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A Comparative Study on Practical Evasive Maneuvers Against Proportional Navigation Missiles

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In this paper, evasive maneuvers of a fighter against a missile employing proportional navigation are investigated. This work is a part of an ongoing project, Visual End-Game Simulation (VEGAS), on implementation, simulation, visualization, and evaluation of effectiveness of proportional navigation against an aircraft performing different evasive maneuvers in an air combat. Two maneuver combinations are examined; Immelmann followed by a barrel roll and split-s followed by a barrel roll. The terminal phase of the encounter is taken into consideration. An extended point-mass aircraft model including orientation kinematics is used to obtain realistic results. Sample encounter scenarios are conducted and flight time analyses are made.

Nomenclature

AR	=	aspect ratio
C_D	=	drag coefficient
C_{D0}	=	zero-lift drag coefficient
C_{Di}	=	lift-induced drag coefficient
C_f	=	skin friction coefficient
C_L	=	lift coefficient
e_o	=	Oswald's efficiency factor
g	=	acceleration of gravity
h	=	altitude
m	=	mass
M	=	Mach number
n	=	load factor
n_c	=	acceleration command
N'	=	effective navigation ratio
q	=	dynamic pressure
S	=	reference wing area
S_{wet}	=	wetted area
T	=	thrust force
u	=	throttle setting
V	=	velocity
x, y	=	horizontal positions
α	=	angle of attack
Y	=	flight path angle
$\dot{\lambda}$	=	line-of-sight rate
Λ_{LE}	=	sweep angle on the leading edge of the wing
μ	=	bank angle
ρ	=	air density
X	=	heading angle
$(\ )_{t,a}$	=	target, aircraft
$(\ )_m$	=	missile

I. Introduction

Since the end of World War II, many different methods for missile guidance have been developed to successfully intercept a stationary, predictable, or even highly maneuvering target[1]. As an expected result of tactical homing missiles' revealing in 1944, it became an obligation to develop target evasive maneuvering tactics against pursuers.

The performance of guidance systems can generally be quantified in terms of the miss distance between the missile and the target. Miss distance is the difference between the target and missile lateral displacements with respect to the reference line of sight (LOS). From a target's point of view, an optimal maneuver means techniques that provide the target with long intercept time and large miss distance.

Since the proportional navigation –in which the missile turning rate is made proportional to the line of sight rate- is the most broadly used guidance method due to its effectiveness. Consequently, in this work, target evasive maneuvers against proportionally navigating guided missiles are studied. A new approach for proportional navigation guidance system proposed by Moran and Altilar[2] is used as the guidance system of the missile.

Up to now, many methods have been studied on optimal maneuvers of an aircraft evading from a proportional navigation guided missile. Some of numerically obtained optimal maneuvers are barrel roll, split-s, and horizontal-s maneuvers.

Although aforesaid maneuvers are numerically accepted, there are many avoidance methods that are performed by fighter pilots under missile threat. In the wake of practicing evasion tactics being quite expensive and time consuming tasks, mathematically modeling and evaluating the performance of other evasive methods is inevitable. Motivated by this fact, we've studied two compositions of notable maneuvers; Immelmann turn followed by a barrel roll and split-s followed by a barrel roll

A new software that is called Visual End-Game Simulation (VEGAS) has been developed to evaluate the motions of a fighter aircraft, and the fighter evasive maneuvers against proportional navigation. All the factors required for gathering realistic results, i.e. aircraft specifications, aerodynamics, kinematics, and notable evasive maneuvers of fighters, are included in this software. On the other hand, the required factors relevant to the missile employing proportional navigation are included in VEGAS. The modular structure of the software has made it completely apt to further developments, such as adding electronic counter-measures (ECM), etc.

Extensive simulation results that are supported by comprehensible visual projections have been obtained by using this new software. Visual C++ and OpenGL are used in simulations. OpenGL's main purpose is to render two and three-dimensional objects into a frame buffer[3]. 3D visualization is included in order to provide the user for a comprehensive understanding about the terminal phase of the encounter.

The paper is organized as follows. In Section II, a brief survey of related studies is expounded. Section III describes the aircraft and missile model. The definitions of practical evasive maneuvers are in Section IV. The structure of the VEGAS is briefly explained in section V. In section VI, sample engagement scenarios are represented, and the efficiency of the evasive maneuvers are compared. Finally the conclusion is stated in Section VII.

II. Review of Related Studies

Imado and Miwa made a comparative study of different evasive maneuvers against proportional navigation missiles in which motions of the vehicles are constrained within a given horizontal plane[4]. It was shown that each maneuver had its advantageous region and the evasion strategy should change depending on the relative geometry, and the altitude.

