APPENDIX B

Analysis of relative DCELL row current error due to the device mismatch

Detailed analysis of relative DCELL row current error with respect to fabrication process noise. DCELL row current can be expressed as follows

$$I_i = \sum_{j=1}^{m} I_{ij}$$

I_i represents i^{th} row current, I_{ij} represents output current of the DCELL placed on i^{th} row and j^{th} column. I_{ij} is a function of random variables β, V_{TP}, C and C_T. Thus I_I and I_{ij} are random variables too. The standard deviation of I_I can be expressed as a sum of standard deviation of each individual DCELL output current. We assume that each DCELL have the same input and template voltage. Thus, the standard deviation of DCELL row current is

$$\sigma^2[I_i] = m\sigma^2[I_{ij}]$$

DCELL output current can be expressed as follows:

$$\begin{aligned} I_{ij} &= \frac{\beta}{2}\left[V_{bias} - \frac{C}{C_T}\left|X_j - T_{ij}\right| - V_{DD} - V_{TP}\right]^2 \\ &= \frac{\beta}{2}\left[\underbrace{V_{DD} + V_{TP} - V_{bias}}_{V} + \underbrace{\frac{C}{C_T}\left|X_j - T_{ij}\right|}_{\Delta V}\right]^2 \\ &= \frac{\beta}{2}\underbrace{\left[V + \Delta V\right]^2}_{M} \\ &= \frac{\beta}{2}M \end{aligned}$$

The standard deviation of DCELL output current is

$$\sigma^2[I_{ij}] = \sigma^2\left[\frac{\beta M}{2}\right] = \frac{1}{4}\sigma^2[\beta M]$$
$$= \frac{1}{4}\left[\sigma^2[\beta]M^2 + \beta^2\sigma^2[M]\right]$$
$$= \frac{1}{4}\left[\sigma^2[\beta][V+\Delta V]^4 + \beta^2\sigma^2\left[(V+\Delta V)^2\right]\right]$$
$$= \frac{1}{4}\left[\sigma^2[\beta][V+\Delta V]^4 + \beta^2 4(V+\Delta V)^2\sigma^2[V+\Delta V]\right]$$
$$= \frac{1}{4}(V+\Delta V)^2\left[\sigma^2[\beta](V+\Delta V)^2 + 4\beta^2\left(\sigma^2[V]+\sigma^2[\Delta V]\right)\right]$$

$$V = V_{DD} + V_{TP} - V_{bias}$$
$$\sigma^2[V] = \sigma^2[V_{TP}]$$

$$\Delta V = \frac{C}{C_T}\left|X_j - T_{ij}\right|$$
$$\sigma^2[\Delta V] = \left|X_j - T_{ij}\right|^2\sigma^2\left[\frac{C}{C_T}\right]$$

$$\sigma^2[I_{ij}] = \frac{1}{4}(V+\Delta V)^2\left[\sigma^2[\beta](V+\Delta V)^2 + 4\beta^2\left[\sigma^2[V_{TP}] + (X_j - T_{ij})^2\sigma^2\left[\frac{C}{C_T}\right]\right]\right]$$
$$= \frac{1}{4}(V+\Delta V)^2\left[\sigma^2[\beta](V+\Delta V)^2 + 4\beta^2\left[\sigma^2[V_{TP}] + (X_j - T_{ij})^2\left[\frac{\sigma^2[C]}{{C_T}^2} + \frac{C^2}{{C_T}^4}\sigma^2[C_T]\right]\right]\right]$$
$$= \frac{1}{4}(V+\Delta V)^2\left[(V+\Delta V)^2\sigma^2[\beta] + 4\beta^2\left[\sigma^2[V_{TP}] + \frac{1}{{C_T}^2}(X_j - T_{ij})^2\left[\sigma^2[C] + \left(\frac{C}{C_T}\right)^2\sigma^2[C_T]\right]\right]\right]$$
$$= \frac{1}{4}(V+\Delta V)^2\left[(V+\Delta V)^2\sigma^2[\beta] + 4\beta^2\left[\sigma^2[V_{TP}] + \left(\frac{C}{C_T}(X_j - T_{ij})\right)^2\left[\underbrace{\frac{\sigma^2[C]}{C^2} + \frac{\sigma^2[C_T]}{{C_T}^2}}_{\frac{2\sigma^2[C]}{C^2}}\right]\right]\right]$$

$$\sigma^2[I_{ij}] = (V+\Delta V)^2\left[\frac{\sigma^2[\beta](V+\Delta V)^2}{4} + \beta^2\left[\sigma^2[V_{TP}] + 2\left(\frac{C}{C_T}(X_j - T_{ij})\right)^2\frac{\sigma^2[C]}{C^2}\right]\right]$$

All DCELLs have the same template and input voltages. Thus

$$\sigma^2[I_i] = m(V+\Delta V)^2\left[\frac{(V+\Delta V)^2}{4}\sigma^2[\beta]+\beta^2\left[\sigma^2[V_{TP}]+2\frac{C^2}{{C_T}^2}(X_j-T_{ij})^2\frac{\sigma^2[C]}{C^2}\right]\right]$$

$$I_i = mI_{ij} = \frac{m\beta}{2}\left(V_{DD}+V_{TP}-V_{bias}+\frac{C}{C_T}\left|X_j-T_{ij}\right|\right)^2 = \frac{m\beta}{2}(V+\Delta V)^2$$

$$I_i^{\,2} = \frac{m^2\beta^2}{4}(V+\Delta V)^4$$

Relative DCELL row current error is

$$\frac{\sigma^2[I_i]}{I_i^{\,2}} = \frac{m(V+\Delta V)^2\left[\frac{(V+\Delta V)^2\sigma^2[\beta]}{4}+\beta^2\left[\sigma^2[V_{TP}]+2\frac{C^2}{{C_T}^2}(X_j-T_{ij})^2\frac{\sigma^2[C]}{C^2}\right]\right]}{\frac{m^2\beta^2}{4}(V+\Delta V)^4}$$

$$\frac{\sigma^2[I_i]}{I_i^{\,2}} = \frac{1}{m}\left[\frac{\sigma^2[\beta]}{\beta^2}+\frac{4\sigma^2[V_{TP}]}{(V+\Delta V)^2}+\frac{8\Delta V^2}{(V+\Delta V)^2}\frac{\sigma^2[C]}{C^2}\right]$$

$$\frac{\sigma^2[I_i]}{I_i^{\,2}} = \frac{1}{m}\left[\frac{\sigma^2[\beta]}{\beta^2}+\left(\frac{2V_{TP}}{V+\Delta V}\frac{\sigma[V_{TP}]}{V_{TP}}\right)^2+2\left(\frac{2\Delta V}{V+\Delta V}\frac{\sigma[C]}{C}\right)^2\right]$$

$$\frac{\sigma^2[I_i]}{I_i^{\,2}} = \frac{1}{m}\left\{\frac{\sigma^2[\beta]}{\beta^2}+\left[\frac{2V_{TP}}{V_{DD}+V_{TP}-V_{bias}+\frac{C}{C_T}\left|X_j-T_{ij}\right|}\frac{\sigma[V_{TP}]}{V_{TP}}\right]^2 +2\left[2\frac{\frac{C}{C_T}\left|X_j-T_{ij}\right|}{V_{DD}+V_{TP}-V_{bias}+\frac{C}{C_T}\left|X_j-T_{ij}\right|}\frac{\sigma[C]}{C}\right]^2\right\}$$