Multi-Hop Space Shift Keying with Path Selection

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Abstract—In this paper, we propose a multi-hop multi-branch multiple-input multiple-output (MIMO) SSK scheme with path selection and investigate its error performance. In this scheme, the best path is selected among multiple branches and a multiple-antenna source (S) communicates with a multiple-antenna destination (D) via the relays of the selected path. Each relay is equipped with multiple transmit and receive antennas. It is assumed that there is no direct link between S and D. Moreover, S and all relays employ SSK modulation to transmit information bits and each relay in each path follows the decode-and-forward principle. A closed-form approximate symbol error rate (SER) expression for the proposed SSK system is derived. Furthermore, an asymptotic SER performance analysis is also performed. The analytical results are verified through Monte Carlo simulations. It is shown that the proposed multi-hop SSK system with path selection outperforms conventional multi-hop S/PSK system with path selection in terms of the SER performance for especially high data rates and sufficient number of receive antennas at the receiving nodes.

Index Terms—Space shift keying, multi-hop relaying, decode-and-forward, path selection.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless communication systems offer significant advantages including improved error performance, high data rates and capacity. However, these advantages bring with it costs such as deployment of multiple transmit radio-frequency (RF) chains, which increases the inter-channel interference (ICI) and the transceiver complexity, and requirement for inter-antenna synchronization (IAS). Promising spatial modulation (SM) [1] and space shift keying (SSK) [2] techniques are proposed as an alternative to traditional MIMO systems to compensate such costs of these systems. In SM and SSK, due to the one-to-one mapping between transmit antenna indices and information bits, only one transmit antenna is activated in a transmission interval and the others remain silent. Hence, SM and SSK ideally require only one transmit RF chain and therefore, ICI is completely avoided, the requirement for IAS is eliminated and transceiver complexity is considerably reduced [1]–[4]. Compared to SM, SSK modulation further decreases the transceiver complexity since I/Q modulation is not performed [2].

On the other hand, cooperative relaying improves the error performance by mitigating the effect of fading efficiently. Furthermore, the coverage is extended as well as the transmit power is reduced by cooperative relaying networks. In recent years, the advantages of both SM/SSK and cooperative relaying networks have attracted the attention of the many researchers and therefore, the concept of SM/SSK have been considered in cooperative relaying networks by many studies [5]–[10]. In [5], the authors investigate bit error rate (BER) performance of the cooperative amplify-and-forward (AF) and decode-and-forward (DF) relaying schemes with SSK. The performance of cooperative AF relaying with SM is studied in [6]. Moreover, in [7], SSK modulation with transmit antenna selection and cooperative DF relays is studied. The authors of [8] analyze the BER performance of a cooperative DF-SSK scheme with relay selection. In [9], a distributed SM protocol, in which the index of the relay conveys information, is proposed. An AF relaying-aided cooperative space-time SSK scheme is proposed in [10].

On the other hand, one of the key performance criteria for wireless networks is the energy efficiency, since the user nodes have limited battery lives. From this aspect, multi-hop transmission further extends the coverage of the wireless networks and therefore, decreases the required transmit power and improves the transmission reliability when especially the transmitter and receiver are away from each other [11]–[13]. Furthermore, in multi-hop multi-branch networks, the signal transmitted from the source reaches to the destination via multiple cooperative multi-hop branches and the destination receives different copies of the source’s transmitted signal from independent branches. Hence, in addition to the advantages of multi-hop relaying, cooperative diversity is achieved in multi-hop schemes [14]–[18]. Considering the advantages of multi-hop and multi-branch networks as well as the SM/SSK techniques, it is important to combine them to further improve the system performance. However, studies on SM/SSK with multi-hop and multi-branch networks are very limited. In the comprehensive study of [19], the performance of multi-hop diversity and multi-hop multi-branch networks for SSK with DF relays are investigated.