The optimality of the horizontal-s maneuver was represented when the final time and miss distance are taken as the cost function in Ref. 5.

Optimal evasive control maximizing the miss distance for very simple two-dimensional missile and constant speed target was considered by Ben-Asher and Cliff (see Ref. 6).

A great number of pursuit-evasion simulations were conducted by giving both aircraft and missile the strategies for combinations of parameter spaces and initial conditions in Ref. 7. Final miss distance was chosen as the performance index of the games; the missile tried to minimize it, while the aircraft tried to maximize it. According to this method, the basic idea laid in giving players a priori optimal or suboptimal feedback strategies, conducting massive simulations in the parameter space of the initial geometries and guidance law parameters, and analyzing the results.

Imado and Uehara discussed the performance of the high-g barrel roll (HGB) maneuver from optimal control of view[8]. The mathematical model for three-dimensional pursuit-evasion problem of the aircraft against proportional navigation missile was considered; some features of the aircraft optimal evasive maneuvers, and high-g barrel roll maneuvers were explained. Finally, the exact numerical solution for the three-dimensional pursuit-evasion problem was illustrated and the non-optimality of the HGB was shown. The relation between the optimal maneuver and the HGB was also discussed.

Realistic target models including the variation of thrust and aerodynamic forces according to the Mach number were used by Ong and Pierson (see Ref. 9). Their work is considering optimal evasive maneuvers against proportional missiles.

Moore and Garcia described the implementation of a genetic programming system that evolved optimized solutions to the extended two dimensional pursuer/evader problems that did not depend upon knowledge of pursuer's current state (see Ref. 10).

Minimum time trajectories to a fixed or moving target were produced with a software called Visual Interactive Aircraft Trajectory Optimization (VIATO) by Virtanen et al. (see Ref. 11). The authors introduced a new approach for the automated solution of optimal flight trajectories. The structure of the aircraft models and the objectives of the problems were specified, and different aircraft types were stored in their model library. The approach was implemented in the VIATO which consists of an optimization server, a model server, and an intuitive, menu-driven, graphical user interface.

Choi et al. considered three-dimensional target optimal evasion problem against a proportionally guided missile[12]. They formulate the optimal evasion problem of an aircraft as a constrained optimization problem whose payoff is the intercept time and constraint is the capture condition.

The optimal avoidance of a missile employing proportional navigation was dealt with in Ref. 13. An extended point mass vehicle model including orientation kinematics was used. The drag, thrust and constraint data of vehicles represented a generic fighter aircraft and a medium range air-to-air missile.

Imado considered different approaches to pursuit-evasion such as giving both players some suboptimal feedback strategies, giving one player a suboptimal feedback strategy and other player an exact one-sided nonlinear optimal control, and giving both players suboptimal feedback strategies dependent on parameters[14].

National Aeronautics and Space Administration (NASA) developed a simulation that is capable of quickly and efficiently supporting flight research requirements and conceptual vehicle studies (see Ref. 15). The simulations in this work operate on UNIX-based platforms and were coded with a FORTRAN shell and C support routines. This simulation software is still used at NASA, within industry, and at several universities, and applicable to a broad range of fixed-wing aircraft including fighters.

Another versatile aircraft simulation study made in the NASA emphasized that realistic aircraft motion was of greatest importance, and accurate roll and pitch dynamics were very significant in developing evasive maneuvers against missiles[16].

This work considers target evasive maneuvers in the terminal phase of missile-aircraft encounter. 3D encounter scenarios are simulated and flight time analysis is made.

III. Model Definitions

A. Aircraft Model

The terminal phase of a missile-aircraft encounter takes a very short time. In a few seconds, the pilot must decide a maneuver to make and perform it. He must choose the convenient maneuver by considering the characteristics of the aircraft. Consequently, if realistic evaluation of the effectiveness of the performed maneuvers is desired, aerodynamic forces and rotational kinematics must be taken into account.

