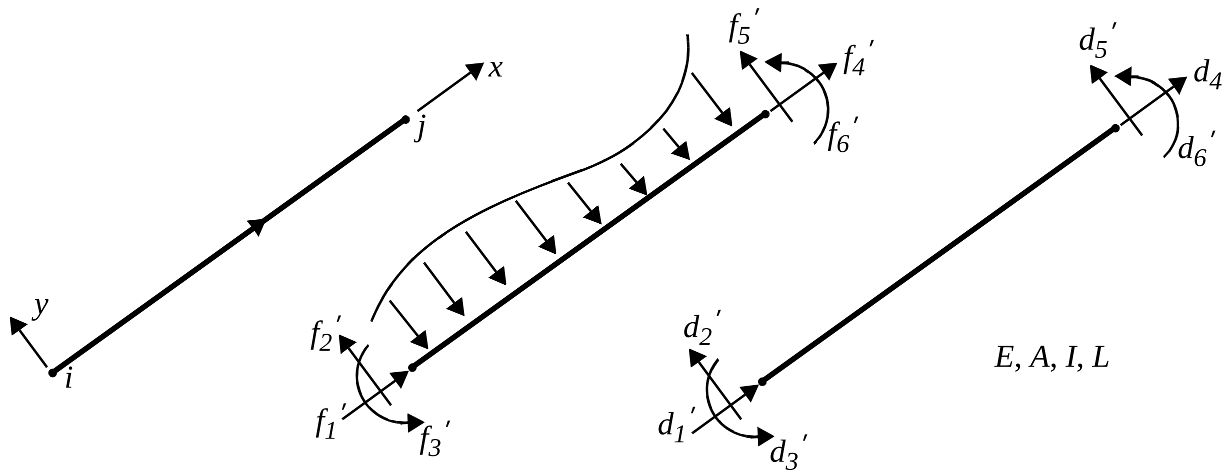
Frame Element in Local Coordinates

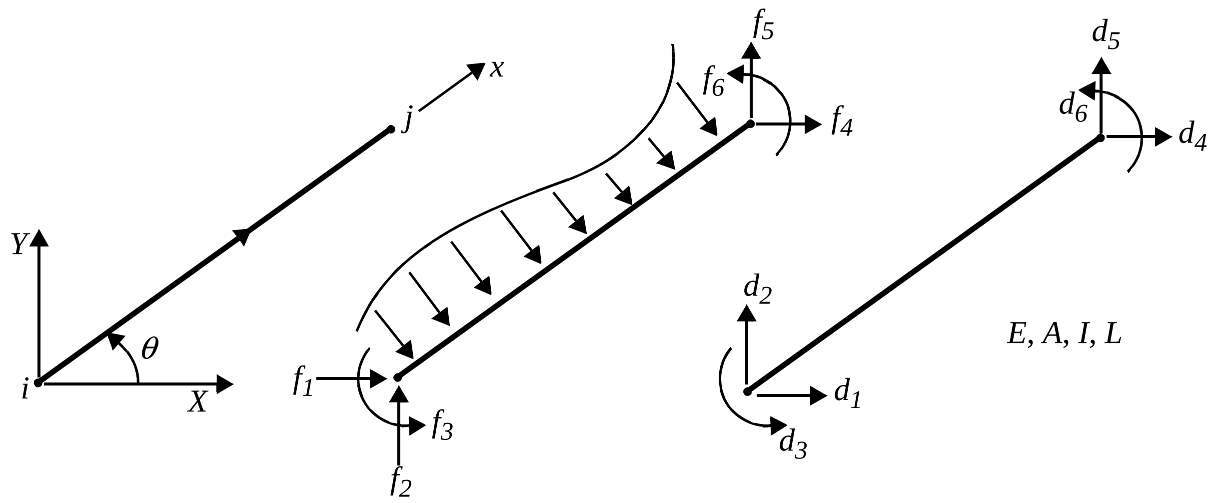


Force Vector: $\mathbf{f}' = \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_4' \\ f_5' \\ f_6' \end{bmatrix}$ Displacement Vector: $\mathbf{d}' = \begin{bmatrix} d_1' \\ d_2' \\ d_3' \\ d_4' \\ d_5' \\ d_6' \end{bmatrix}$ Fixed End Force Vector: $\mathbf{p}' = \begin{bmatrix} p_1' \\ p_2' \\ p_3' \\ p_4' \\ p_5' \\ p_6' \end{bmatrix}$

