# An Engagement Simulation to Evaluate Fighter Evasive Maneuvers Combined with Chaff Utilization against Missiles

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**Keywords: fighter evasive maneuvers, engagement simulation, missile countermeasures, chaff.**

## Abstract

In this paper, evasive actions of a fighter a missile employing proportional navigation are investigated. This paper is a part of our ongoing academic project, the Visual End-Game Simulation. In three dimensions, realistic, extended point-mass, generic fighter is modeled as the target of a missile that employs proportional navigation guidance system. Simulations of engagement scenarios are conducted for a fighter inside the beam width of a missile seeker. Some evasive tactics are experimented to cause large miss distances by applying the suitable maneuvers and dispensing chaffs to make the fighter fall outside the seeker beam and mislead the missile gradually to lock off, and then executing additional evasive maneuvers. The terminal phase of the encounter is taken into consideration. Taking into account both the state and control variable inequality constraints, the three-dimensional evasion trajectories are sought. Example trajectories and control histories of scenarios are represented. Effectiveness of performing horizontal-s maneuver and the evasive maneuver that was proposed before by the authors is analyzed for two different scenarios. The utilization of dispensing chaff followed by additional maneuvers is shown.

## 1. INTRODUCTION

Fighter pilots are trained on the reactions they will take against hostile actions when they face enemy fighters and the incoming guided missiles. In particular, they are expected to have master level skills in basic fighter maneuvers (BFM). In a possible air combat, a pilot tries to gain and maintain positional dominance over the hostile fighter. Also, executing the necessary and the suitable evasive maneuvers against an incoming guided missile is essential for survival. In this paper, the evasive maneuvers of a fighter (target) against incoming air-to-air missile are examined. Generally, the performance of the maneuver is measured in terms of the miss distance and the flight time. Terminal miss distance is the range between the target and missile at the closest approach. The object of the target is to perform the best maneuver to provide itself with long terminal flight time and large miss distance. In the terminal phase of a guided missile and fighter encounter, a fighter may have higher possibility of avoidance if it can force the missile to make harsh turns. Especially, in cases where the missile runs out of fuel, it will lose energy that it will never gain. This will increase the miss distance and the flight time, which, in turn, will cause the missile to go beyond the effective range. Some of well-known optimal maneuvers are split-s and horizontal-s maneuvers. Although high-g barrel roll is stated as ‘not optimal’, it’s, also, one of the effective evasive maneuvers performed against missiles. A detailed study on performing optimal barrel roll is considered by Takano and Baba [Takano and Baba 2004].

Defensive BFMs are not the only measures to be taken by a pilot against a guided missile in an encounter. At this point, electronic counter measures come up. These measures are flares, jammers, and chaffs, etc. Chaff among all is the counter measure that has been used the longest time. Chaff is a kind of a decoy used against RF tracking radars. Chaffs consist of glass fiber or metal strips and they are dispensed from fighters either by ejection or series of drops. A chaff bundle has thousands of strips and these strips turn out to be a cloud that acts as reflectors to a variety of radar frequencies [Sleven and Sheridan 1966]. According to the position of the chaff cloud relative to the missile, the chaff cloud either obscures the fighter from the missile or decoys the tracker of the missile by appearing as another target. It degrades the performance of tracking radar as it tracks a fighter. The common opinion that comes out of extensive studies both on the structure of Chaff and its effects on radars is that, it cannot be undervalued as a electronic counter measure even how much the technologies in counter-counter measures (ECCM) improve.