The equations of motion are (Ref. 17)

$$\dot{x}_a = v_a \cos \Upsilon_a \cos \mathrm{X}_a \tag{1}$$

$$\dot{y}_a = v_a \cos \Upsilon_a \sin \mathrm{X}_a \tag{2}$$

$$h_a = v_a \sin \Upsilon_a \tag{3}$$

$$\dot{\Upsilon}_a = \frac{1}{m_a v_a}\left[\left(C_L\left(\alpha, M\left(h_a, v_a\right)\right) S q\left(h_a, v_a\right) + u T_{\max}\left(h_a, M_a\left(h_a, v_a\right)\right) \sin\alpha\right) \cos\mu - m_a g \cos\Upsilon_a\right] \tag{4}$$

$$\dot{\mathrm{X}}_a = \frac{1}{m_a v_a \cos \Upsilon_a}\left[C_L\left(\alpha, M\left(h_a, v_a\right)\right) Sq\left(h_a, v_a\right) + uT_{\max}\left(h_a, M\left(h_a, v_a\right)\right)\sin\alpha \right]\sin\mu \tag{5}$$

$$\dot{v}_a = \frac{1}{m_a}\left(uT_{\max}\left(h_a, M\left(h_a, v_a\right)\right)\cos\alpha - C_D Sq\left(h_a, v_a\right)\right) - g\sin\Upsilon_a \tag{6}$$

x_a, y_a , h_a , Υ_a , X_a ,and v_a are x and y range, the altitude, the flight path angle, the heading angle, and the velocity, respectively. g is the acceleration of gravity, m_a is the mass of the aircraft, and both assumed constant. S is the reference wing area. The dynamic pressure is denoted by $q\ (h_a,\ v_a) = 1/2\rho v^2$. Here, ρ is the density of the air at a specific altitude. $T_{max}\ (h_a, M_a(h_a, v_a))$ denotes the maximum available thrust. The fighter in our model is assumed to have a turbofan engine with afterburner. Hence, the maximum available thrust is approximated by the following equation (see Ref. 18):

$$T_{\max} = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right)\left(1 + 0.7M\right) \tag{7}$$

where T_{SL}, ρ_{SL}, and M are thrust at sea level, air density at sea level and the Mach number, respectively. C_D is the overall drag which is separated into two components; the zero-lift drag coefficient, C_{D0}, and the induced drag coefficient, C_{Di}. The details of derivation of the total drag coefficient are given in the Appendix. The control variables are the angle of attack α, the throttle setting u, and the bank angle μ. There are some constraints for the angle of attack. These are

- The lift coefficient $C_L(\alpha, M(h_a,\ v_a)$ must not exceed the aircraft-specific quantity, $C_{L,max}(M)$
- The load factor $n(\alpha,\ h_a,\ v_a)$ must not exceed the aircraft-specific quantity, $N_{\max}$
- The pitch rate must not exceed $P_{\max}$.

The state variables and the limitations regarding the control variables are detailed in Ref. 13.

B. Missile Model

The missile's guidance system is the Proportional Navigation. Theoretically, this law issues acceleration commands, perpendicular to the instantaneous missile-aircraft line-of-sight, which are proportional to the line-of-sight rate and closing velocity[1], V_C. The guidance law can be stated as

$$n_c = N' V_C \dot{\lambda} \tag{8}$$

where n_c is the acceleration command, N', a unitless gain (usually in range of 3-5) known as the effective navigation ratio, and $\dot{\lambda}$ the line-of-sight rate. In this paper, the proposed implementation of proportional navigation guidance law in Ref. 2 is used as the pursuing missile's guidance system.

IV. Definition of Practical Evasive Maneuvers

When a missile is fired at an aircraft in the aft quadrant, the best way to defeat it is with a maximum rate turn to put the missile on the beam, which is called 3/9 line. This kind of a maneuver gives a proportional guidance missile the most difficult guidance solutions possible problems. In this position, the evading aircraft will be at 90° of aspect with respect to the missile, and it will have the worst possible line-of-sight rate problem to solve. Missiles fly lead pursuit courses to the target in order to achieve maximum range. If the missile is held on 3/9 line, it will make the missile pull the maximum amount of lead.

The pilots perform different maneuvers to avoid the missiles in the wake of the principal mentioned above. The following sections briefly describe the maneuvers implemented in this paper. For details, see Ref. 19.

A. The Barrel Roll Maneuver

Similar to a roll, the pilot applies back pressure on the stick while he is rolling to the left or right. This makes the aircraft plane fly in a corkscrew pattern and is used to make the missile deal with the most difficult guidance problems. The trajectory of a fighter which performs a barrel roll maneuver is represented in Fig. 1.

Figure 1. Barrel Roll Maneuver

B. The Immelmann Maneuver

The Immelmann Maneuver is named after Max Immelmann, a World War I German ace who reportedly invented the maneuver. The Immelmann is essentially a maneuver for repositioning. Although the Immelmann is a very effective tool for setting up for an engagement used by attacker aircrafts, it also can be applied in defensive situations. Basically the Immelmann is a quick way to change direction while increasing altitude.

