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Three Plane Approach for 3D True Proportional Navigation

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Abstract

A new three-dimensional (3D) guidance approach is developed in this paper. It is named as Three Plane Approach (TPA). The performance of TPA has been tested visually via developed simulation environment. Although this approach is based on True Proportional Navigation (TPN), it diverges when computing accelerations special to TPN, and putting 3D acceleration commands into practice in Cartesian coordinates. Proposed approach, TPA works by separating 3D space with axes x, y and z to three perpendicular plane xy, xz and yz respectively; after solving the pursuit-evasion problem analytically in these planes and computing required accelerations, rejoining the solutions to three-dimensional environment with respect to the geometric relationships. Trajectory and performance analysis are performed in simulation software, VEGAS (Visual End-Game Simulation). It is verified that the performance of proposed approach is robust and effective in terms of the miss distance and interception time for the high-g capacity aerial targets employing evasive manoeuvres. The adaptive approach proposed in this paper can be applied also to the other Proportional Navigation types.

Key words: Homing missiles, missile guidance, true proportional navigation

1. Introduction

While looking into air-to-air missiles, it is realized that the most crucial phase of an air encounter is terminal phase, the last seconds of it; because the success or failure of entire mission is determined in this phase.

We developed a new approach for missile guidance in the terminal phase. Missile guidance law presented in this paper is based on the TPN [1]. TPA works by separating 3D space with axes x, y and z to three perpendicular plane xy, xz and yz respectively; after solving the pursuit-evasion problem analytically in these planes and computing required accelerations, rejoining the solutions to three-dimensional environment with respect to the geometric relationships. Review of the work and motivation is explained in Section II. Three plane geometry, the angular relationships and analytical solution of TPA for the interception of a maneuvering target are introduced in Section III. To demonstrate the effectiveness of our approach we constructed numerical simulations with visual elements. We named our simulation software as VEGAS (Visual End-GAme Simulation). In VEGAS, two-player pursuit-evaser problem is implemented wherein a missile guided with TPA is commanded to intercept an aerial target employing basic fighter aircraft evasive manoeuvres. Simulation definitions, including missile model and design features of VEGAS are explained in Section IV. In Section V, evaluation of our approach has been constructed with respect to the performance metrics. Numerical results and visual elements from the simulation are shown in this section.

2. Background

A guidance system acquires data from various on-board or external sensors and generates relevant signals or set points for its control system. Guidance issues are mainly determined by the characteristics and the location of both target and the missile, and the environmental conditions. Many of the current operational guided missiles employ PN as the guidance law for the terminal phase. PN has been proved to be a useful guidance scheme in many air-to-air and surface-to-air homing systems for the interception of airborne targets [1, 2]. The major advantage of PN is its simplicity of implementation in missile systems. PN requires low level of target information thus simplifying missile sensor requirements and improving effectiveness. Theoretically, True Proportional Navigation (TPN) guidance law generates acceleration commands perpendicular to the instantaneous missile-target line-of-sight (LOS), and proportional to the line-of-sight rate (LOSR) and closing velocity, V_c.

Proportional Navigation (PN) has been known since World War II. and applied by the Germans at *Peenemünde Research Center*. The "*Lark*" missile, which was successfully tested in 1950, was the first missile to use PN. Proportional Navigation was studied by C. Yuan et al. at RCA Laboratories during under the support of the U.S. Navy [3].

PN was studied at *Hughes Aircraft Company* and implemented for a tactical missile using a pulsed radar system. It was examined at *Raytheon* and implemented in a tactical continuous wave radar homing missile [4]. After World War II., the U.S. work on PN was declassified and first appeared in [5]. Today, guidance commands proportional to the LOS angle rate are generally used by most of the high-speed missiles to correct the missile course in the guidance loop [6,7]. In PN, the acceleration of the missile is, perpendicular to the velocity of the missile (PPN) or perpendicular to the line-of-sight (TPN), and proportional to the observed line-of-sight rate (LOSR) and the closing velocity, V_c. The line-of-sight rate (LOSR) is the angular velocity of the line connecting the missile and the target. Hence, the change in the heading of the missile is also proportional to the LOSR.

In other words, velocity vector of the missile is rotated at a rate that is proportional to the rotation rate of the line joining the missile and the target (LOSR). In essence, PN is simply a proportional controller that regulates the LOSR to zero. The idea is that if LOSR is zero; the target and the missile are on collision course.

Actually in an encounter, the missile seeker attempts to track the target and measures the line-of-sight angle (LOS) and the closing velocity, V_c. A guidance command is generated, based on the Proportional Navigation guidance law. The flight control system enables the missile to manoeuvre in such a way that the achieved acceleration matches the acceleration commands from the guidance law.

PN is the most common and effective technique that seeks to nullify the angular velocity of the LOS angle. The missile heading rate is made proportional to the LOS rate. The rotation of the LOS is measured by a sensor either onboard or located at a ground station, which causes commands to be generated to adjust the direction of the missile in the direction of the target. Mathematically PN law can be stated as:

$$n_c = N' . V_c . \dot{\lambda} \quad (1)$$

where, n_c is the acceleration command, N' is the effective navigation ratio, V_c is the closing velocity, $\dot{\lambda}$ is the LOS angle rate. In tactical active homing missiles that using PN as guidance law; seeker provides measurement of the line-of-sight rate, $\dot{\lambda}$ and radar provides closing velocity, V_c. Computed Proportional Navigation acceleration commands are implemented by tactical missile's control surfaces to obtain the required lift for the missile. A two-dimensional missile-target engagement geometry for Proportional Navigation is shown in Fig. 1.