$$\text{Stiffness Matrix: } \mathbf{k}' = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Force Displacement Relation: $\mathbf{f}' = \mathbf{k}'\mathbf{d}' + \mathbf{p}'$

Frame Element in Global Coordinates



Force Vector: $\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix}$ Displacement Vector: $\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix}$ Fixed End Force Vector: $\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix}$

Force Displacement Relation: $\mathbf{f} = \mathbf{kd} + \mathbf{p}$

Stiffness Matrix, $C = \cos(\theta)$ and $S = \sin(\theta)$

$$
\mathbf{k} = \left[\begin{array}{c:c:c:c:c:c}
C^2\frac{EA}{L} + S^2\frac{12EI}{L^3} & CS\frac{EA}{L} - CS\frac{12EI}{L^3} & -S\frac{6EI}{L^2} & -C^2\frac{EA}{L} - S^2\frac{12EI}{L^3} & -CS\frac{EA}{L} + CS\frac{12EI}{L^3} & -S\frac{6EI}{L^2} \\
CS\frac{EA}{L} - CS\frac{12EI}{L^3} & S^2\frac{EA}{L} + C^2\frac{12EI}{L^3} & C\frac{6EI}{L^2} & -CS\frac{EA}{L} + CS\frac{12EI}{L^3} & -S^2\frac{EA}{L} - C^2\frac{12EI}{L^3} & C\frac{6EI}{L^2} \\
-S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{4EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{2EI}{L} \\
-C^2\frac{EA}{L} - S^2\frac{12EI}{L^3} & -CS\frac{EA}{L} + CS\frac{12EI}{L^3} & S\frac{6EI}{L^2} & C^2\frac{EA}{L} + S^2\frac{12EI}{L^3} & CS\frac{EA}{L} - CS\frac{12EI}{L^3} & S\frac{6EI}{L^2} \\
-CS\frac{EA}{L} + CS\frac{12EI}{L^3} & -S^2\frac{EA}{L} - C^2\frac{12EI}{L^3} & -C\frac{6EI}{L^2} & CS\frac{EA}{L} - CS\frac{12EI}{L^3} & S^2\frac{EA}{L} + C^2\frac{12EI}{L^3} & -C\frac{6EI}{L^2} \\
-S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{2EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{4EI}{L}
\end{array}\right]
$$

Relations between the Forces and Displacements in Local and Global Coordinates

Member End Displacements and Rotations: $\mathbf{d}' = \mathbf{T}_D\mathbf{d} = \mathbf{T}\mathbf{d}$

Member End Forces and Moments: $\mathbf{f} = \mathbf{T}_F\mathbf{f}' = \mathbf{T}^T\mathbf{f}'$

Fixed End Forces and Moments: $\mathbf{p} = \mathbf{T}_F\mathbf{p}' = \mathbf{T}^T\mathbf{p}'$

Member Stiffness Matrix: $\mathbf{k} = \mathbf{T}_F\mathbf{k}'\mathbf{T}_D = \mathbf{T}^T\mathbf{k}'\mathbf{T}$

where

Transformation Matrices: $\mathbf{T}_D = \mathbf{T} = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, $\mathbf{T}_F = \mathbf{T}^T = \begin{bmatrix} C & -S & 0 & 0 & 0 & 0 \\ S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & -S & 0 \\ 0 & 0 & 0 & S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

and, $C = \cos(\theta)$ and $S = \sin(\theta)$