APPENDIX D

An alternative method to analyze the resolution of the WTA

An alternative method to analyze the resolution of the Winner-Takes-All cell. In the following analysis, transistor M1 represents the transistor M4, transistor M2 represents the transistor M3 and transistor M3 represents the transistor M5 in Figure 3.3.1.

$$V_{TP(M1)}, V_{TP(M2)} < 0 \qquad ; V_{TN(M3)} > 0$$

Drain current of transistor M1 is

$$I_{D(M1)} = I_{TAIL} = \frac{\beta_1}{2}\left(V_{TP(M1)} - V_{GS(M1)}\right)^2$$

We substitute $V_{GS(M1)}$ by $V_{GS(M1)} = V_{OUT} - V_{CSN}$

$$I_{TAIL} = \frac{\beta_1}{2}\left(V_{TP(M1)} + V_{CSN} - V_{OUT}\right)^2$$

Thus, V_{CSN} can be expressed as

$$V_{CSN} = V_{OUT} + \sqrt{\frac{2I_{TAIL}}{\beta_1}} - V_{TP(M1)}$$

Drain current of transistor M2 is

$$I_{D(M2)} = \frac{\beta_2}{2}\left(V_{TP(M2)} - V_{GS(M2)}\right)^2$$

We substitute $V_{GS(M2)}$ by $V_{GS(M2)} = V_{CSN} - V_{DD}$

$$I_{D(M2)} = \frac{\beta_2}{2}\left(V_{TP(M2)} + V_{DD} + V_{TP(M1)} - V_{OUT} - \sqrt{\frac{2I_{TAIL}}{\beta_1}}\right)^2$$

Drain current of transistor M2 can be expressed as follows too:

$$I_{D(M2)} = I_{D(M3)} - I_{IN} = \frac{\beta_2}{2}\left(V_{BIAS} - V_{TN(M3)}\right)^2 - I_{IN}$$

Thus, cell input current can be expressed as

$$I_{IN} = \left[\frac{\beta_3}{2}\left(V_{BIAS} - V_{TN(M3)}\right)^2\right] - \frac{\beta_2}{2}\left(V_{DD} + V_{TP(M1)} + V_{TP(M2)} - \sqrt{\frac{2I_{TAIL}}{\beta_1}} - V_{OUT}\right)^2$$

Standard deviation of input current with respect to random variables β_1, β_2, β_3, $V_{TP(M1)}$, $V_{TP(M2)}$ and $V_{TN(M3)}$ can be expressed as follows:

$$\sigma^2[I_{IN}] = \sigma^2\left[\frac{\beta_3}{2}\left(V_{BIAS} - V_{TN(M3)}\right)^2\right] + \sigma^2\left[\frac{\beta_2}{2}\left(V_{DD} + V_{TP(M1)} + V_{TP(M2)} - \sqrt{\frac{2I_{TAIL}}{\beta_1}} - V_{OUT}\right)^2\right]$$

$$= \frac{1}{4}\left[\sigma^2[\beta_3]\left(V_{BIAS} - V_{TN(M3)}\right)^4 + \beta_3^{\,2}\sigma^2\left(V_{BIAS} - V_{TN(M3)}\right)^2\right]$$

$$+\sigma^2[\beta_2]\left(V_{DD} + V_{TP(M1)} + V_{TP(M2)} - \sqrt{\frac{2I_{TAIL}}{\beta_1}} - V_{OUT}\right)^4$$

$$+\beta_2^{\,2}\sigma^2\left[\left(V_{DD} + V_{TP(M1)} + V_{TP(M2)} - \sqrt{\frac{2I_{TAIL}}{\beta_1}} - V_{OUT}\right)^2\right]$$

$$\sigma^2\left[\left(V_{BIAS} - V_{TN(M3)}\right)^2\right] = 4\left(V_{BIAS} - V_{TN(M3)}\right)^2\sigma^2\left[V_{TN(M3)}\right]$$

$$\sigma^2\left[\left(V_{DD} + V_{TP(M1)} + V_{TP(M2)} - \sqrt{\frac{2I_{TAIL}}{\beta_1}} - V_{OUT}\right)^2\right] = 4\left(V_{DD} + V_{TP(M1)} + V_{TP(M2)} - \sqrt{\frac{2I_{TAIL}}{\beta_1}} - V_{OUT}\right)^2 \cdot \sigma^2\left[V_{DD} + V_{TP(M1)} + V_{TP(M2)} - \sqrt{\frac{2I_{TAIL}}{\beta_1}} - V_{OUT}\right]$$

$$\sigma^2\left[V_{DD} + V_{TP(M1)} + V_{TP(M2)} - \sqrt{\frac{2I_{TAIL}}{\beta_1}} - V_{OUT}\right] = \sigma^2\left[V_{TP(M1)}\right] + \sigma^2\left[V_{TP(M2)}\right] + \sigma^2\left[\sqrt{\frac{2I_{TAIL}}{\beta_1}}\right]$$

$$\sigma^2\left[\sqrt{\frac{2I_{TAIL}}{\beta_1}}\right] = \frac{I_{TAIL}}{2\beta_1^{\,3}}\sigma^2[\beta_1]$$

Thus, relative input current error (precision) is

$$\frac{\sigma^2\left[I_{\dot{I}N}\right]}{I_{\dot{I}N}^2}=\frac{\begin{gathered}\left\{\frac{\sigma^2\left[\beta_2\right]}{\beta_3{}^2}\beta_3{}^2\left(V_{BIAS}-V_{TN(M3)}\right)^4+\beta_3{}^2\,4\left(V_{BIAS}-V_{TN(M3)}\right)^2 V_{TN(M3)}{}^2\,\frac{\sigma^2\left[V_{TN(M3)}\right]}{V_{TN(M3)}{}^2}\right.\\+\frac{\sigma^2\left[\beta_2\right]}{\beta_2{}^2}\beta_2{}^2\left(V_{DD}+V_{TP(M1)}+V_{TP(M2)}-\sqrt{\frac{2I_{TAIL}}{\beta_1}}-V_{OUT}\right)^4\\+\beta_2{}^2\,4\left(V_{DD}+V_{TP(M1)}+V_{TP(M2)}-\sqrt{\frac{2I_{TAIL}}{\beta_1}}-V_{OUT}\right)^2\\\left.\left[V_{TP(M1)}{}^2\,\frac{\sigma^2\left[V_{TP(M1)}\right]}{V_{TP(M1)}{}^2}+V_{TP(M2)}{}^2\,\frac{\sigma^2\left[V_{TP(M2)}\right]}{V_{TP(M2)}{}^2}+\frac{I_{TAIL}}{2\beta_1}\frac{\sigma^2\left[\beta_1\right]}{\beta_1{}^2}\right]\right\}\end{gathered}}{\left[\beta_3\left(V_{BIAS}-V_{TN(M3)}\right)^2-\beta_2\left(V_{DD}+V_{TP(M1)}+V_{TP(M2)}-\sqrt{\frac{2I_{TAIL}}{\beta_1}}-V_{OUT}\right)^2\right]^2}$$