As stated above, the effective electronic counter measures, combined with evasive maneuvers within a case against guided missile threat could provide a significant success. Consequently, it presents importance to train fighter pilots about taking other measures, besides bringing their talents on executing defensive BFMs to master-level. On the other hand, it’s appreciated that training in real time

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with real equipment are quite expensive and time consuming. This fact makes it very significant to modeling the scenarios in a simulation environment to examine the effects and efficiency of the evasive maneuvers and electronic counter-measures. This is our main motivation to develop new software to be used in pilot training and in aeronautical research studies. In our academic project, Visual End-Game Simulation Project, we intended to examine and observe the effects of fighter evasive maneuvers against a missile which is employing proportional navigation missiles. In our first paper of this project, results from different experiments in which the effects of some combinations of practical maneuvers performed by fighter pilots, such as horizontal-s, split-s, and barrel roll were presented[Akdag and Altilar 2005]. In the following paper, an evasive maneuver tactic aimed at exploiting the limitations of a missile seeker was proposed [Akdag and Altilar 2006]. In this paper we depict the effects of proper use of chaff while executing effective evasive maneuvers. The terminal phase of the encounter is taken into consideration. At the beginning of engagement scenarios, a proportional navigation missile is considered to already have a fighter aircraft tracked. Later in the scenarios, the fighter dispenses a chaff bundle and performs evasive maneuvers. In order to show the efficiency of using chaff, maneuver-only engagement scenarios are conducted. In three dimensions, realistic, extended point-mass, generic fighters are modeled as the target of a missile that employs proportional navigation guidance system. Three-dimensional realistic vehicle models are used to obtain reliable results. The simulation results that are supported by comprehensible visual projections have been obtained by using this new software. The software is implemented in C++ using OpenGL. Using the Visual Eng-Game Simulation software, it is possible to analyze all the important parameters, such as aerodynamic forces, etc. of the missile and the aircraft during an engagement scenario applicable.

The paper is organized as follows. In Section 2, a brief survey of related studies is expounded. Section 3 describes the aircraft and missile model. Brief information of implementation issues is given in Section 4. The explanation of the fighter vs. guided missile encounter scenarios, and remarkable results from missile-fighter engagement simulation runs are represenred Section 5. The paper is concluded Secction 6.

## 2. RELATED STUDIES

For years, extensive studies have been made on the topic of evasive maneuvers of fighters against guided missiles. In our prior work, two combinations of three notable evasive maneuvers, Immelmann, split-s, and barrel roll, are observed and flight time analyses were made [Akdag and Altilar 2005].

The authors of this paper proposed an evasive action that tries to exploits the seeker limitations of an RF missile. For a fighter in the cone of a missile seeker, computing the closest point of the seeker cone of that missile and applying the suitable commands to evade out on that point was chosen as the objective [Akdag and Altilar 2006].

Imado and Miwa made a comparative study of different evasive maneuvers against proportional navigation missiles in which motions of the vehicles are constrained within a given horizontal plane. It was shown that each maneuver had its advantageous region and the evasion strategy should change depending on the relative geometry, and the altitude [Imado and Miwa 1985]. The optimality of the horizontal-s maneuver was shown when the cost function is the miss distance and the final time [Imado and Miwa 1986].

Ben-Asher et. al., studied the optimal evasion with a terminal path angle constraint for the evader, and the optimal evasion more than one pursuer. The optimal controls are represented to be bang-bang type, with the quantities of the switches depend on the missile's navigation gain and the particular constraints of each case [Ben-Asher et al. 1988].

Realistic target models including the variation of thrust and aerodynamic forces according to the Mach number were used by Ong and Pierson [Ong and Pierson 1996]. Their work is considering optimal evasive maneuvers against proportional missiles.

Minimum time trajectories to a fixed or moving target were produced with the software called Visual Interactive Aircraft Trajectory Optimization15 (VIATO) by Virtanen, et al. The authors introduced a new approach for the automated solution of optimal flight trajectories. The structure of the aircraft models and the objectives of the problems were specified, and different aircraft types were stored in their model library. The approach was implemented in the VIATO, which consists of an optimization server, a model server, and an intuitive, menu-driven, graphical user interface [Virtanen et al 1999].

Mentz et.al describes the development and application of an air-to-air combat simulation system where all contestants are controlled by a single computer. The human user can choose between a variety of tools to configure aircraft, weapon systems, and countermeasures, and set up engagements. High-level control depends on the course of the engagement [Mentz et al. 2006].

