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Horizontal-s and Barrel Roll Maneuvers Against Proportional Navigation Missiles

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Abstract

In this paper, evasive maneuvers of a fighter against a missile employing proportional navigation are investigated. This work is a part of an ongoing project, Visual End-Game Simulation (VEGAS), on implementation, simulation, and visualization of proportional navigation against an aircraft performing different evasive maneuvers in an air combat. The performances of two maneuvers are examined; barrel roll and horizontal-s. The terminal phase of the encounter is taken into consideration. An extended point-mass aircraft model including orientation kinematics is used to obtain realistic results. Sample encounter scenarios are conducted and flight time analysis is made.

Keywords: Visual end-game simulation, guidance systems, homing missiles, target models

1. Introduction

Since the end of World War II, many different methods for missile guidance have been developed to successfully intercept a stationary, predictable, or even highly maneuvering target [1]. As an expected result of tactical homing missiles' revealing in 1944, it became an obligation to develop target evasive maneuvering tactics against pursuers.

The performance of guidance systems can generally be quantified in terms of the miss distance between the missile and the target. Miss distance is the difference between the target and missile lateral displacements with respect to the reference line of sight (LOS). From a target's point of view, an optimal maneuver means techniques that provide the target with long intercept time and large miss distance.

Since the proportional navigation –in which the missile turning rate is made proportional to the line of sight rate- is the most broadly used guidance method due to its effectiveness. Consequently, in this work, target evasive maneuvers against proportionally navigating guided missiles are studied. The implementation of proportional navigation guidance system proposed by Moran and Altilar [2] is used as the pursuing missile's guidance system.

Up to now, many methods have been studied on optimal maneuvers of an aircraft evading from a proportional navigation guided missile. Some of numerically obtained optimal maneuvers are barrel roll, split-s, and horizontal-s maneuvers. In this paper, we've studied two of notable maneuvers; horizontal-s and barrel roll.

New software that is called Visual End-Game Simulation (VEGAS) has been developed to evaluate the motions of a fighter aircraft and the fighter evasive maneuvers against proportional navigation. All the factors required for gathering realistic results, i.e. aircraft specifications, aerodynamics, kinematics, and notable evasive maneuvers of fighters, are included in this software. On the other hand, the required factors relevant to the missile employing proportional navigation are included in VEGAS. Also the modular structure of the software has made it completely apt to further developments, such as adding electronic counter-measures (ECM), etc.

Extensive simulation results that are supported by comprehensible visual projections have been obtained by using this new software. Visual C++ and OpenGL [3] are used in simulations. 3D visualization is included in order to provide the user for a comprehensive understanding about the terminal phase of the encounter.

The paper is organized as follows. In Section 2, a brief survey of related studies is expounded. Section 3 describes the aircraft and missile models. The definitions of evasive maneuvers are in Section 4, and details of implementation are in Section 5. Results of sample engagement scenario are represented in Section 6. Finally, the conclusions are stated in Section 7.

2. Review of Related Studies

Choi et al. [4] considered three-dimensional target optimal evasion problem against a proportionally guided missile. They formulate the optimal evasion problem of an aircraft as a constrained optimization problem whose payoff is the intercept time and constraint is the capture condition.

Imado and Miwa [5] represented the optimality of the horizontal-s maneuver when the final time and miss distance are taken as the cost function.

Optimal evasive control maximizing the miss distance for very simple two-dimensional missile and constant speed target was considered by Ben-Asher and Cliff [6].

A great number of pursuit-evasion simulations were conducted by giving both aircraft and missile the strategies for combinations of parameter spaces and initial conditions in Ref. [7]. Final miss distance was chosen as the performance index of the games; the missile tried to minimize it, while the aircraft tried to maximize it. According to this method, the basic idea laid in giving players a priori optimal or suboptimal feedback strategies, conducting massive simulations in the parameter space of the initial geometries and guidance law parameters, and analyzing the results.

Imado and Uehara [8] discussed the performance of the high-g barrel roll (HGB) maneuver from optimal control of view. The mathematical model for three-dimensional pursuit-evasion problem of the aircraft against proportional navigation missile was considered; some features of the aircraft optimal evasive maneuvers, and high-g barrel roll maneuvers were explained. Finally, the exact numerical solution for the three-dimensional pursuit-evasion problem was illustrated and the non-optimality of the HGB was shown. The relation between the optimal maneuver and the HGB was also discussed.

Realistic target models including the variation of thrust and aerodynamic forces according to the Mach number were used by Ong and Pierson [9]. Their work is considering optimal evasive maneuvers against proportional missiles.