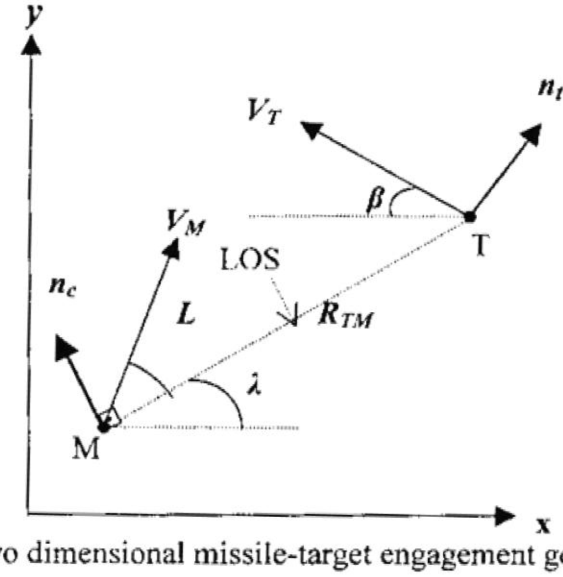


Figure 1. Two dimensional missile-target engagement geometry for TPN.

In Fig.1, the capital M and T denotes the missile and the target respectively. The imaginary line connecting the missile to the target is the line-of-sight (LOS). LOS makes an angle of λ with respect to the x-axe. The length of the LOS called range and denoted R_{TM}. Missile velocity vector, V_M makes an angle of L with respect to LOS angle. The angle L is called the missile lead angle. V_T is the target velocity vector. β is the flight path angle of the target and n_c is the acceleration magnitude which is generated by PN guidance law.

Considering the geometry drawn in Fig.1; details of the missile-target engagement model in two-dimensional (2D) space are presented below. The effective navigation ratio, N', is related to the relative velocity between the missile and the target and derived from Eq.2 [8]:

$$3\,\frac{V_M - V_T}{V_M} < N' < 3\,\frac{V_M + V_T}{V_M} \tag{2}$$

The missile will stay on the collision triangle if target does not change its heading or speed in this time interval, once L is computed. The point of closest range of the missile and the target is miss distance. It is desired to make the miss distance zero or acceptable non-zero values that will keep the target in explosion impact range.

The initial angle of the missile velocity vector with respect to the line-of-sight (LOS) i.e. the missile lead angle L can be computed by applying of the law of sine:

$$L = \arcsin\,\frac{V_T.\,\sin\,(\beta+\lambda)}{V_M} \tag{3}$$

The components of the target velocity vector, V_T on x and y axis are given in Eq.4 and Eq.5 respectively. Negative sign in the term V_{Tx} comes from the projection of V_T on to the x-axe as seen in Fig.1.

$$V_{Tx} = -\,V_T.\,\cos\,\beta \tag{4}$$

$$V_{Ty} = \;\; V_T.\,\sin \tag{5}$$

As the first derivative of the displacement (position) vector gives the velocity vector and consecutively the first derivative of the velocity vector gives the acceleration vector, the following differential equations having the components of the target and missile position can be derived.

Note that subscripts T and M indicate target and missile where x and y indicate the related axis. Considering target position components, differential equations are:

$$\dot{P}_{Tx} = V_{Tx} \tag{6}$$

$$\dot{P}_{Ty} = V_{Ty} \tag{7}$$

Similarly, considering the missile position components:

$$\dot{P}_{Mx} = V_{Mx} \tag{8}$$

$$\dot{P}_{My} = V_{My} \tag{9}$$

And the missile velocity components:

$$\dot{V}_{Mx} = a_{Mx} \tag{10}$$

$$\dot{V}_{My} = a_{My} \tag{11}$$

where a_{Mx} and a_{My} are the components of missile acceleration, n_c, which will be obtained by the PN law.

Considering that any vector constitutes two projections on two perpendicular axes. (x and y for this case), R_{TM} can be defined as follows:

$$R_{TM} = \sqrt{P^2{}_{TMx} + P^2{}_{TMy}} \tag{12}$$

where, relative position components are P_{TMx} and P_{TMy} are:

$$P_{TMx} = P_{Tx} - P_{Mx} \tag{13 a}$$

$$P_{TMy} = P_{Ty} - P_{My} \tag{13 b}$$

Assuming that R_{TM} is the absolute distance between the missile and the target; closing velocity, V_c is defined as the negative change rate of the distance between the target and the missile.

Therefore;

$$V_c = -\acute{R}_{TM} \tag{14}$$

When the first derivative of Eq.16 is taken which is equal to Eq.18:

$$V_c = -\acute{R}_{TM} = \frac{P_{TMx} \, . \, V_{TMx} + P_{TMy} \, . V_{TMy}}{R_{TM}} \tag{15}$$

where, relative velocity components are:

$$V_{TMx} = V_{Tx} - V_{Mx} \tag{16}$$

$$V_{TMy} = V_{Ty} - V_{My} \tag{17}$$

Considering the projections of R_{TM} on x and y axes, the line-of-sight angle, λ is:

$$\lambda = arctan \frac{P_{TMy}}{P_{TMx}} \tag{18}$$

and the first derivative of λ is:

$$\dot{\lambda} = \frac{P_{TMx} \,.\, V_{TMy} - P_{TMy} \,.\, V_{TMx}}{R^2_{TM}} \tag{19}$$

when the variables in Eq.1 are replaced with the ones in Eq.15 and Eq.19, the magnitude of missile acceleration can be defined in terms of target-missile distance.

$$n_c = N' \frac{-(R_{TMx} \,.\, V_{TMx} + R_{TMy} \,.\, V_{TMy})}{R_{TM}} \; \frac{(R_{TMx} \,.\, V_{TMy} - R_{TMy} \,.\, V_{TMx})}{R^2_{TM}} \tag{20}$$

Since n_c is perpendicular to the instantaneous line-of-sight (LOS), missile acceleration components for x and y axes can be derived as follows:

$$a_{Mx} = - n_c \,.\, sin\, \lambda \tag{21}$$

$$a_{My} = \quad n_c \,.\, cos\, \lambda \tag{22}$$

In practice, the missile is not launched on a collision triangle, since the expected intercept point is not known precisely.