An optimal three dimensional pursuit-evasion (PE) problem between two aerial vehicles is applied using parallel evolutionary programming (PEP) algorithm. This work shows that the good solution is found to be highly dependent on the initial condition, the number of generation, the size of population and the number of CPUs used in such an approach [Nusyirwan and Bil 2008].

Karelahti et al. proposed an approach for producing realistic near-optimal aircraft trajectories via computational optimal control and inverse simulation is introduced and implemented in a software named Ace. They showed 3-DOF aircraft model affords a tractable model for optimizing the flight paths using direct multiple shooting [Karelahti et al. 2008].

Results of experiments on spectral characteristics of radar echoes from aircraft-dispensed chaff were described in [Estes et al. 1985]. They introduced two terms, “new chaff” and “mature chaff”, referring to chaff dipoles being agitated by the turbulent associated with the wake of the aircraft, and chaff dipoles that are affected only by natural winds, respectively.

A chaff simulator for the purpose of training radar operators was developed be [Seleven and Sheridan 1966]. The authors gave the equations of motion for chaff bundle in this work.

## 3. MODEL DEFINITIONS

### 3.1. Aircraft Model

The aircraft used in this work is a generic fighter with high-g capability. Its motions, as is of the missile, are in three dimensions. Effects of aerodynamic forces acting on the fighter along with kinematics, and dynamics gain importance because of the instantaneous hard turning maneuvers in the terminal phase of an engagement. Thus the following equations of motion are used which satisfies these considerations :

$$\dot{x}_a = v_a \cos \Upsilon_a \cos \mathrm{X}_a$$

$$\dot{y}_a = v_a \cos \Upsilon_a \sin \mathrm{X}_a$$

$$h_a = v_a \sin \Upsilon_a$$

$$\dot{\Upsilon}_a = \frac{1}{m_a v_a}\Big[\big(C_L\left(\alpha, M\left(h_a, v_a\right)\right) Sq\left(h_a, v_a\right) + uT_{\max}\left(h_a, M_a\left(h_a, v_a\right)\right)\sin\alpha\big)\cos\mu - m_a g\cos\Upsilon_a\Big]$$

$$\dot{\mathrm{X}}_a = \frac{1}{m_a v_a \cos\Upsilon_a}\Big[C_L\left(\alpha, M\left(h_a, v_a\right)\right) Sq\left(h_a, v_a\right) + uT_{\max}\left(h_a, M\left(h_a, v_a\right)\right)\sin\alpha\Big]\sin\mu$$

$$\dot{v}_a = \frac{1}{m_a}\Big(uT_{\max}\left(h_a, M\left(h_a, v_a\right)\right)\cos\alpha - C_D Sq\left(h_a, v_a\right)\Big) - g\sin\Upsilon_a$$

$x_a$, $y_a$ , $h_a$ , $\Upsilon_a$ , $\mathrm{X}_a$ ,and $v_a$ are $x$ and $y$ range, the altitude, the flight path angle, the heading angle, and the velocity, respectively. g is the acceleration of gravity, $m_a$ is the mass of the aircraft, and both assumed constant. $S$ is the reference wing area. The dynamic pressure is denoted by $q\ (h_a,\ v_a) = 1/2\rho v^2$. Here, $\rho$ is the density of the air at a specific altitude. $T_{max}\ (h_a, M_a(h_a, v_a))$ denotes the maximum available thrust which is a function of $T_{SL}$, $\rho_{SL}$, $\rho$ and $M$, which are thrust at sea level, air density at sea level, air density at a specific altitude and the Mach number, respectively. $C_L$ is the coefficient of lift, $C_D$ is the overall coefficient of drag, which is separated into two components; the zero-lift drag coefficient, $C_{D0}$, and the induced drag coefficient, $C_{Di}$. The lift coefficient, is Complicated sequence for approximation of $C_L$ and $C_D$ that include a number of variables are omitted. For details, see [Miele 1962] and [Brandt et al 1997].