Moore and Garcia [10] described the implementation of a genetic programming system that evolved optimized solutions to the extended two dimensional pursuer/evader problems that did not depend upon knowledge of pursuer's current state.

Minimum time trajectories to a fixed or moving target were produced with an MS compatible software called Visual Interactive Aircraft Trajectory Optimization (VIATO) by Virtanen et al. [11]. The authors introduced a new approach for the automated solution of optimal flight trajectories. The structure of the aircraft models and the objectives of the problems were specified, and different aircraft types were stored in their model library. The approach was implemented in the VIATO which consists of an optimization server, a model server, and an intuitive, menu-driven, graphical user interface.

The optimal avoidance of a missile employing proportional navigation was dealt with by Raivio and Ranta [12]. An extended point mass vehicle model including orientation kinematics was used to obtain realistic results. The drag, thrust and constraint data of vehicles represented a generic fighter aircraft and a medium range air-to-air missile.

Imado [13] considered different approaches to pursuit-evasion such as giving both players some suboptimal feedback strategies, giving one player a suboptimal feedback strategy and other player an exact one-sided nonlinear optimal control, and giving both players suboptimal feedback strategies dependent on parameters.

National Aeronautics and Space Administration (NASA) developed a simulation that is capable of quickly and efficiently supporting flight research requirements and conceptual vehicle studies (see Ref. [14]). The simulations in this work operate on UNIX-based platforms and were coded with a FORTRAN shell and C support routines. This simulation software is still used at NASA, within industry, and at several universities, and applicable to a broad range of fixed-wing aircraft including fighters.

Another versatile aircraft simulation study made in the NASA [15] emphasized that realistic aircraft motion was of greatest importance, and accurate roll and pitch dynamics were very significant in developing evasive maneuvers against missiles.

This work considers target evasive maneuvers in the terminal phase of missile-aircraft encounter. 3-D encounter scenarios are simulated and flight time analysis is made.

3. Model Definitions

a. Aircraft Model

The terminal phase of a missile-aircraft encounter takes a very short time. In a few seconds, the pilot must decide a maneuver to make and perform it. He must choose the convenient maneuver by considering the characteristics of the aircraft. Consequently, if realistic evaluation of the effectiveness of the performed maneuvers is desired, aerodynamic forces and rotational kinematics must be taken into account.

The equations of motion are (see Ref. [16])

$$\dot{x}_a = v_a \cos \Upsilon_a \cos \mathrm{X}_a \tag{1}$$

$$\dot{y}_a = v_a \cos \Upsilon_a \sin \mathrm{X}_a \tag{2}$$

$$h_a = v_a \sin \Upsilon_a \tag{3}$$

$$\dot{\Upsilon}_a = \frac{1}{m_a v_a} \begin{bmatrix} \left(C_L \left(\alpha, M \left(h_a, v_a \right) \right) S q \left(h_a, v_a \right) + u T_{\max} \left(h_a, M_a \left(h_a, v_a \right) \right) \sin \alpha \right) \\ \cos \mu - m_a g \cos \Upsilon_a \end{bmatrix} \tag{4}$$

$$\dot{\mathrm{X}}_a = \frac{1}{m_a v_a \cos \Upsilon_a} \begin{bmatrix} C_L \left(\alpha, M \left(h_a, v_a \right) \right) S q \left(h_a, v_a \right) \\ + u T_{\max} \left(h_a, M \left(h_a, v_a \right) \right) \sin \alpha \end{bmatrix} \sin \mu \tag{5}$$

$$\dot{v}_a = \frac{1}{m_a} \left(u T_{\max} \left(h_a, M \left(h_a, v_a \right) \right) \cos \alpha - C_D S q \left(h_a, v_a \right) \right) - g \sin \Upsilon_a \tag{6}$$

x_a, y_a, h_a, Y_a, X_a ,and v_a are *x* and *y* range, the altitude, the flight path angle, the heading angle, and the velocity, respectively. g is the acceleration of gravity, m_a is the mass of the aircraft, the both assumed constant. *S* is the reference wing area and the dynamic pressure $q\ (h_a,\ v_a) = 1/2\rho v^2$. Here, ρ is the density of the air at a specific altitude. $T_{max}\ (h_a, M_a(h_a, v_a))$ denotes the maximum available thrust. The fighter in our model is assumed to have a turbofan engine with afterburner. Hence, the maximum available thrust is approximated by the following equation (see Ref. [17]):

$$T_{\max} = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right)(1+0.7M) \tag{7}$$

where T_{SL}, ρ_{SL}, and M are thrust at sea level, air density at sea level and the Mach number, respectively. C_D is the overall drag which is separated into two components; the zero-lift drag coefficient, C_{D0}, and the induced drag coefficient, C_{Di}. The details of derivation of the total drag coefficient are given in Appendix-1. The control variables are the angle of attack α, the throttle setting u, and the bank angle μ. There are some constraints for the angle of attack. These are