Any angular deviation of the missile from the collision triangle is called heading error and denoted ***HE***. Accordingly, initial missile velocity components can be expressed as:

$$V_{Mx}(0) = V_M \,.\, cos\,(\, L + HE + \lambda \,) \tag{23}$$

$$V_{My}(0) = V_M \, . \, \sin \ (L+HE+\lambda) \tag{24}$$

Zero terms in the equations denote the initial conditions. The differential equations derived above are sufficient to model missile-target engagement in two-dimensions (2D) for True Proportional Navigation.

3. The Three Plane Approach (TPA) for 3D True Proportional Navigation

Keeping the essentials of two-dimensional TPN approach explained in Section II. in mind, Three Plane Approach (TPA) has been developed and is explained in detail in this chapter. In the TPA; three dimensional (3D) engagement space is projected onto three perpendicular planes: S_{xy}, S_{xz} and S_{yz}. (Figure 2) It is assumed that the missile and the target are point mass and having the velocity vectors V_T, V_M respectively. The projection of such two point masses' relative motion geometry to S_{xy}, S_{xz} and S_{yz} planes are shown in Fig. 3 (a),(b),(c).

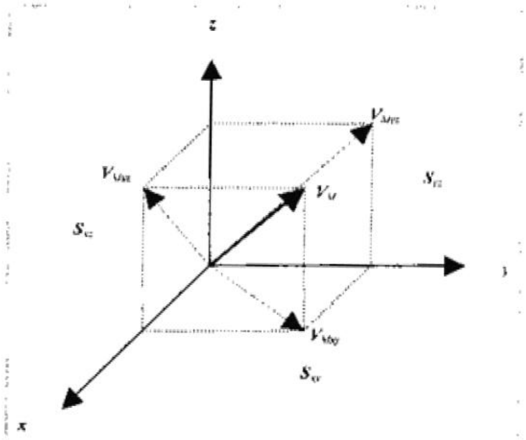


Figure 2. Projections of missile velocity vector on to three planes.

The projections of missile velocity vector on to the planes are shown in Fig.2. In a pursuit-evasion scenario, there are two main vectors to deal with: target and missile velocity vectors.

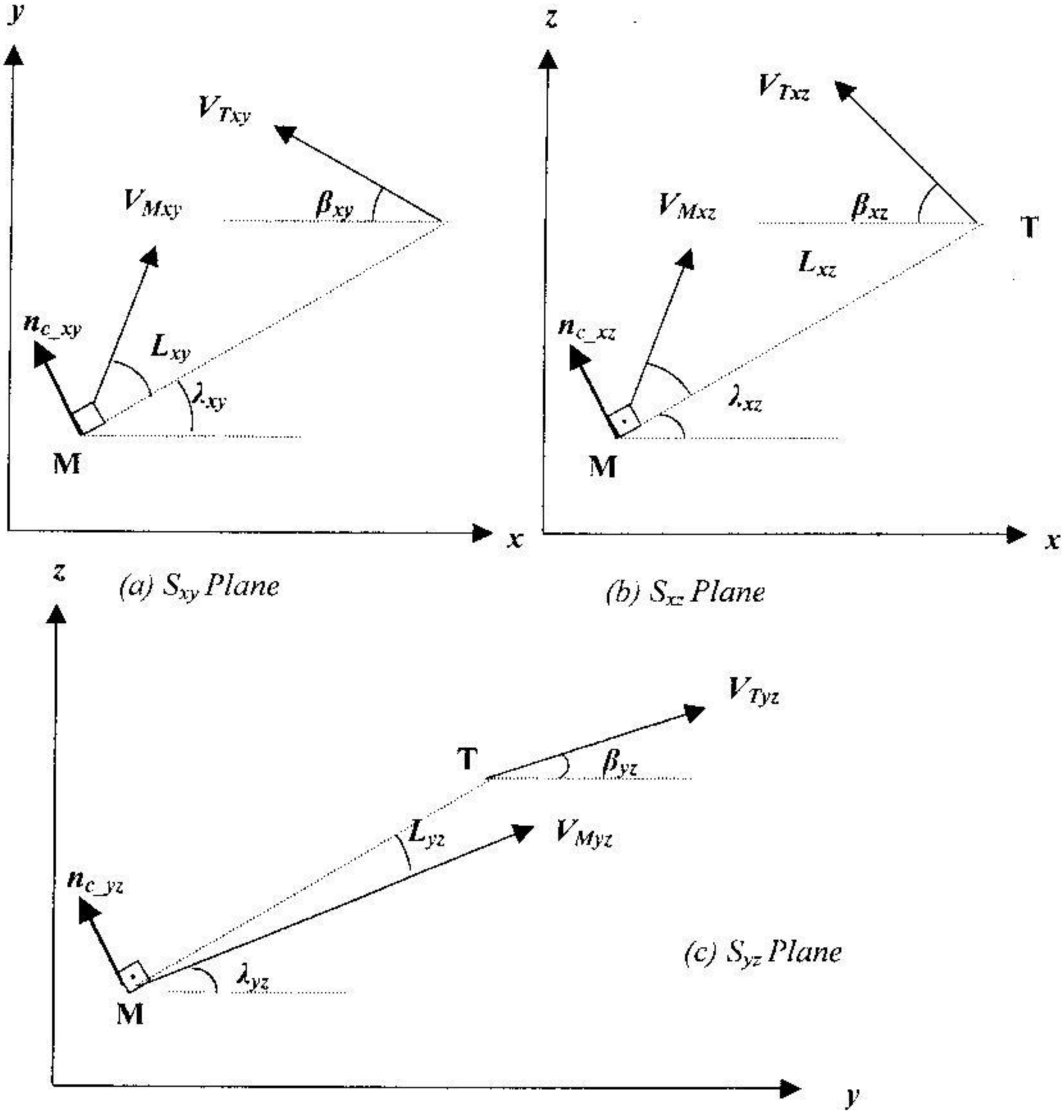


Figure 3. The projections of target's and missile's relative motion onto $\mathbf{S}_{xy}$, $\mathbf{S}_{xz}$ and $\mathbf{S}_{yz}$ planes.