The control variables are the angle of attack $\alpha$, the throttle setting $u$, and the bank angle $\mu$. There are some constraints for the angle of attack. These are

- The lift coefficient $C_L(\alpha, M(h_a, v_a))$ must not exceed the aircraft-specific quantity, $C_{L,max}(M)$
- The load factor $n(\alpha, h_a, v_a)$ must not exceed the aircraft-specific quantity, $N_{\max}$
- The pitch rate must not exceed $P_{\max}$, where

$$\mathrm{P} = \alpha + (\gamma\ \ * \cos(\mu)) + (\mathrm{X}\ \ * \cos(\gamma) * \sin(\mu))$$

and, maximum allowed pitch rate is 20 degrees per second for our model.

### 3.2. Missile Model

The missile’s guidance system is Proportional Navigation. Theoretically, this law issues acceleration commands, perpendicular to the instantaneous missile-aircraft line-of-sight (LOS), which are proportional to the LOS rate and closing velocity[1], $V_C$. The guidance law can be stated as follows

$$n_c = N' * V_C * \dot{\lambda}$$

where $n_c$ is the acceleration command, $N'$ , a unitless gain (usually in range of 3-5) known as the effective navigation ratio, and $\dot{\lambda}$ the LOS rate. The maneuvers of the missile is implemented using three-plane approach (TPA) for 3D True Proportional Navigation [Moran and Altilar 2005].

## 4. IMPLEMENTATION ISSUES

The evaluation of fighter evasive maneuvers and the usage of chaff are made with the Visual End-Game Simulation software. The main modules of the software are the referee, pursuer, evader, radar, aero modules. The referee module serves as the manager of the simulation which initializes the simulation, checks if the termination condition is met, handles visual representation.

The “pursuer” module is what handles the maneuvers of the guided missile. A missile pursuing its target by employing proportional navigation guidance is taken into consideration. The positional information of the sides of the engagement are held in the “radar” module. In this work, we assume that the vehicles have perfect information about positions of each other.

The computation of parameters that effect aerial motion of vehicles, aerodynamic calculations, the air density and the Mach numbers corresponding to the vehicle velocities and altitudes are made by the “aero” module.

In the “evader” module, a generic aircraft with high-g capability and its maneuvers are modeled.

In addition to the aforementioned modules, another module is implemented to compute the closest point of the missile seeker cone to the fighter. Remember, for all scenarios, the fighter is assumed to be in the beam of the missile seeker. The modular structure of the software helps us easily integrate new algorithms, new scenarios, etc.

## 5. ENGAGEMENT SCENARIOS

The earth is assumed to be flat, and the sideslip angle is assumed to be zero in the simulations. The fighter is hit if it falls in the capture radius of the missile, i.e., 10 m., and it succeeds to evade if the Angle-Off-Tail (AOT) exceeds the limit of the cone of the missile seeker. Both the fighter and the missile are assumed to have exact knowledge of the state of each other. In three dimensions, the fighter performs bank-to-turn maneuvers in order to change direction, is dependent of aerodynamic forces, changes its flight path angle by applying angle of attack, α, commands, changes its heading angle by applying bank angle, μ, commands, and applies desired thrust force by adjusting throttle setting, u, where $0<u<1$.

We demonstrate 4 different engagement scenarios in order to examine the diverse effects of both the evasive maneuvers and dispensing Rapid Bloom Chaff. Tracker of an RF missile sees its target as a cluster of RF emitters, especially in close ranges. Thus, when it catches a fighter, it tracks the centroid of the cluster. The structure of a chaff cloud exhibits similar blip in a radar tracker. At the time that a tracker lock-on its target, chaff dispensed by the fighter will disturb the centroid that the tracker locks on. It will probably degrade the accuracy of tracking and cause a missile break. Approximately half a second after the RBC is dispensed from a fighter; the chaff cloud starts to mislead the missile by causing its track-centroid to slide towards the cloud. After a second or two, the chaff cloud appears as if it were a secondary target. Executing an appropriate evasive maneuver right after dispensing the chaff may increase the possibility that the missile is mislead. In the scenarios, we tried to find the most appropriate time to dispense chaff, and combining this with an accurate evasive maneuver to increase the possibility to successful avoidance.