- The lift coefficient $C_L(\alpha, M(h_a, v_a))$ must not exceed the aircraft-specific quantity, $C_{L,max}(M)$
- The load factor $n(\alpha, h_a, v_a)$ must not exceed the aircraft-specific quantity, $N_{\max}$
- The pitch rate must not exceed $P_{\max}$.

The state variables and the limitations regarding the control variables are detailed in Ref. [12].

b. Missile Model

The missile's guidance system is the Proportional Navigation. Theoretically, this law issues acceleration commands, perpendicular to the instantaneous missile-aircraft line-of-sight, which are proportional to the line-of-sight rate and closing velocity, V_C (see Ref.[1]). The guidance law can be stated as

$$n_c = N' V_C \dot{\lambda} \tag{8}$$

where n_c is the acceleration command, N', a unitless gain (usually in range of 3-5) known as the effective navigation ratio, and $\dot{\lambda}$ the line-of-sight rate. In this paper, the proposed implementation of proportional navigation guidance law in Ref. [2] is used as the pursuing missile's guidance system.

4. Definition of Evasive Maneuvers

When a missile is fired at an aircraft in the aft quadrant, the best way to defeat it is with a maximum rate turn to put the missile on the beam, which is called 3/9 line. This kind of a maneuver gives a proportional guidance missile the most difficult guidance solutions possible problems. In this position, the evading aircraft will be at 90 degrees of aspect with respect to the missile, and it will have the worst possible line-of-sight rate problem to solve. Missiles fly lead pursuit courses to the target in order to achieve maximum range. If the missile is held somewhere on 3/9 line, it will make the missile pull the maximum amount of lead.

The pilots perform different maneuvers to avoid the missiles in the wake of the basic principals mentioned above. The following subsections briefly describe the maneuvers implemented in this paper. For details, see Ref. [18].

a. The Barrel Roll Maneuver

Similar to a roll, the pilot applies back pressure on the stick while he is rolling to the left or right. This makes the aircraft plane fly in a corkscrew pattern and is used to make the missile deal with the most difficult guidance problems. The flight path of a fighter performing a barrel roll maneuver is represented in Figure 1.

Figure 1. Barrel roll maneuver

b. The Horizontal-s Maneuver

A notable maneuver to avoid a missile is the break turn maneuver. This maneuver results in a tight turn with high roll angle. If this maneuver is performed repeatedly, then it becomes a horizontal-s maneuver and it is commonly performed against missiles. The horizontal-s is a high energy consuming maneuver. The flight path of a fighter performing a horizontal-s maneuver is represented in Figure 2.

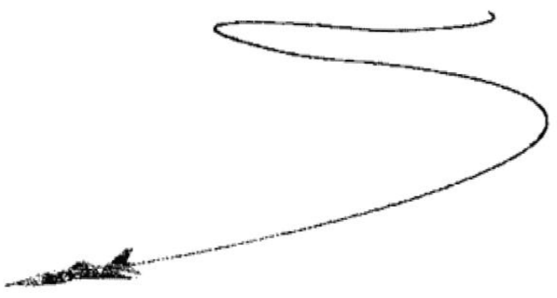

Figure 2. Horizontal-s maneuver

5. Implementation

The 3D simulation of horizontal-s and barrel roll maneuvers is implemented. Visual C++ and Open GL are used in simulations.

There are five main modules in the program. These are main, pursuer, evader, radar and aero modules. The main module serves as the manager of the simulation which initializes the simulation, calls the pursuer and evader modules, and handles drawings. The pursuer module is the program where the motion of the missile is modeled. A missile that employs proportional navigation guidance system is taken into consideration. The implementation of the missile model is detailed in Ref. [2].

The third module in the simulation is the evader module. In this part of the simulation, a generic aircraft with high-g capability is modeled. An extended point-mass aircraft model including orientation kinematics is used to obtain realistic results. In this paper, two of well-known aircraft evasive maneuvers, horizontal-s and barrel roll, are simulated.

The coordinates of the missile and the target is provided for the pursuer and evader modules by the radar module.