The modern version of the Immelmann is a vertical climb or half loop with a half-roll at the top. Its main value lies in using the vertical plane to change the direction of flight in the smallest possible horizontal space. Horizontal turns costs high turn radiuses at fighting speeds. Using the vertical plane enables the fighter to turn square corners in relation to its position above the ground. This maneuver makes repositioning for meeting a threat, much easier than horizontal maneuvers. Figure 2 shows the phases of the Immelmann maneuver.

Figure 2. Immelmann Maneuver

C. The Split-S Maneuver

The Split S is the Immelmann in reverse, and consists of a half-roll followed by a tight vertical loop downwards. One important prerequisite of this maneuver is a fairly high altitude. The turn begins with rolling 180 degrees; once upside down, pulling back on the stick to execute a vertical U-turn. By performing this maneuver, a pilot can reverse directions and gain a lot of speed.

V. Implementation

The 3-D simulation of barrel roll following an Immelmann turn and barrel roll following a split-s turn is implemented. Visual C++ and Open GL are used in simulations. A 3-D visualization is performed in order to provide the user for a comprehensive understanding about the terminal phase of the encounter. Also, miss distance analysis is made and graphically presented to the user.

There are four main modules in the program. These are referee, missile, target, and radar modules. The "referee" module serves as the manager of the simulation which initializes the simulation, calls the functions of pursuer's and evader's maneuvers, handles drawings. The "missile" module is the program of missile's seeker model and maneuvers. A missile pursuing the aircraft with proportional navigation guidance is taken into consideration. The missile seeker implementation is detailed in Ref. 2.

The third module in the simulation is the "target" and is the subject of this paper. In this part of the simulation, a generic aircraft with high-g capability is mathematically modeled. An extended point-mass aircraft model including orientation kinematics is used to obtain realistic results. Three of well-known aircraft evasive maneuvers, barrel roll, Immelmann and split-s, are simulated. Both Immelmann and split-s maneuvers combined with a barrel roll turn are selected because of their contrary nature. The coordinates of the missile and the target is provided for the "missile" and "target" modules by the "radar" module.

In this paper, different initial positional geometries are selected for performance evaluation purposes.

VI. Engagement Scenarios

Two engagement scenarios where a fighter performs the mentioned evasive maneuvers against a missile employing proportional navigation are considered in this section. The earth is assumed to be flat, and the side-slip angle is assumed to be zero n the simulations. Capture radius is selected as 10m and program terminates if the

Table 1.Vehicle Parameters

Fighter	
Mass m_t	13607.7711 kg
Thrust at Sea Level T_{SL}	77.84 kN
Wetted Area S_{wet}	138.89 m^2
Wing Area S	27.8709 m^2
Aspect Ratio AR	4.08
Wing Sweep Angle Λ_{LE}	40°
Maximum Speed V_{tmax}	Mach 2.0
Maximum Load Factor n_{max}	8
Skin Friction Coefficient C_f	0.0035
Wave Drag Coefficient C_{Dwave}	0.0261
Lift Curve Slope at M=0 $c_{l\alpha(M=0)}$	5.73 /degree
Max./Min Angle of Attack α	-2° / +25°
Missile	
Mass m_m (before 7 s)	100 kg
Mass m_m (after 7 s)	79 kg
Thrust T_m (before 7 s)	10 kN
Thrust T_m (after 7 s)	0
Maximum Acceleration a_{max}	30 G
Maximum Speed V_{mmax}	Mach 4.0

fighter evades out of the cone of the missile seeker, which is 60° in this study. The vehicle specifications are tabulated in Table 1.

The initial positions and the velocities of the vehicles are fixed in scenario 1. The initial values for this scenario are as follows:

$x_m = 0$ m. $y_m = 0$ m. $z_m = 2000$ m. $V_m = 800$ m/s
$x_t = 14000$ m. $y_t = 0$ m. $z_t = 4000$ m. $V_t = 290$ m/s

Table 2. Split-s - Barrel Roll

Initial Heading of the Fighter, X_{to} \ Initial Heading of the Missile, X_{mo}	-30°	-15°	0°	15°	30°
0°	-	-	-	-	-
10°	+	-	-	-	-
20°	+	-	-	-	-
30°	+	-	-	-	-
40°	+	-	-	-	-
50°	+	-	-	-	-
60°	+	-	-	-	-
70°	+	-	-	-	-
80°	+	-	-	-	-
90°	+	-	-	-	+
100°	+	-	-	-	+
110°	+	-	-	-	+
120°	+	-	-	+	+
130°	+	+	-	+	+
140°	+	+	+	+	+
150°	+	+	+	+	+
160°	+	+	+	+	+
170°	+	+	+	+	+
180°	+	+	+	+	+