Our approach to solve guidance problem in 3D space is to project 3D geometry onto 3 perpendicular planes such as, S_{xy}, S_{xz} and S_{yz}; to solve the problem in each plane independently for two-dimensional True Proportional Navigation (TPN) and to produce a 3D solution by combining these 2D solutions.

Considering Fig.2 and Fig.3, the components of 2D solutions and equations to combine those to provide 3D solutions are derived as follows:

The distance between the target and the missile is:

$$R_{TM} = \sqrt{P^2_{TMx} + P^2_{TMy} + P^2_{TMz}} \tag{25}$$

The line of sight (LOS) angles:

$$\lambda_{xy} = \arctan \frac{P_{TMy}}{P_{TMx}} \tag{26 a}$$

$$\lambda_{xz} = \arctan \frac{P_{TMz}}{P_{TMx}} \tag{26 b}$$

$$\lambda_{yz} = \arctan \frac{P_{TMz}}{P_{TMy}} \tag{26 c}$$

Target flight-path angles:

$$\beta_{xy} = \arctan \frac{\Delta P_{Ty}}{\Delta P_{Tx}} \tag{27 a}$$

$$\beta_{xz} = \arctan \frac{\Delta P_{Tz}}{\Delta P_{Tx}} \tag{27 b}$$

$$\beta_{yz} = \arctan \frac{\Delta P_{Tz}}{\Delta P_{Ty}} \tag{27 c}$$

Projection of target velocity vector onto S_{xy}, S_{xz} and S_{yz} planes:

$$V_{Txy} = \sqrt{V^2_{Tx} + V^2_{Ty}} \quad (28\ a)$$
$$V_{Txz} = \sqrt{V^2_{Tx} + V^2_{Tz}} \quad (28\ b)$$
$$V_{Tyz} = \sqrt{V^2_{Ty} + V^2_{Tz}} \quad (28\ c)$$

Missile lead angles, L_{xy}, L_{xz} and L_{yz} for each plane (i.e., S_{xy}, S_{xz}, S_{yz}) have to be computed considering Eq.26-Eq.29 c. In order to find the current leading angles for all possible engagement schemes:

$$L_{xy} = \arcsin \frac{V_{Txy} \cdot \sin(\beta_{xy} + \lambda_{xy})}{V_M} \quad (29\ a)$$

$$L_{xz} = \arcsin \frac{V_{Txz} \cdot \sin(\beta_{xz} + \lambda_{xz})}{V_M} \quad (29\ b)$$

$$L_{yz} = \arcsin \frac{V_{Tyz} \cdot \sin(\beta_{yz} + \lambda_{yz})}{V_M} \quad (29\ c)$$

It can be seen from Eq.1 and Eq.20 that to produce the required acceleration commands for each plane; their closing velocity (V_c) and line of sight (LOS) change rate ($\dot{\lambda}$) values must be computed. From Eq.55 (a)-(c); change rates of LOS angles can be derived as:

$$\dot{\lambda}_{xy} = \frac{P_{TMx} V_{TMy} - P_{TMy} V_{TMx}}{P^2_{TMx} + P^2_{TMy}} \quad (30\ a)$$

$$\dot{\lambda}_{xz} = \frac{P_{TMx} V_{TMz} - P_{TMz} V_{TMx}}{P^2_{TMx} + P^2_{TMz}} \quad (30\ b)$$

$$\dot{\lambda}_{yz} = \frac{P_{TMy} V_{TMz} - P_{TMz} V_{TMy}}{P^2_{TMy} + P^2_{TMz}} \quad (30\ c)$$

To compute the closing velocities(V_{Cxy}, V_{Cxz}, V_{Cyz}) for each plane; P_{TMxy}, P_{TMxz}, P_{TMyz} values must be differentiated just like in Eq.15, thus,

$$V_{Cxy} = -\frac{P_{TMx} . V_{TMx} + P_{TMy} . V_{TMy}}{(P^2_{TMx} + P^2_{TMy})^{1/2}} \quad (31\text{ a})$$

$$V_{Cxz} = -\frac{P_{TMx} . V_{TMx} + P_{TMz} . V_{TMz}}{(P^2_{TMx} + P^2_{TMz})^{1/2}} \quad (31\text{ b})$$

$$V_{Cyz} = -\frac{P_{TMy} . V_{TMy} + P_{TMz} . V_{TMz}}{(P^2_{TMy} + P^2_{TMz})^{1/2}} \quad (31\text{ c})$$

where, relative velocity components are:

$$V_{TMx} = V_{Tx} - V_{Mx} \quad (32\text{ a})$$
$$V_{TMy} = V_{Ty} - V_{My} \quad (32\text{ b})$$
$$V_{TMz} = V_{Tz} - V_{Mz} \quad (32\text{ c})$$

Hence,

$$n_{c\_xy} = N' . V_{Cxy} . \dot{\lambda}_{xy} \quad (33\text{ a})$$
$$n_{c\_xz} = N' . V_{Cxz} . \dot{\lambda}_{xz} \quad (33\text{ b})$$
$$n_{c\_yz} = N' . V_{Cyz} . \dot{\lambda}_{yz} \quad (33\text{ c})$$

acceleration commands for S_{xy}, S_{xz} and S_{yz} planes are derived.

Missile acceleration components (a_{Mx}, a_{My}, a_{Mz}) for x, y and z axis can be computed by combining two components sharing the same axis. Fig.3 indicates that one axe' acceleration component is interacted by 2 planes' acceleration commands. By the help of trigonometric relationships, unified missile acceleration components of axes x, y and z can be founded as below:

$$a_{Mx} = -n_{c\_xy} . \sin \lambda_{xy} - n_{c\_xz} . \sin \lambda_{xz} \quad (34\text{ a})$$
$$a_{My} = n_{c\_xy} . \cos \lambda_{xy} - n_{c\_yz} . \sin \lambda_{yz} \quad (34\text{ b})$$
$$a_{Mz} = n_{c\_xz} . \cos \lambda_{xz} + n_{c\_yz} . \cos \lambda_{yz} \quad (34\text{ c})$$

In the literature, [9, 10, 11, 12] while implementing the acceleration commands as control variables in equations of motion, they are broken into vertical and horizontal components, named as a_{pitch} and a_{yaw}.