The required values for aerodynamic calculations, the air density and the Mach number values are computed in the aero module.

6. Engagement Scenario

In this scenario, the initial positions of the vehicles are fixed. The altitude of the missile is some hundred meters higher than the fighter's. The initial conditions are as follows:

$x_m = 0$ m. $\quad y_m = 0$ m. $\quad z_m = 3300$ m.
$V_m = 950$ m/sec $\quad X_m = 0^\circ$
$x_t = 8000$ m. $\quad y_t = 0$ m. $\quad z_t = 3000$ m.
$V_t = 240..300$ m/sec $\quad X_t = 0^\circ..180^\circ$

x, y, z are the coordinates of the vehicles. V and X are the velocity, and the heading angle of the vehicles, respectively. Subscript 'm' denotes the missile, and subscript't' denotes the target. Both vehicles take level flight initially. The heading angle of the missile is fixed. The velocity of the fighter varies from 240 m/sec to 300 m/sec with the interval of 20 m/sec. For each initial velocity of the target, its heading angle varies from 0° to 180° with the interval of 5°. The results of this scenario for horizontal-s and barrel roll maneuvers are represented in Figures 3 and 4, respectively. The mark * in the figures denote the failure of the missile.

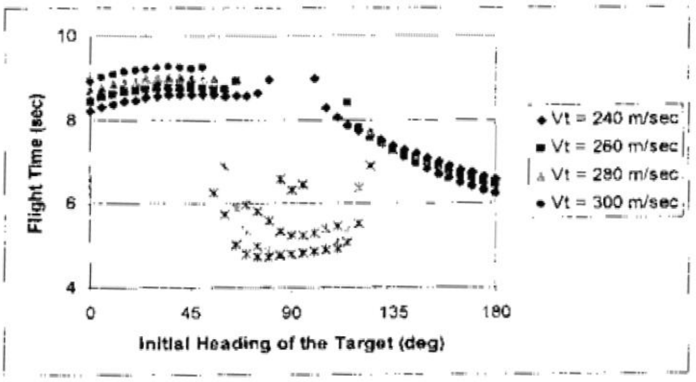

Figure 3.flight time values for horizontal-s maneuver

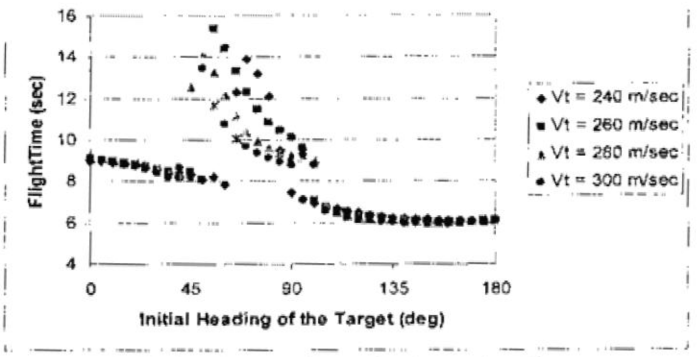

Figure 4.flight time values for barrel roll maneuver

It's noted from the figures that the initial heading of the fighter makes significant changes on the flight time for the cases which the missile achieves to hit the fighter. For instance, when the fighter performs horizontal-s maneuver, initial heading angles which makes fighter go far from the missile, i.e. $0° < X_t < 90°$, favor flight time for the fighter. On the other hand, the resulting flight time values decrease as the fighter is oriented towards the missile.

It's also noted that, in general, higher initial velocities causes the flight time to increase only when the initial heading of the fighter is between 0 and 90 degrees. For initial heading angle values exceeding 90 degrees, lower initial velocities seem to obtain better flight time results for the fighter.

The most important point for this scenario is the flight time values when the initial heading angles are near 90 degrees. In this case, it's seen that higher initial velocities favors evasion performance. The best performance is obtained by the horizontal-s maneuver.

The barrel roll maneuver is seemed noteworthy due to its efficiency between 45 and 90 degrees. By performing this maneuver, long flight times are obtained for any of the initial velocities. It has been seen that this maneuver may be performed at any initial speed to gain time when the initial headings are between 45 and 90 degrees.

Note that the initial heading of the missile is 0^0 in all cases of the scenario. It means that its heading is always towards the fighter regardless of its heading at the beginning of the engagement scenario. According to the proportional navigation guidance system, the missile steers towards the anticipated position of its target. Hence, the initial heading of the missiles is set to $+5^0$ and the flight time values are observed. Figure 5 represents the results under these conditions when the initial velocity of the fighter is 280 m/sec.