In this paper, we propose a multi-hop MIMO DF-SSK scheme in which the path selection is performed. Our contributions are summarized as follows. A novel MIMO scheme combining multi-hop relaying and SSK modulation is proposed. Our system model differs from that of [19] in the following aspects: First, we consider the error propagation in multi-hop DF relaying. Second, a path is selected among available branches and transmission occurs via the selected path instead of activating all of the branches. Our system
model is inspired by the path selection scheme in multi-hop DF protocol adopted in [16] and [20]. These works have analyzed the performance for conventional $M$-PSK modulation; however, we consider the SSK modulation in each transmitting node and analyze the SER performance for SSK modulation. It is shown that the proposed multi-hop SSK system with path selection outperforms the conventional multi-hop $M$-PSK system in terms of the SER performance for especially high data rates and sufficient number of receive antennas at the receiving nodes. Moreover, unlike [16] and [20], our scheme is a more general MIMO scheme with arbitrary number of receive antennas. Furthermore, the proposed SSK system completely avoids ICI, eliminates the requirement of IAS in a multi-hop network and can be implemented with a very simple hardware that does not require I/Q modulation.

II. SYSTEM MODEL

![Fig. 1. System model of the multi-hop SSK system with path selection.](image)

The system model of the proposed multi-hop SSK system is given in Fig. 1. We consider a multi-hop multi-branch system with a source (S) equipped with $N_t$ transmit antennas and a destination (D) equipped with $N_r$ receive antennas. Furthermore, there are $L$ branches and each branch consists of $K$ relays, which are equipped with $N_t$ transmit and $N_r$ receive antennas. We denote the $r$th relay in the $p$th branch by $R_{p,m}$ ($1 \leq m \leq K, 1 \leq p \leq L$).

In such a system, S communicates with D via half duplex DF relays of the best path as follows: The overall transmission occurs in $K+1$ phases. In the first phase, a group of information bits are mapped to a transmit antenna index at S according to the SSK modulation. Due to the SSK mapping, only one transmit antenna is activated during the transmission and active antenna transmits the signal with energy of $E_S$. In $K-1$ hops, after the first phase, the relays on the selected path decode their received signals according to ML detection and forwards them with energy $E_S$ using SSK modulation as in the first phase. Hence, information is sent hop by hop until it reaches D over the selected path. With $l_i$ denoting the active transmit antenna index at the $i$th hop ($i = 1, \ldots, K+1$), the received signal vector at the $i$th hop of the $p$th branch can be given as

$$y_{p,i} = \sqrt{E_S}h_{p,i} + n_{p,i}$$  \hspace{1cm} (1)

where $h_{p,i}$ is the $i$th column of $H_{p,i}$, which is the channel matrix at the hop $i$ and path $p$. Note that the elements of $H_{p,i}$ are distributed as $\mathcal{CN}(0,1)$. $n_{p,i}$ is the additive Gaussian noise vector at the $i$th hop of the $p$th branch whose elements are distributed as $\mathcal{CN}(0,N_0)$. Since, the relays decode their received signals and then, forward them applying the SSK modulation, the system is exposed to error propagation. Finally, at the last phase of the transmission, D receives the signal from the last relay of the selected path and then decodes its received signal according to ML detection rule.

The pairwise error probability (PEP) of the end-to-end SSK systems depends on the Euclidean distances between channel fading coefficients corresponding to the transmit antennas [2]. Hence, we consider these Euclidean distances to perform the path selection in our system. Since each transmitting node uses SSK modulation to send information bits, PEP for the hop $i$ of the $p$th branch, or in other words, the probability that $l_i$ is detected erroneously as $\hat{l}_i$, can be given as [2]

$$P\left(l_i \to \hat{l}_i \right) = \int_0^\infty Q \left( \sqrt{r} \right) f_{\gamma_{p,i}}(r) \, dr$$  \hspace{1cm} (2)

where $\gamma_{p,i} = \frac{E_S}{2N_0} \| h_{p,i} - h_{p,\hat{l}_i} \|^2$. Here, $h_{p,i}$ denotes the channel fading coefficients vector corresponding $l_i$th transmit antenna of the $i$th hop of the $p$th branch where $l_i \neq \hat{l}_i$. The best
path is selected considering the squared Euclidean distances between channel fading coefficients for each hop as follows:

$$\gamma_{\text{sel}} = \max_{p=1,\ldots,L} \left\{ \min_{l_1,i_1=1,\ldots,N_t, l_2,i_2 \neq l_1,i_1} \gamma_{l_1,i_1}^{(p)} \right\}.$$  \hspace{1cm} (3)

Note that such path selection procedure can be performed by a central controller, in which the CSI of all the paths is available, as in [17], [18].