The vertical acceleration component, a_{pitch} is directed perpendicular to the velocity vector of the missile and upwards, and the horizontal component, a_{yaw} is perpendicular to both the velocity vector and the vertical acceleration component. The suggested vertical and horizontal accelerations generated by Proportional Navigation are defined as:

$$a_{pitch} = N' \cdot V_c \cdot \dot{\lambda}_{pitch} + g \cdot \cos \gamma_M \tag{35}$$

$$a_{yaw} = N' \cdot V_c \cdot \dot{\lambda}_{yaw} \tag{36}$$

where, γ_M is flight path angle between the velocity vector and its projection onto the xy plane; $\dot{\lambda}_{pitch}$ and $\dot{\lambda}_{yaw}$ are the line-of-sight rates (LOSR) for relative vertical and horizontal motion respectively. These acceleration commands can not exceed the maximum achievable acceleration limits of the missile imposed by the structural limits.

In our study, vertical and horizontal components of acceleration commands,

a_{pitch} and a_{yaw} are computed in a different method. As seen on Fig. 13 and Fig.14, vertical component of missile acceleration command can be derived as:

$$a_{pitch} = a_{Mz} \cdot \cos \gamma_M + g \cdot \cos \gamma_M \tag{37}$$

and the lateral component of missile acceleration command:

$$a_{yaw} = a_{My} \cdot \sin \left(\frac{\pi}{2} - \chi_M \right) - a_{Mx} \cdot \sin \chi_M \tag{38}$$

These acceleration components, a_{pitch} and a_{yaw} , will be used as control variables of the missile in the equations of motion.

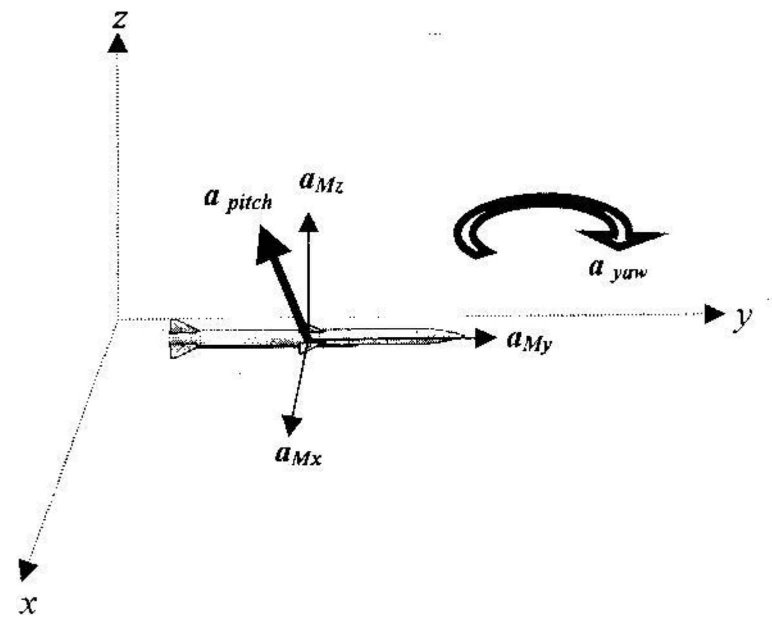


Figure 4. Components of missile acceleration commands in 3D environment.

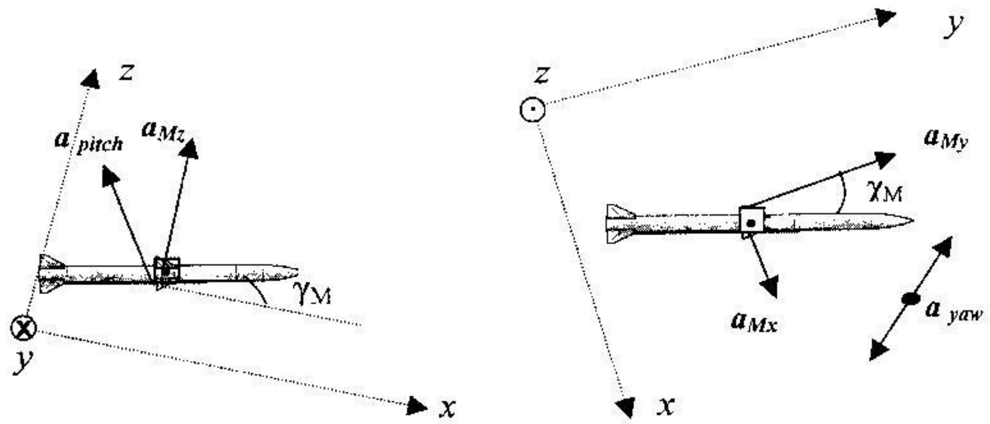


Figure 5. Projections of missile acceleration commands onto **xz** and **xy** planes.

To summarize TPA; 3D engagement space geometry is projected onto 3 perpendicular planes such as, S_{xy}, S_{xz} and S_{yz}. Guidance problem is solved in each plane independently for two-dimensional True Proportional Navigation (TPN); acceleration commands are generated for each plane, and a 3D solution is produced by combining these 2D solutions.

4. Vegas: Visual Endgame Simulation

To evaluate the performance of guidance law proposed in this paper; a simulation software, VEGAS is implemented, in which the missile and the target are independent modules. Hence it would be possible to evaluate the effectiveness of both sides. This simulation software is named as, Visual End-Game Simulation, VEGAS. The last seconds of the encounter is also called End-Game. The VEGAS software is implemented in Visual C++ programming language using OpenGL library.