Although the chaff is an effective counter measure against RF missiles, the improvement of radar and missile technologies also has lead improvement of counter-counter measure techniques. For this reason, it's possible that a missile tracker will recognize the clutter in a few seconds and will lock on its target. Our simulations are based on these assumptions.

The specifications of the generic fighter and missile are tabulated in Table 1.

**Table 1.** Vehicle specifications.

| **Fighter** | |
|---|---|
| Mass $m_t$ | 13607.7711 kg |
| Thrust at Sea Level $T_{SL}$ | 77.84 kN |
| Wetted Area $S_{wet}$ | 138.89 $m^2$ |
| Wing Area $S$ | 27.8709 $m^2$ |
| Aspect Ratio $AR$ | 4.08 |
| Wing Sweep Angle $\Lambda_{LE}$ | 40° |
| Maximum Speed $V_{tmax}$ | Mach 2.0 |
| Maximum Load Factor $n_{max}$ | 8 |
| Skin Friction Coefficient $C_f$ | 0.0035 |
| Wave Drag Coefficient $C_{Dwave}$ | 0.0261 |
| Lift Curve Slope at M=0 $c_{l\alpha(M=0)}$ | 5.73 /degree |
| Max./Min Angle of Attack $\alpha$ | -2° / +25° |
| **Missile** | |
| Mass $m_m$ (before 7 s) | 100 kg |
| Mass $m_m$ (after 7 s) | 79 kg |
| Thrust $T_m$ (before 7 s) | 10 kN |
| Thrust $T_m$ (after 7 s) | 0 |
| Maximum Acceleration $a_{max}$ | 30 G |
| Maximum Speed $V_{mmax}$ | Mach 4.0 |

### 5.1. Scenario 1

In the first scenario, the fighter and the missile take level flight initially. The initial position of the fighter is x=9000 m., y=1000 m., and z=2500 m. The velocity of the fighter is 260 m/sec. Its flight path angle, heading angle and angle of attack values are all 0 degrees. The missile starts the terminal phase in position x=0 m., y=1000 m., and z=2800 m. with initial speed of 900 m/sec. Its flight path angle and heading angle is also 0 degrees. Note, altitude values of the vehicles are their positions on the z-plane. First, we conduct the simulation without dispensing any chaff. The fighter only executes a maneuver to evade out of the seeker cone of the missile. At each time step, it computes the closest point on the boundary of the seeker cone of the missile and apply bank angle and angle of attack commands to move towards that point. The derivation of the computation of the evasion point is detailed in [Akdag and Altilar 2006]. The resulting trajectories of the fighter and the missile are shown in Figures 1-3

The resulting flight time of the engagement is 10.04 seconds and the terminal miss distance is observed 3.67 m. Next, we start the simulation with the same initial conditions. The difference is that the chaff is dispensed when the displacement between the vehicles comes to 2500 meters. For this scenario, 7.41 seconds after the beginning of the simulation, the fighter dispenses the chaff and continues to execute the evasive maneuvers.