Table 3. Immelmann - Barrel Roll

Initial Heading of the Fighter, X_{to} \ Initial Heading of the Missile, X_{mo}	-30°	-15°	0°	15°	30°
0°	+	-	-	-	+
10°	+	-	-	-	+
20°	+	-	-	-	-
30°	+	-	-	-	-
40°	+	-	-	-	-
50°	+	-	-	-	-
60°	+	-	-	-	-
70°	+	-	-	-	-
80°	+	-	-	-	-
90°	+	-	-	-	-
100°	+	-	-	-	-
110°	+	-	-	-	-
120°	-	-	-	-	-
130°	-	-	-	-	-
140°	-	-	-	-	-
150°	-	-	-	-	-
160°	-	-	-	-	-
170°	-	-	-	-	-
180°	-	-	-	-	-

Both the missile and the fighter take level flight initially. The heading angle of the missile varies from -30° to +30° with the interval of 10°, and the heading angle of the fighter from 0° to 180° with the interval of 10°. The throttle setting, u, is 1.0 at all time steps in “Immelmann - barrel roll” combination, but it is adjusted during the “split-s - barrel roll” combination in order to keep the velocity of the fighter near the corner velocity, which is 230 m/s for our model.

Table 2 and Table 3 represent the engagement results when the fighter performs split-s followed by a barrel roll, and Immelmann followed by a barrel roll, respectively. The mark “+” denotes “success” and “-” denotes “failure” of the fighter for scenario 1. The “split-s - barrel roll” combination will be called as maneuver 1, and the “Immelmann - barrel roll” combination as maneuver 2 for clarity.

It’s noted from Tables 2 and 3 that the maneuver 1 gives better performance than the maneuver 2 does. The most important reason for this result is the rapid energy dissipation of the fighter during the Immelmann maneuver. While the available thrust decreases with the increasing altitude, the velocity decreases below the corner velocity, excessively. This causes the fighter to start the barrel roll at a quite low speed. The velocity of the fighter goes down to 178 m/s until it reaches the top of the turn, and this negatively affects the performance of the fighter for following maneuvers.

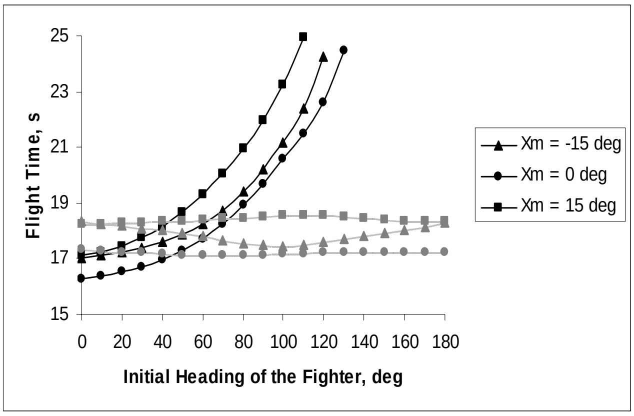

Figure 3. Comparison of Flight Times for the Cases of Failure

When the cases of failure of the fighter is observed, it’s seen that the maneuver 1 shows better performance if the flight time is taken as the performance metric. Longer flight times are obtained by performing the maneuver 1. The flight time comparison of the maneuvers for failure cases in scenario 1 are represented in Fig. 3. The black-straight lines and the gray-dashed lines denote the flight time values for the maneuver 1 and the maneuver 2, respectively.

As seen in Fig. 3, the performance of the maneuver 1 doesn’t change significantly as the initial heading of the fighter increases, besides; the fighter is captured at all initial aspect angles. On the other hand, the performance of the maneuver 1 increases gradually, and the fighter succeeds in evading the missile in cases where its heading becomes nearly aligned with the missile after it completes the split-s, i.e., at beginning of the barrel roll.

One important point is that the maneuver 1 is less effective than the maneuver 2 where the initial heading of the fighter is between 0° and 40°. As the maneuver 1 starts with a split-s turn, the fighter flies down to the initial altitude of the missile and turns towards it. Consequently, after 7 seconds, the missile doesn’t lose much of its velocity while it follows the motions of the target. This causes the flight time obtained by maneuver 1 to be less than maneuver 2.

Along with the results of the scenario 1, it’s considered that the “split-s - barrel roll” combination is more effective under given initial conditions.