III. PERFORMANCE ANALYSIS

In this section, closed-form approximate and asymptotic SER expressions of the proposed SSK system are derived.

A. Approximate Symbol Error Rate Analysis

It is considerably hard to derive exact mathematical results for the proposed multi-hop SSK system with path selection since we need to consider all error events occurred at each node. However, to simplify the analysis, the worst case PEP of the selected path can be used to determine approximate SER of the proposed system [20]. In SM/SSK systems, the value of PEP is related with the difference of channel fading coefficients. We can define the Euclidean distance between channel fading coefficients corresponding to the $i$th and $l$th transmit antennas as $\lambda_{p,i}^{(l)} = \left\| \mathbf{h}_{p,i} - \mathbf{h}_{p,l} \right\|^2$. Since $\mathbf{h}_{p,i}$ and $\mathbf{h}_{p,l}$ follow complex Gaussian distribution, $\lambda_{p,i}^{(l)}$ follows chi-square distribution and its CDF is given as [21]

$$F_{\lambda_{p,i}^{(l)}}(r) = 1 - e^{-\frac{r}{2}} \sum_{z=0}^{N_r-1} \frac{1}{z!} \left( \frac{r}{2} \right)^z.$$  \hspace{1cm} (4)

The minimum Euclidean distance for the $i$th hop can be expressed as

$$\lambda_{p,i} = \min_{l \neq i} \lambda_{p,l}.$$  \hspace{1cm} (5)

Since we have $\binom{N_t}{2}$ squared Euclidean distances in each hop, CDF of $\lambda_{p,i}$ can be written with the help of order statistics as [22, (2.1.2)]

$$F_{\lambda_{p,i}}(r) = 1 - \left[ 1 - F_{\lambda_{p,i}^{(l)}}(r) \right]^{\binom{N_t}{2}} = 1 - \left[ e^{-\frac{r}{2}} \sum_{z=0}^{N_r-1} \frac{1}{z!} \left( \frac{r}{2} \right)^z \right]^{\binom{N_t}{2}}.$$  \hspace{1cm} (6)

On the other hand, the minimum Euclidean distance for the $p$th branch can be expressed as

$$\lambda_p = \min_{i=1,\ldots,K+1} \lambda_{p,i}. $$  \hspace{1cm} (7)

Therefore, the CDF of $\lambda_p$ can be written as

$$F_{\lambda_p}(r) = 1 - \left[ 1 - F_{\lambda_p}(r) \right]^{K+1} = 1 - \left[ e^{-\frac{r}{2}} \sum_{z=0}^{N_r-1} \frac{1}{z!} \left( \frac{r}{2} \right)^z \right]^{(K+1)\binom{N_t}{2}}.$$  \hspace{1cm} (8)

Considering (3), the selected path has the lowest minimum squared Euclidean distance among all branches. Hence, we can define this distance ($\lambda_{\text{sel}}$) as follows

$$\lambda_{\text{sel}} = \max_{p=1,\ldots,L} \lambda_p.$$  \hspace{1cm} (9)

Therefore, the CDF of $\lambda_{\text{sel}}$ can be written as

$$F_{\lambda_{\text{sel}}}(r) = \left[ F_{\lambda_p}(r) \right]^L = \left[ 1 - \left( 1 - F_{\lambda_p}(r) \right)^{\binom{N_t}{2}(K+1)} \right]^L = \left[ 1 - e^{-\frac{r}{2}} \sum_{z=0}^{N_r-1} \frac{1}{z!} \left( \frac{r}{2} \right)^z \right]^L.$$  \hspace{1cm} (10)

By applying binomial expansion, the CDF of $\lambda_{\text{sel}}$ can be rewritten as

$$F_{\lambda_{\text{sel}}}(r) = \sum_{\nu=0}^{L} \binom{L}{\nu} (-1)^\nu C_{\nu} (N_r,N_t,\nu) e^{-\frac{r}{2}} \sum_{z=0}^{N_r-1} \frac{1}{z!} \left( \frac{r}{2} \right)^z.$$  \hspace{1cm} (11)

where $M_C = (N_r-1)\binom{N_t}{2}(K+1)\nu$ and $C_{\nu} (N_r,N_t,\nu)$ is the coefficient of $r^\nu$ in the expansion of

$$\sum_{z=0}^{N_r-1} \frac{1}{z!} \left( \frac{r}{2} \right)^z.$$  \hspace{1cm} (12)