The terminal phase of the encounter between the missile and the aircraft is considered in the VEGAS. These last seconds of the engagement are the most important period since its success or failure determines the success or failure of the entire mission. Since the scenarios of this simulation tool starts at the beginning of the terminal phase, the missile and the aircraft are assumed to have initial velocities and some kilometres from each other. All the aerodynamic and physical parameters of both missile and target aircraft are included in VEGAS. It is realized that the developed guidance approach works effectively against high-g capacity fighter aircrafts.

5. Design Features

The VEGAS is comprised of five basic modules:

- Main
- Evader
- Pursuer
- Radar
- Aero

The overall flow chart of Visual End-Game Simulation is given in Fig.6. As seen on the chart, the simulation steps are as follows:

1. Target makes a step of evasion manoeuvre in 3D environment with respect to aerodynamic considerations.
2. Missile takes the position data of target from the "radar" module.
3. Missile guidance law generates the acceleration commands with respect to the engagement geometry.
4. By using the parameters coming from "aero" module, aerodynamic forces such as drag, thrust, weight are computed, limits are controlled. Translational movement in 3D is derived from equations of motion with all these values.

These steps are repeated while the range between target and missile is larger than the capture radius R_C and the target is in the missile seeker cone.

Since VEGAS is a discrete-time simulation, motions of the missile and the target are performed in fixed time steps. The size of time step is assumed equal to the missile's guidance system time constant which is the total lag of guidance system.

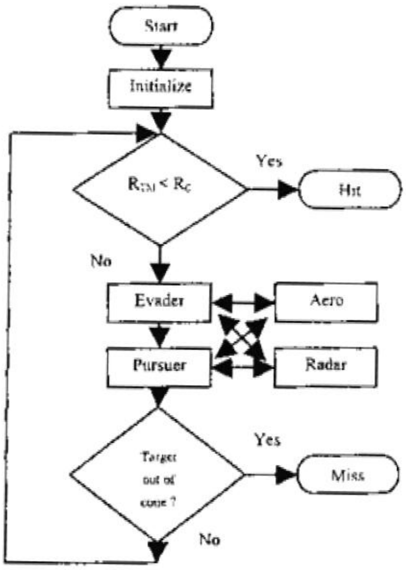


Figure 6. Visual end game simulation flow chart.

6. Performance Evaluation

A missile guided with the TPA approach presented in Section III and a target fighter aircraft are considered in this section. The air combat between the fighter aircraft employing evasive manoeuvres and the missile guided by proposed approach to intercept this aircraft is examined. VEGAS, described in the previous section, is used as the discrete-time simulation software.

The time constants of both missile and aircraft are assumed as 0.1 second. Aircraft and missile models used in the simulations are based on the extended point mass assumptions.

The earth is assumed flat because the relative distance between the missile and target is short in the terminal guidance phase and thus the curvature of earth is considered not to be able to affect the dynamics of the flight. The velocity vector, reference line of the vehicle, the thrust and drag forces are all assumed parallel. Also wind is ignored in the missile and target models hence the side-slip angle is assumed to be zero. The dynamic model of the missile and the aircraft and their guidance dynamics are presented below.

a. Missile Model

The evaluation of the missile flight trajectory requires consideration of degrees of freedom (DOF) to be simulated. The simplest and the acceptable model for the conceptual design of the high speed missiles, one degree-of-freedom is considered to model the missile. One degree-of-freedom requires only thrust, weight and drag forces of the missile [13].

In one degree-of-freedom modelling, heading angle and flight path angles are used as state variables. The position of the missile in the three-dimensional (3D) space is defined by three state variables, which are coordinates x and y range and altitude z.

The missile is directly controlled with commanded accelerations a_{pitch} and a_{yaw}, those are generated by the guidance law developed in Section III.

The mass of the vehicle and its change due to fuel consumption are also included into simulation as a state variable. The propellant mass at the terminal phase is assumed equal to 30 kg. and change rate of propellant mass, so the total mass rate of the missile during this period is:

$$\dot{m}_M = -3 \text{ kg/sec.} \tag{39}$$

b. Target Aircraft Model

In our study, high-g capacity fighter aircraft is considered as the target. Motion modelling and implementation of the target evasive manoeuvres are examined by Akdağ [14].

c. Simulation Scenarios

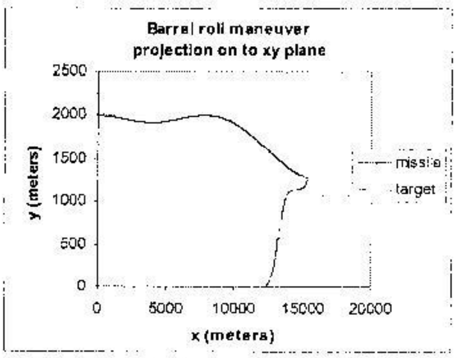
In all of the simulation scenarios, the final period, terminal phase of the encounter is considered, where the missile is initially flying with a supersonic velocity on collision course within some kilometres from the target aircraft.

Scenario 1:

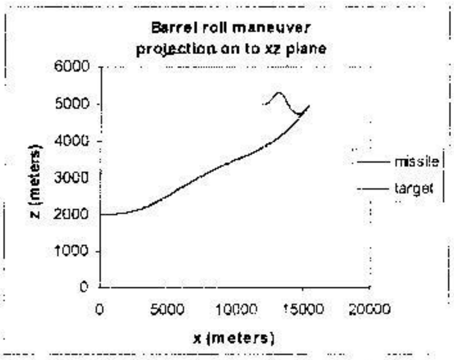
Initial engagement conditions for Scenario 1 are given as:

Missile	Target
positions on x, y, z:	positions on x, y, z:
(0, 2000m, 2000m)	(12000m, 0,5000m)
heading, $\chi_M = 0°$	heading, $\chi_T = 0°$
flight path angle, $\gamma_M = 0°$	flight path angle, $\gamma_T = 0°$
initial velocity = 1000 m/sec.	initial velocity = 300 m/sec.