Figures. 4-6 represents the trajectories of vehicles for the scenario 1 where the initial heading angles of both the fighter and the missile are 0°. The fighter performs the “split-s - barrel roll” combination. The split-s maneuver takes 11.3 seconds and then the barrel roll maneuver starts. Since the throttle setting is adjusted during the split-s maneuver, the velocity of the fighter is reduced to 240 m/s before it starts the barrel roll maneuver. The velocity variation is shown in Figure 7. The angle of attack rate is controlled at each time step in order to hold the pitch rate and load factor between the aircraft-specific limitations. The maximum pitch rate is chosen 20 degree/second. Another important limitation for the fighter is the load factor. During the maneuver, the load factor must not exceed the aircraft-specific quantity. A load factor limit of 9 g is chosen for the fighter that’s modeled in this study. The alternation of the angle of attack values arises from these limitations (see, Fig 8). The bank angle is applied so that the roll rate of the fighter never exceeds 180 degree/second. This value agrees with the roll rate values of the contemporary fighters. The excepted values are between 100° and 200°. Time history of bank angle is represented in Fig. 9.

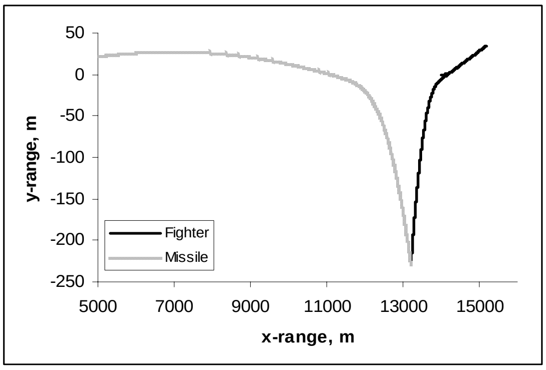

Figure 4. Vehicle Trajectories in the x-y Plane

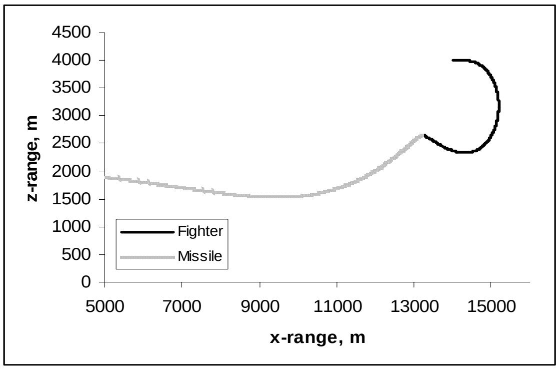

Figure 5. Vehicle Trajectories in the x-z Plane

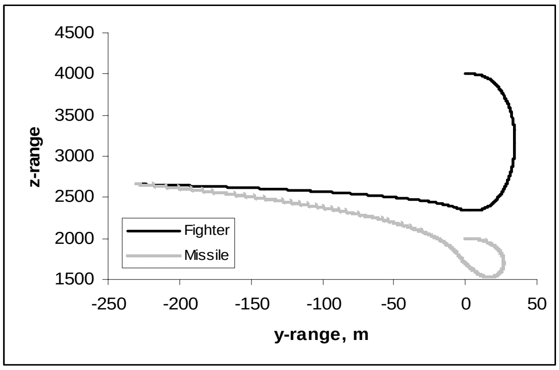

Figure 6. Vehicle Trajectories in the y-z Plane

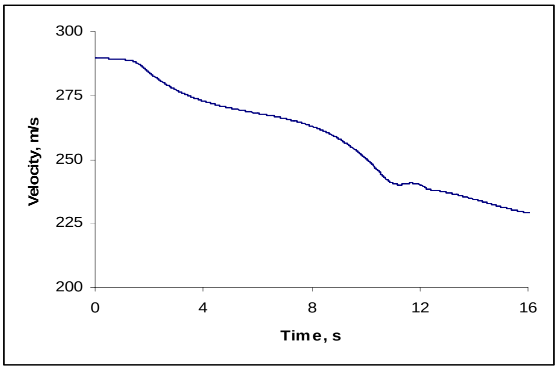

Figure 7. Time History of Velocity

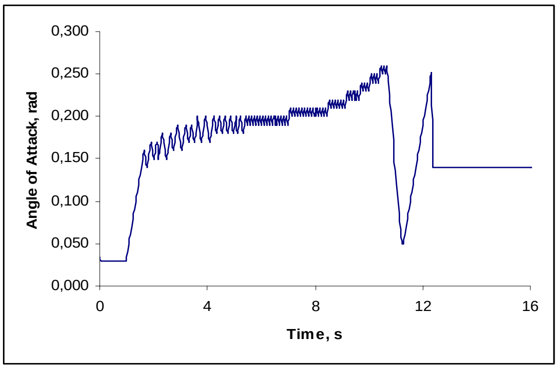

Figure 8. Time History of Angle of Attack

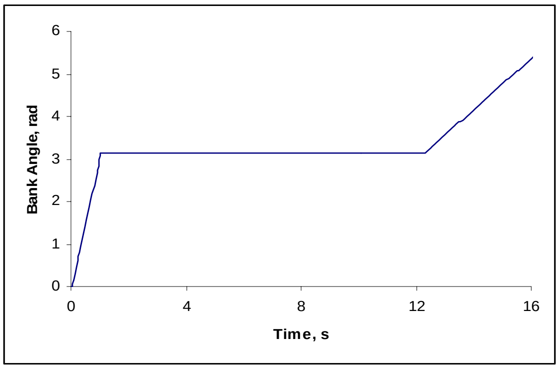

Figure 9. Time History of Bank Angle

In the scenario 1, it's assumed that the pilot starts the maneuver combinations at the beginning of the scenario. This caused the fighter to have very few seconds for the barrel roll. Now, the encounter is initiated by assuming that the pilot has already completed an Immelmann turn which started at 3500 m. with 290 m/s, and he is just at the beginning of the barrel roll maneuver. With these assumptions, the initial conditions of scenario 2 are as follows:

$x_m = 0$ m. $y_m = 0$ m. $z_m = 4800$ m. $V_m = 700$ m/s

$x_t = 8000$m. $y_t = 0$ m. $z_t = 4500$ m. $V_t = 190$ m/s

The initial heading of the missile is fixed at 0° on a collision course with the fighter, and that of the fighter varies from 0° to 180° with the interval of 10 degrees. The resulting flight time values for this scenario are represented in Fig. 10. The mark "*" denotes the success of the fighter in evasion. It's seen that the fighter succeeds in evading the missile when its initial heading angles are perpendicular with that of the missile. It also obtains long flight time by the initial heading of 60°. The performance of the fighter can be increased by starting the barrel roll maneuver at higher velocities. Figure 11 is represented to show the effects of the initial velocities when starting a barrel roll maneuver. As the velocity gets closer to the corner velocity, probability of survival increases. It must be noted that

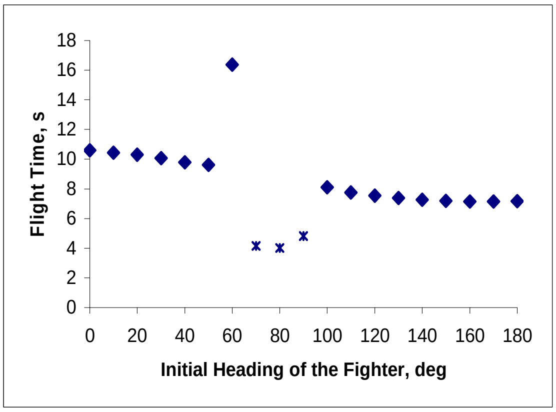

Figure 10. Flight Time Values for Scenario 2

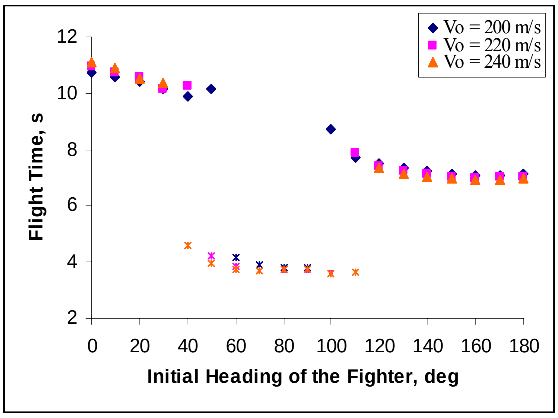

Figure 11. Flight Time Values for Various Initial Velocities at 4500 m

the fighter must start the Immelmann turn at supersonic speed to perform the barrel roll maneuver near corner velocity for the altitude in scenario 2.

A simulation similar to the scenario 2 is conducted with the assumption that the fighter has completed a split-s turn. The initial conditions (x, y, z, V) for scenario 3 are (8000m, 0, 1300m, 240 m/s) for the fighter and (0, 0, 1600m, 700m) for the missile. The initial heading angle of the missile is 0°. When the results of this scenario are compared to the scenario 2, it's seen that the fighter shows better performance when it performs barrel roll after a split-s turn. The higher velocity and the relatively low altitude are considered as the most important reasons for this result. Figure 12 shows the flight time values for these initial conditions. The pilot will have more chance to avoid the missile (between 40° and 110°), and he will prolong the flight time between the initial heading angles of 0° and 30°.