By substituting (14) into (13) and evaluating the integral with the help of [24, eq. (3.63)], approximate SER of the proposed system can be obtained in the closed-form as

$$P_s \approx 2N_t \int_0^{\infty} \sqrt{\frac{E_s}{2N_0}} f_3_{\lambda_{\text{sel}}}(r) dr.$$  \hspace{1cm} (13)

The PDF $f_3_{\lambda_{\text{sel}}}(r)$ is obtained by taking the derivative of $F_{\lambda_{\text{sel}}}(r)$ as

$$f_3_{\lambda_{\text{sel}}}(r) = \sum_{\nu=0}^{L} \binom{L}{\nu} (-1)^\nu C_{\nu} (N_r,N_t,\nu) e^{-\frac{r}{2}} \sum_{z=0}^{N_r-1} \frac{1}{z!} \left( \frac{r}{2} \right)^z.$$  \hspace{1cm} (14)

By substituting (14) into (13) and evaluating the integral with the help of [24, eq. (3.63)], approximate SER of the proposed system can be obtained in the closed-form as

$$P_s \approx \frac{2}{N_t} \sum_{l=1}^{L} \sum_{\nu=0}^{(N_r-1)K_C} \binom{L}{\nu} (-1)^\nu C_{\nu} (N_r,N_t,\nu) \frac{1}{K_C}$$

$$\times \left[ 1 - \left( \frac{2K_C}{E_s/N_0} + 1 \right)^{-\frac{1}{2}} \right]^{\nu-1} \left[ 1 - \left( \frac{2K_C}{E_s/N_0} + 1 \right) \right]^{\nu-1/2}$$

$$\times \sum_{u=0}^{l-1} 2^{-u} \binom{l+u}{u} \left[ 1 + \left( \frac{2K_C}{E_s/N_0} + 1 \right) \right]^{-\frac{u}{2}}$$

$$\times \sum_{u=0}^{l-1} 2^{-u} \binom{l-1+u}{u} \left[ 1 + \left( \frac{2K_C}{E_s/N_0} + 1 \right) \right]^{-1/2}.$$  \hspace{1cm} (15)
B. Asymptotic Symbol Error Rate Analysis

Considering the well-known behavior of the PDFs of the direct and relaying links around the origin [25], the diversity and coding gains of the system, \( G_d \) and \( G_c \), respectively, can be derived. Hence, the asymptotic SER of the proposed system at high SNR values can be given as [25]

\[
\bar{P}_s(\epsilon) \approx (G_cE_S/N_0)^{-G_d}.
\]  

(16)

Taking the derivative of (8), the PDF of \( \lambda_p \) can be written as

\[
f_{\lambda_p}(r) = (K + 1) \frac{N_t}{2} \left[ e^{-r} \sum_{n=0}^{N_r-1} \frac{1}{n!} \left( \frac{r}{2} \right)^n \right]^{(K+1)/2}.
\]  

(17)

The PDF of \( \lambda_p \) around the origin can be written as

\[
f_{\lambda_p}(r) = \frac{(K + 1) \left( \frac{N_t}{2} \right)^{r/2} \Gamma\left(\frac{N_r}{2}\right)}{\Gamma\left(\frac{N_t}{2}\right)2^{N_r}} + \text{HOT}, \quad r \to 0^+.
\]  

(18)

where \( \text{HOT} \) stands for the higher order terms. Note that \( \lambda_p \) denotes the minimum squared Euclidean distance in the \( p \)-th branch. Using (18) and [25], the diversity and coding gains provided by the \( p \)-th branch can be given as \( G_d^p = N_r \) and \( G_c^p = \frac{1}{2} \left( \frac{N_t}{2} \right)^{K+1} \Gamma\left(\frac{N_t+1}{2}\right) \frac{1}{N_c} \), respectively. Since we consider the path with the largest minimum Euclidean distance, using [25, Eq. (15)], the diversity and coding gains provided by the proposed system can be written respectively as

\[
G_d = \sum_{p=1}^{L} G_d^p = \sum_{p=1}^{L} N_r = LN_r,
\]  