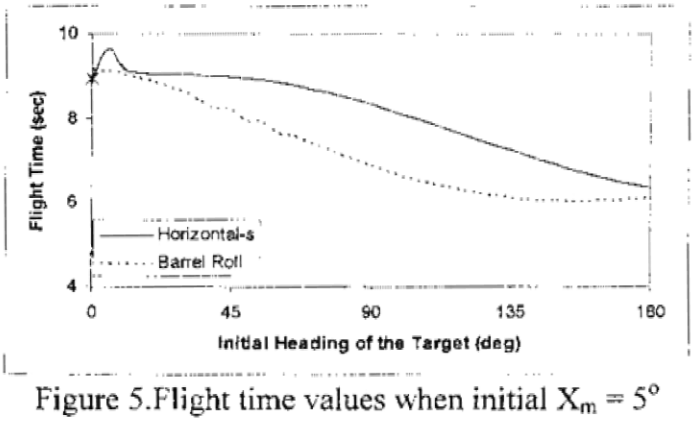

Figure 5.Flight time values when initial $X_m = 5^o$

It's noted from Figure 5 that; by performing the horizontal-s maneuver, better results than the barrel roll are obtained. It's observed that the initial heading angles beyond +55° decreases evasion performance of the fighter.

Consequently, under given conditions, it can be concluded that the horizontal-s maneuver is the most convenient evasive maneuver against proportional navigation when we take the flight time as the performance metric.

The figures that are represented for the scenario show another important fact. If the headings of the vehicles are towards each other initially, none of the evasive maneuvers that the fighter performs can change the flight time significantly. So, a fighter pilot must try not to fall in such a positional geometry.

7. Conclusion

In an air combat, the maneuvers performed by the fighters are of crucial importance. Understanding the current tactical situation, choosing the convenient maneuver, applying correct commands are vital matters for a fighter pilot. Especially, when an incoming missile is detected, a pilot has very few seconds to think and to make a move. So, he must know the characteristics, the limitations and the abilities of his fighter. This is only possible with training. As practicing the maneuvers with real fighters are very expensive and time consuming task, it is inevitable to model realistic fighters and to simulate their maneuvers with computers. Namely, a pilot must know what may happen where and when he performs an evasive maneuver before he takes off for a mission. It will be invaluable for an air force whose pilots are illuminated with this knowledge.

By the simulation runs that have been conducted, it is noted that the performance of a particular evasive maneuver may vary according to the initial positional geometry. The performance can be improved by making some changes to applied commands which will change the turn radius, the load factor, and the velocity, etc. of the fighter. It is also noted that there are significant effects of aerodynamic forces on the attitudes of the fighter.

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Appendix-1

The wetted area method is used to estimate the zero-lift drag coefficient in the subsonic region. According to this method, a uniform skin friction coefficient can be assumed for the different surfaces of the aircraft [16]. Thus, the zero-lift drag coefficient becomes

$$C_{D0} = C_f \left(\frac{S_{wet}}{S} \right) \tag{9}$$

The skin friction coefficient is denoted by C_f, and all the surface area over which air flows is denoted by S_{wet}. The wave drag coefficient is added to the zero-lift drag coefficient at speeds exceeding Mach 1.

Drag polar is the variation of the drag coefficient of a fighter with its lift coefficient. It is a function of the lift coefficient, C_L, and can be approximated by the following equation:

$$C_{Di} = k_1 C_L^2 \tag{10}$$

The variable k_1 denotes an aircraft-specific constant characteristic, and can be expressed by

$$k_1 = \frac{1}{\pi e_0 AR} \tag{11}$$

for subsonic speeds. Aspect ratio of the wing is denoted by AR. The variable e_0 is the Oswald's efficiency factor which is in range of 0.6 and 0.9. In this paper, the variable e_0 is calculated by the equation obtained with a curve fit of wind tunnel data (see Ref. [17]):

$$e_0 = 4.61(1 - 0.045 AR^{0.68})(\cos \Lambda_{LE})^{0.15} - 3.1 \tag{12}$$

where Λ_{LE} is the sweep angle on the leading edge of the wing. The following equation is used to calculate the variable k_1 for supersonic speeds:

$$k_1 = \frac{AR\left(M^2 - 1\right)}{\left(4AR\sqrt{M^2 - 1}\right) - 2} \cos \Lambda_{LE} \tag{13}$$

Finally, the overall drag coefficient is expressed by the following equation:

$$C_D = C_{D0} + C_{Di} \tag{14}$$