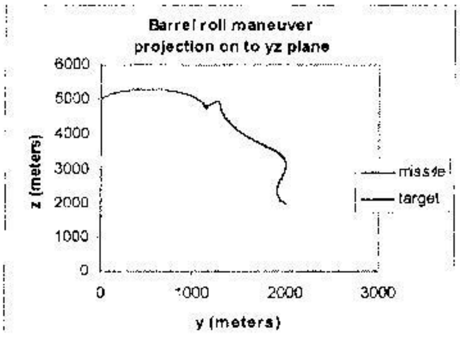
Missile and target trajectories are evaluated with the initial conditions given above and while the target is employing Barrel Roll evasive manoeuvre.



(a) Trajectory Projections onto xy-Plane



(b) Missile-Target Trajectory Projections onto xz-Plane



(c) Missile-Target Trajectory Projections onto yz-Plane

Figure 7. Missile-target trajectories of scenario 1, barrel roll.

The effects of Barrel Roll manoeuvres on the missile-target trajectories are observed below for Scenario 1. It is realized that developed guidance approach works effectively against this basic evasive manoeuvre.

As mentioned before, PN guidance law works by regulating the line-of-sight rate (LOSR) to zero. Missile line-of-sight angles (λ_{xy}, λ_{xz} and λ_{yz}) and deviation of line-of-sight rates ($\tilde{\lambda}_{xy}$, $\tilde{\lambda}_{xz}$ and $\tilde{\lambda}_{yz}$) during missile flight are shown in Fig.8 and Fig.9 for our approach, TPA. The target employs Horizontal-S manoeuvre in this engagement scenario.

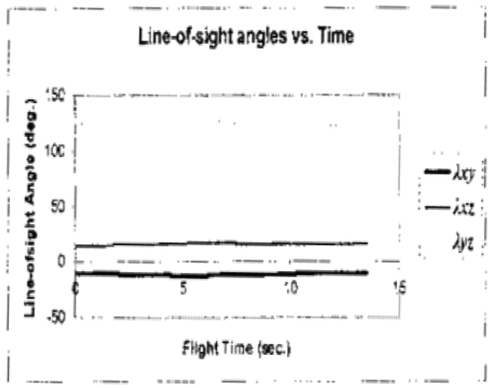


Figure 8. Line-of-sight angles due to flight time

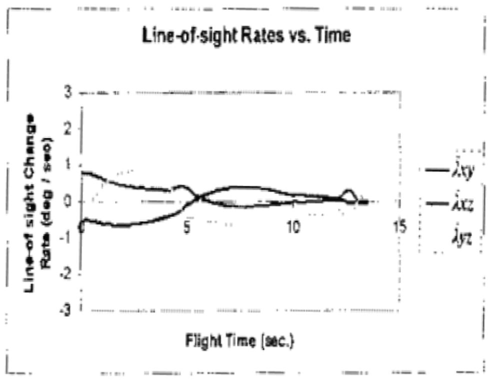


Figure 9. Line-of-sight change rates due to flight time.

From Fig.8 and Fig.9, it can be seen that line-of-sight rates (LOSR) come near to zero during the missile flight. This means the target and the missile are on collision course.

Scenario 2:

For an anti-air missile; the meaning of initial conditions such as initial position and target line-of-sight are very critical in an engagement. To evaluate the results of possible situations of both missile and the target, heading angles of both side are varied in Scenario 2.

The target employs Barrel Roll manoeuvre. Missile's heading angle varies from $-30°$ to $+30°$ with the intervals of $10°$ while the target's heading angle varies from 0° to 180° with the intervals of 15°. Other initial conditions are given below:

<u>Missile</u>
positions on x, y, z:
(0, 0, 2000m)
missile heading, $= -30° < \chi_M < 30°$
flight path angle, $\gamma_M = 0°$
initial velocity = 1000 m/sec.

<u>Target</u>
positions on x, y, z:
(9000m, 0,2000m)
target heading, $0° < \chi_T < 180°$
flight path angle, $\gamma_T = 0°$
initial velocity = 300 m/sec.

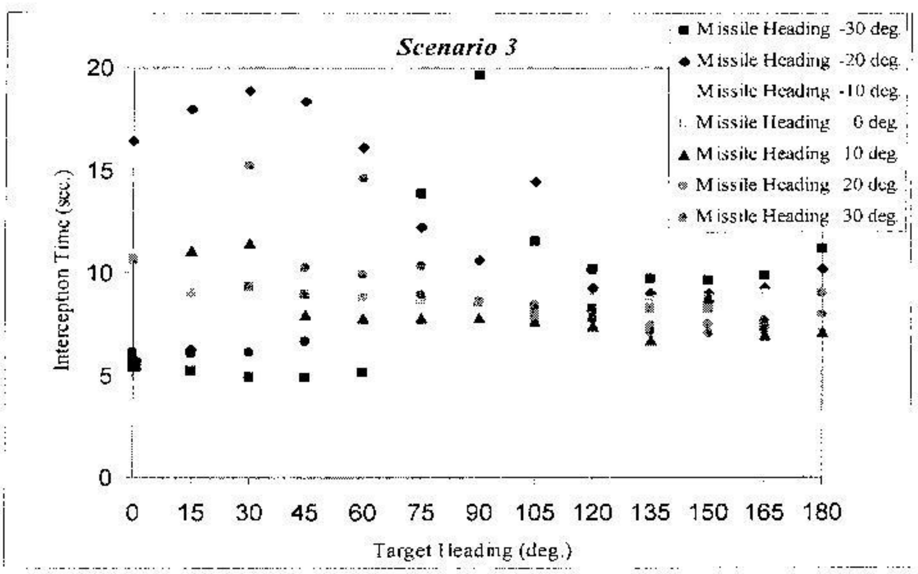


Figure 10. Interception time due to missile and target heading angles.