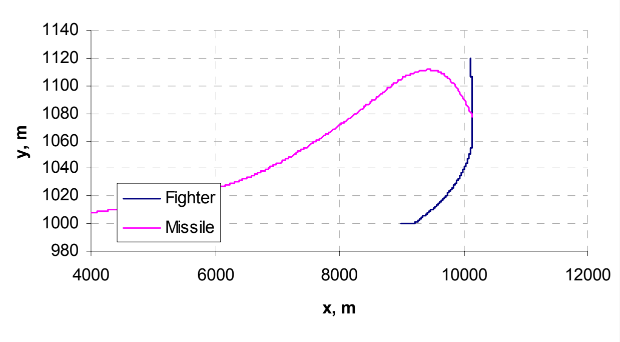

**Figure 1.** Vehicle trajectories on xy plane.

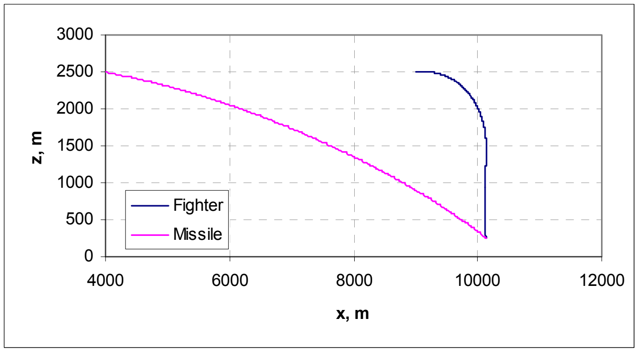

**Figure 2.** Vehicle trajectories on xz plane.

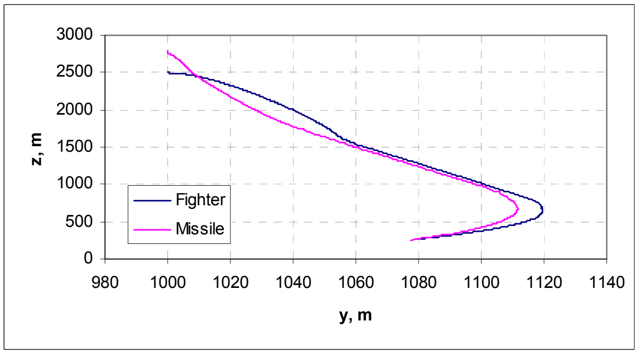

**Figure 3.** Vehicle trajectories on yz plane.

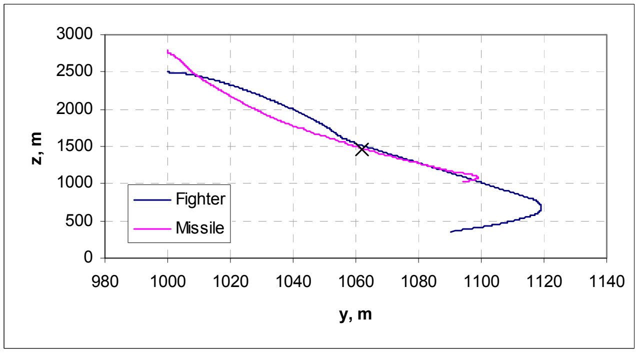

**Figure 4**. Vehicle trajectories on yz plane. (Chaff used)

The point where the chaff is released is shown with “X” in Figure 4. When we compare Fig. 4 to Fig 3., we see the slip of the missile out of the correct route, i.e. the route to the fighter.

After about 1.5 seconds after chaff dispensed, the missile tracker filters out the chaff cloud. In order to head for the fighter, it makes a harsh turn which will force its structural limits. At the time of 9.76 seconds after the beginning of th scenario, it loses track of its target. The pitch acceleration commands applied to the missile are shown in Figure 5.

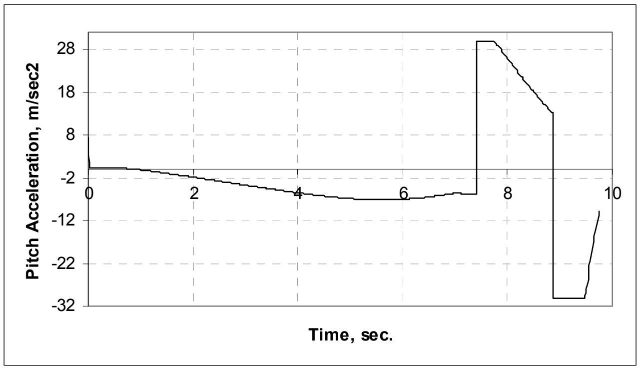

**Figure 5**. History of missile pitch acceleration.

Figure 5 seems to be worthwhile to show the importance of forcing the missile make hard turns. When the chaff first dispensed, the pitch acceleration of the missile saturated on one limit, and when its seeker filters out the chaff cloud, its pitch acceleration saturates on the other extreme. While it turns from one side to the other, it loses much of its energy. Consequently, its velocity decreases and fails to keep the fighter inside its seeker beam.