(19)

\[
G_c = \frac{\left( \frac{N_t}{2} \right)^{L-1} \Gamma\left(\frac{(L-1)/2}{2}\right) \Gamma\left(\frac{LN_r+1/2}{2}\right)}{(G_c^p)^{LN_r} \left( N_r + 1/2 \right)^L}.
\]  

(20)

IV. Numerical Results

In this section, the analytical expressions given in the previous section are verified through Monte Carlo simulations. Moreover, the SER comparisons are performed with the classical multi-hop SIMO schemes in which the \( M \)-PSK modulation is used instead of SSK modulation in each hop, the transmitting and receiving nodes are equipped with one transmit and multiple receive antennas, respectively. We consider the system models of [16] and [20] for the classical multi-hop SIMO scheme; however, the only difference between the classical multi-hop SIMO scheme and the system models of [16] and [20] is that the receiving nodes are equipped with single receive antennas for the given references. The SER results of the proposed SSK system are provided for different number of transmit antennas \( N_t \), branches \( L \), relays \( K \) and receive antennas \( N_r \).

Fig. 2 depicts the SER performance of the proposed multi-hop SSK system with path selection. The curves in Fig. 2 are given for \( L \in \{1, 2, 3\} \), \( K+1 = 5 \), \( N_t \in \{2, 4\} \) and \( N_r = 3 \). As seen from Fig. 2, computer simulation results match the analytical SER and diversity order results given in the previous section. Moreover, Fig. 2 indicates that the SER performance is improved and a diversity gain is obtained when the number of branches increases. According to the asymptotic analysis, the asymptotic diversity orders of the curves corresponding to the proposed SSK systems for \( L = 1, 2 \) and 3 are calculated as \( LN_r = 3, 6 \) and 9, respectively. It can be verified from the slopes of the SER curves given in Fig. 2 that these values are consistent with the computer simulation results.

In Fig. 3, we compare the SER performance of the proposed multi-hop SSK and classical SIMO schemes with path selection for different data rates \( \eta \in \{2, 3\} \) bits/sec/Hz, i.e.,
$N_t, M \in \{4, 8\}$, where the derived theoretical SER curves are shown with dashed lines. Note that a mapping between information bits and amplitude and/or phase modulated symbols is not performed in the SSK modulation, consequently, the number of the transmit antennas is used to determine the data rate in SSK systems. The curves in Fig. 3 are given for $L = 4$, $K + 1 = 4$, $N_l, M \in \{4, 8\}$ and $N_r = 4$. As seen from Fig. 3, the derived approximate SER expression is considerably accurate for especially high SNR region and the effectiveness of the proposed multi-hop SSK scheme with path selection against the classical multi-hop SIMO [16], [20] scheme is observed at higher data rates. Fig. 3 shows that the classical multi-hop SIMO scheme outperforms the proposed multi-hop SSK scheme for $\eta = 2$ bits/sec/Hz, i.e., $N_l = M = 4$, by approximately 1.8 dB; however the proposed multi-hop SSK scheme outperforms the classical multi-hop SIMO scheme for $\eta = 3$ bits/sec/Hz, i.e., $N_l = M = 8$, by approximately 1.8 dB at a SER value of $10^{-4}$.

V. Conclusion

A multi-hop SSK scheme with path selection has been proposed in this paper. In this scheme, a MIMO structure, in which the transmitting and receiving nodes are equipped with the multiple transmit and receive antennas, respectively, is considered. The transmission occurs hop-by-hop via the relays of the best path, which is selected among available branches. SSK is applied at all transmitting nodes, i.e., $S$ and relays. Approximate and asymptotic SER expressions for the proposed multi-hop DF-SSK system with path selection have been derived. Our analytical results are validated by computer simulation results. It has been shown that the proposed multi-hop SSK system outperforms the classical SIMO system, in which the $M$-PSK modulation is applied, for especially high data rates and sufficient number of receive antennas.

Acknowledgment

This work was supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under Grant no. 114E607.

References