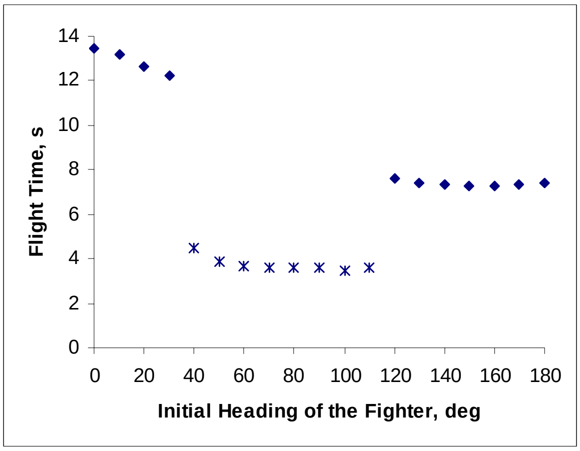

Figure 12. Flight Time Values for Scenario 3

VII. Conclusion

When the fighter has the advantage of altitude over the missile (scenario 1), a split-s turn followed by a barrel roll (maneuver 1) gives better results than the "Immelmann-barrel roll" combination (maneuver 2) does. The effectiveness of the maneuver 1 increases for the initial heading angles of the fighter exceeding 90°. Maneuver 2 is seen advantageous only in a few cases. For instance, longer flight times are obtained by performing this maneuver when the initial heading of the fighter is between 0° and 40°.

In the other two scenarios, the pilot assumed to start the maneuver combinations before the beginning of the encounter. With this assumption, there is more time to perform a barrel roll maneuver, thus, the pilot has more chance to gain advantages of this maneuver. It's seen that the barrel roll maneuver increases the probability to survive when the fighter is near perpendicular to the missile initially. It's also noted in scenario 2 that, performing a barrel roll maneuver after an Immelmann turn which is initiated at subsonic speeds degrades the evasion performance of the fighter.

As a conclusion, the split-s followed by a barrel roll is better than the other maneuver combination. Nevertheless, a pilot must consider the current states before performing an evasive maneuver. For instance, the split-s maneuver can not be performed at low altitudes. Immelmann turns started at high altitudes and low speeds may cause the fighter to lose energy which is crucial in an air combat.

In this study, a modern fighter is given two sub-optimal evasion strategies against a missile employing proportional navigation. The analyses are made with the software called the Visual End-Game Simulation. Currently, all the parameters regarding the missiles and the fighters in an air combat can be observed with this software by providing it with the required vehicle-specific data.

In the near future, the effects of electronic counter-measures to the guidance system of a missile will be studied an added to the software. Also, development of a novel algorithm for optimizing the evasion trajectory of a fighter, and implementation of an autopilot which will take the control of the fighter when an incoming missile is detected is considered as the future work.

Appendix

The wetted area method is used to estimate the zero-lift drag coefficient in the subsonic region. According to this method, a uniform skin friction coefficient can be assumed for the different surfaces of the aircraft[18]. Thus, the zero-lift drag coefficient becomes

$$C_{D0} = C_f \left(\frac{S_{wet}}{S} \right) \tag{9}$$

The skin friction coefficient is denoted by C_f, and all the surface area over which air flows is denoted by S_{wet}. The wave drag coefficient is added to the zero-lift drag coefficient at speeds exceeding Mach 1.

Drag polar is the variation of the drag coefficient of a fighter with its lift coefficient. It is a function of the lift coefficient, C_L, and can be approximated by the following equation:

$$C_{Di} = k_1 C_L^2 \tag{10}$$

The variable k_1 denotes an aircraft-specific constant characteristic, and can be expressed by

$$k_1 = \frac{1}{\pi e_0 AR} \tag{11}$$

for subsonic speeds. Aspect ratio of the wing is denoted by AR. The variable e_0 is the Oswald's efficiency factor which is in range of 0.6 and 0.9. In this paper, the variable e_0 is calculated by the equation obtained with a curve fit of wind tunnel data (see Ref. 18):

$$e_0 = 4.61(1 - 0.045 AR^{0.68})(\cos \Lambda_{LE})^{0.15} - 3.1 \tag{12}$$

The following equation is used to calculate the variable k_1 for supersonic speeds:

$$k_1 = \frac{AR\left(M^2 - 1\right)}{\left(4AR\sqrt{M^2 - 1}\right) - 2} \cos \Lambda_{LE} \tag{13}$$

Finally, the overall drag coefficient is expressed by the following equation:

$$C_D = C_{D0} + C_{Di} \tag{14}$$

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