### 5.2. Scenario 2

The scenario 2 is chosen to highlight the misuse of chaff, when a wrong evasive maneuver is combine with using it. In this scenario, the initial position of the fighter is x=9000 m., y=0 m., and z=2500 m. Its speed is 250 m/sec and flight path angle, heading angle and angle of attack values are all 0 degrees. The missile starts the terminal phase in position x=0 m., y=50 m., and z=2000 m. with initial speed of 850 m/sec. Its flight path angle is 0 degrees and heading angle is also 2 degrees to the right, so that it moves towards the fighter. There is approximately an initial separation of 9000 m. between them. The fighter performs horizontal-s maneuver. Fighter dispenses a bundle of RBC when the distance goes down to 2500 m. Starting from this point, the centroid of the volume that the missile locks on slides towards the chaff cloud. After approximately 1 second, the echoes from the chaff bundle is assumed to be filtered out by the Doppler filter of the missile, and missile locks on the fighter again. The trajectories of the fighter and the missile are shown in figures 6-8. As we can see from Fig. 6 that the missile is mislead starting from where it comes at 8250 m on x axis. After 1 second, at about 9800 m. on x axis, it locks on the fighter again and intercepts it. This could have been a correct action if it had combined with a suitable evasive maneuver. But, in this scenario, the maneuver performed, i.e. the horizontal-s maneuver, keeps the missile from acceleration saturation and helped it successfully intercept the fighter. The correct maneuver must be chosen to make the missile deal with the worst guidance problem and force it to perform sharp turns that, in turn, cause acceleration saturation and energy loss. Figure 9 represents the missile yaw acceleration history and Figure 10 represents the Angle off Tail (the angle that the missile sees its target) for scenario 1.

By combining the vehicle trajectories with the Figures 9 and 10, it's seen that the fighter dispenses a bundle of chaff at the time of 8.3 seconds from the beginning, and the missile starts to follow the wrong target because the centroid computed by the missile begins sliding. At this point, the missile makes a turn towards the new centroid. Meanwhile, the fighter goes on executing horizontal turn maneuver, then, at the time of 9.8 seconds, the missile filters out the chaff impulses and tracks the fighter again. This forces it to make a harsh turn and to apply very high acceleration commands. Maximum acceleration allowed for the missile in our simulations is 30 G. This limitation will be very useful for a fighter if exploited. In this scenario, the fighter fails to exploit this limitation of the missile and, also, fails to evade.

