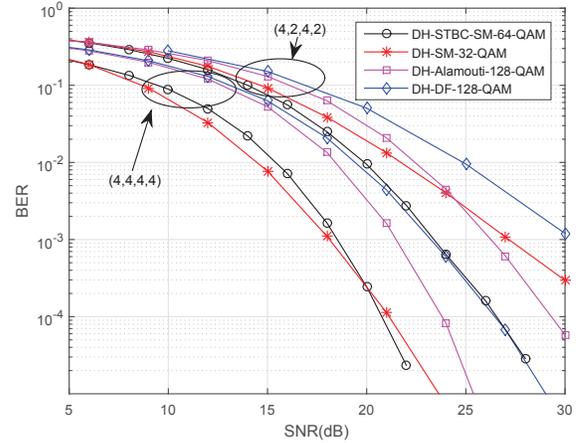


Fig. 7. DH BER performances for 7 bits/s/Hz per link.

conventional DH-DF, at a BER of 10^{-3} , respectively. By decreasing the number of antennas from four to two at the receivers, we have nearly 7 dB SNR loss for our protocol. Moreover, the gap between DH-STBC-SM and conventional DH-SM is increased by decreasing the number of receive antennas due to the fact that the performance of SM is mostly dependent on the receive antennas.

In Fig. 6, the BER performance for the proposed DH-STBC-SM 16-QAM scheme is compared with the corresponding reference schemes. As seen from Fig. 6, at a BER of 10^{-4} for DH-STBC-SM with 16-QAM, we have nearly 2, 2, 6 dB SNR gains over conventional DH-SM with 8-QAM, DH-Alamouti and conventional DH-DF with both 32-QAM for the (4,4,4,4) case and 4, 1 and 5 dB SNR gains for (4,2,4,2) case, respectively.

In Fig. 7, the simulation studies are extended to 7 bits/s/Hz transmission per link for the DH-STBC-SM with 64-QAM. In this case, a comparable, 3 and 6 dB SNR gains are provided at a BER value of 10^{-4} for the (4,4,4,4) case and 4, 3, 7 dB SNR gains at a BER value of 10^{-3} for the (4,2,4,2) case over the conventional DH-SM system with 32-QAM, DH-Alamouti and conventional DH-DF both with 128-QAM, respectively.

V. CONCLUSION

In this paper, a new system, which applies the STBC-SM to multi-hop MIMO networks, has been introduced. The theoretical upper bound for the proposed scheme has been analytically derived and the results have been supported via computer simulations for different number of transmit/receive antennas and data rates. The performance results have been compared with reference systems such as conventional SM, classical Alamouti and conventional DH-DF schemes for different spectral efficiencies and varying numbers of transmit and receive antennas. Significant improvements for the BER performance have been provided compared with the reference systems at different data rates.

REFERENCES

- [1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.
- [2] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.
- [3] R. Y. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228–2241, July 2008.
- [4] J. Jeganathan, A. Ghrayeb, and L. Szczecinski, "Spatial modulation: Optimal detection and performance analysis," *IEEE Commun. Lett.*, vol. 12, no. 8, pp. 545–547, Aug 2008.
- [5] J. Jeganathan, A. Ghrayeb, L. Szczecinski, and A. Ceron, "Space shift keying modulation for MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3692–3703, July 2009.
- [6] E. Basar, Ü. Aygözü, E. Panayirci, and H. V. Poor, "Space-time block coded spatial modulation," *IEEE Trans. Commun.*, vol. 59, no. 3, pp. 823–832, Mar. 2011.
- [7] M. O. Hasna and M. S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov 2003.
- [8] J. Zhang and T. M. Lok, "Performance comparison of conventional and cooperative multihop transmission," in *IEEE Wireless Commun. and Networking Conf.*, 2006. WCNC 2006., vol. 2, April 2006, pp. 897–901.
- [9] S. Amara and H. Boujemaa, "Multihop multibranch DF relaying for cooperative systems," in *Veh. Technol. Conf. (VTC Spring), 2011 IEEE 73rd*, May 2011, pp. 1–5.
- [10] A. Ribeiro, X. Cai, and G. B. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 1264–1273, May 2005.
- [11] R. Mesleh, S. S. Ikki, and M. Alwakeel, "On the performance of dual-hop space shift keying with single amplify-and-forward relay," in *2012 IEEE Wireless Commun. and Networking Conf. (WCNC)*, April 2012, pp. 776–780.
- [12] A. Stavridis, S. Sinanovi, M. D. Renzo, and H. Haas, "A power saving dual-hop architecture based on hybrid spatial modulation," in *2012 Conf. Record of the Forty Sixth Asilomar Conf. on Signals, Systems and Computers (ASILOMAR)*, Nov 2012, pp. 1366–1370.
- [13] P. Som and A. Chockalingam, "Performance analysis of space-shift keying in decode-and-forward multihop MIMO networks," *IEEE Trans. Veh. Technol.*, vol. 64, no. 1, pp. 132–146, Jan 2015.
- [14] N. Serafimovski, S. Sinanovic, M. D. Renzo, and H. Haas, "Dual-hop spatial modulation (Dh-SM)," in *Veh. Technol. Conf. (VTC Spring), 2011 IEEE 73rd*, May 2011, pp. 1–5.
- [15] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*. John Wiley & Sons, 2005, vol. 95.
- [16] G. L. Turin, "The characteristic function of hermitian quadratic forms in complex normal variables," *Biometrika*, vol. 47, no. 1/2, pp. 199–201, 1960. [Online]. Available: <http://www.jstor.org/stable/2332977>