From Fig.10, it can be easily seen that interception time increases with relative heading angle magnitude. For example, when the missile heading is equal to zero, means the target is in front of the missile, interception time is very small whatever the target heading is. But when the missile heading is equal to −30° or −20° the interception time increases or the mission results with miss. Black points in Fig.10 represent the misses.

Scenario 3:

In this scenario, the performance of developed approach, TPA, is compared with the PN method widely used in the literature [9,10,11,12]. As mentioned in Section III, in this method, vertical and lateral acceleration components are computed differently from those computed in TPA. The scenario is given as:

Missile	Target
positions on y, z:	positions on x, y, z:
(2000m, 2000m)	(14000m, 0,2000m)
heading, $\chi_M = 0°$	heading, $\chi_T = 0°$
flight path angle, $\gamma_M = 0°$	flight path angle, $\gamma_T = 0°$
initial velocity = 1000 m/sec.	initial velocity = 300 m/sec.

To compare the results of both methods, TPA and PN; the x position of missile is varied from origin to 10000 meters with the intervals of 250 meters. The simulation is run for TPA and PN for the target employing basic evasive manoeuvres. The results are given in Fig.11-Fig.14.

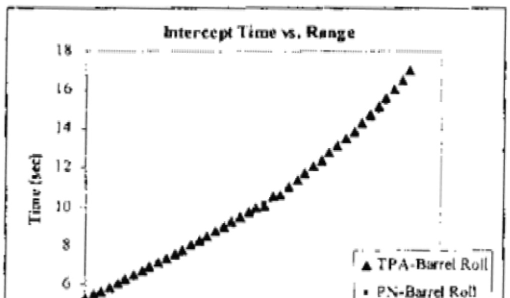


Figure 11. Evaluation of TPA with PN, barrel roll manoeuvre.

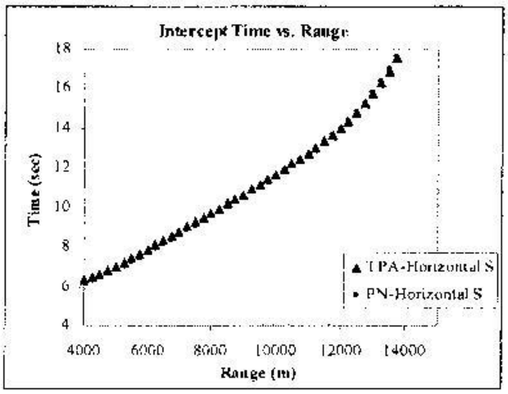


Figure 12. Evaluation of TPA with PN, horizontal S manoeuvre.

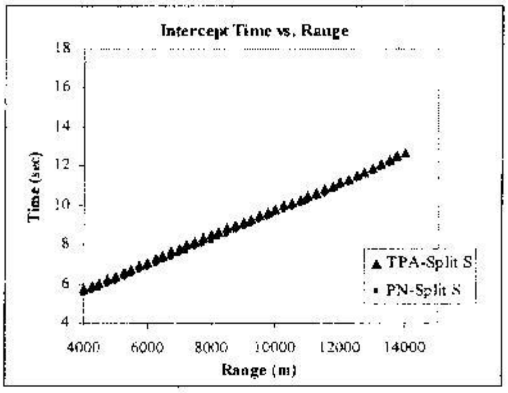


Figure 13. Evaluation of TPA with PN, split S manoeuvre

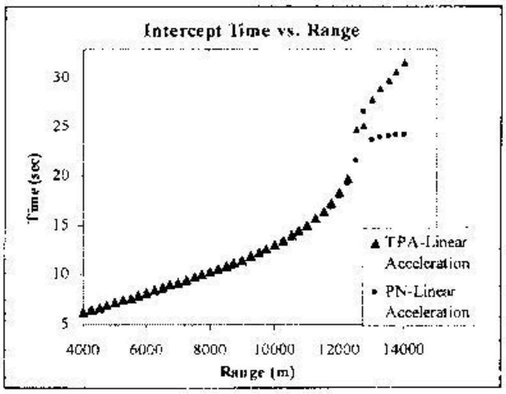


Figure 14. Evaluation of TPA with PN, linear acc. manoeuvre.

Black points in Fig.14 represent the miss conditions. It could be seen from the simulation results that; the interception time due to range magnitudes for PN and TPA methods are very close to each other. (Fig.11-Fig.14)

7. Conclusion and Further Research

A novel guidance approach for 3D missile guidance is presented in this paper, which is effective against high-g capability fighter aircrafts that employ evasive manoeuvres. The performance of this approach has been tested both visually and analytically via developed simulation software, VEGAS (Visual End-Game Simulation).

When compared with classical PN approach, it is verified that the performance of proposed approach is robust and effective for high-g capacity aerial targets employing evasive manoeuvres.

In this study, the missile is assumed to have perfect knowledge of target position. In a real situation, there are likely to be measurement errors in the missile's measurement of the target aircraft data such as position, closing velocity etc. Although "radar" module produces perfect map of engagement space upon the requests from missile, the module structure is designed to support producing signal superimposed with noise.

Only the missile motion dynamics of the encounter are considered in this study. However, in a real encounter, the target aircraft is probable to have countermeasures, such as chaff. In such a situation, missile guidance system is expected to filter the fake data by using extended techniques like Kalman Filtering and to guide the missile to the real target position to intercept.

Additionally, in a possible missing occurrence, the missile should have re-attack capability by using an extra algorithm if its propellant quantity is sufficient for a new attack.

Trajectory learning and optimization using neural networks will also be included into the guidance system. Missile guidance system will be trained via specific targmanoeuvrever data. Hence the missile and its guidance system will be ready to make the best manoeuvre decision for pre-defined target manoeuvres.

The modelling of the electronic counter-countermeasures, the re-attack function, trajectory learning and optimizing the use of such procedures are our directions of further research.

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