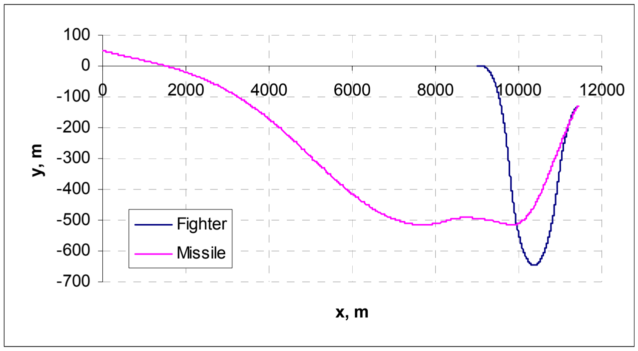

**Figure 6.** Vehicle trajectories on xy plane.

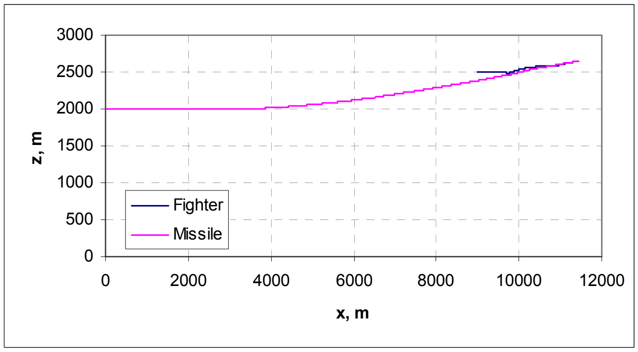

**Figure 7.** Vehicle trajectories on xz plane.

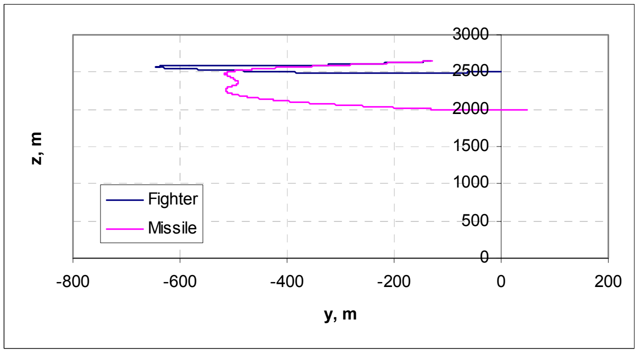

**Figure 8.** Vehicle trajectories on yz plane.

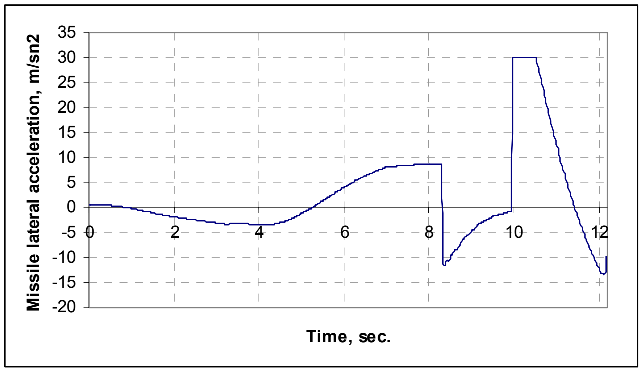

**Figure 9.** History of missile yaw acceleration

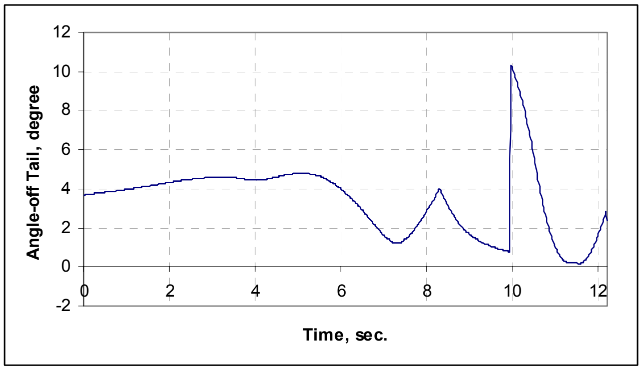

**Figure10.** History of angle off tail

## 6. CONCLUSIONS

The effects of chaff, one of the most widely used electronic counter-measures against RF guided missile threat, are investigated and represented. The simulation results we obtain shows that chaff may be efficiently used in air combat while executing correct evasive maneuvers against airborne missiles. Two of evasive maneuver tactics are experimented in two different engagement scenarios. In the first scenario, the maneuver towards the closest point on the boundary of the seeker cone of the missile is observed as an effective evasion tactic with dispensing chaff before some appropriate time from approximate impact time. In the second scenario, the significance of choosing the correct evasive maneuver when dispensing chaff is depicted. It is shown that an erroneously chosen maneuver will result in failure for a fighter. It must also be considered that the initial engagement geometry between the vehicles may chance the results of the engagements scenarios.

In the wake of the difficulties faced in training pilots in the real world, the indispensability of utilizing the simulation techniques will be well appreciated. Thus, it will be invaluable for a pilot candidate to recognize the possible outcomes of choosing different counter-measures in different engagement scenarios.

In this paper, we used 3-DOF vehicle models including orientation kinematics to model the terminal phase of fighter-guided missile engagement scenarios. It's seen that, realistic results may be obtained by using simplified models and making legitimate assumptions. The Visual End-Game Simulation software is thought to be used in flight schools for educational purposes.

Engagement scenarios in which multiple fighters take evasive actions against missile threat will be examined in the near future. The vehicles will be modeled as 6-DOF models to obtain more realistic results. Also, optimization of the time that chaff is dispensed is being planned as a future work.

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