LEGENDRE'S DIFFERENTIAL EQUATOINS AND POLINOMIALS

Legendre's differential equation is in form as below [1]:

$$\left(1-x^2\right)y''-2xy'+n\left(n+1\right)y=0 \qquad \text{or} \qquad \left[\left(1-x^2\right)y'\right]'+n\left(n+1\right)y=0$$

By power series solution for the Legendre's differential equation two solution y_1, y_2 are obtained as below [1]:

$$y_1=c_1\left[1-\frac{n(n+1)}{2!}x^2+\frac{n(n-2)(n+1)(n+3)}{4!}x^4-\frac{n(n-2)(n-4)(n+1)(n+3)(n+5)}{6!}x^6 +\frac{n(n-2)(n-4)(n-6)(n+1)(n+3)(n+5)(n+7)}{8!}x^8-\ldots\right]$$

$$y_2=c_2\left[x-\frac{(n-1)(n+2)}{3!}x^3+\frac{(n-1)(n-3)(n+2)(n+4)}{5!}x^5 -\frac{(n-1)(n-3)(n-5)(n+2)(n+4)(n+6)}{7!}x^7+\ldots\right]$$

for special cases and $P_n(x=1)=1$ the polynomials are obtained as below [1]:

$$P_0(x)=1$$

$$P_1(x)=x$$

$$P_2(x)=\frac{1}{2}\left(3x^2-1\right)$$

$$P_3(x)=\frac{1}{2}\left(5x^3-3x\right)$$

$$P_4(x)=\frac{1}{8}\left(35x^4-30x^2+3\right)$$

$$P_5(x)=\frac{1}{8}\left(63x^5-70x^3+15x\right)$$

$$P_6(x)=\frac{1}{16}\left(231x^6-315x^4+105x^2-5\right)$$

$$P_7(x)=\frac{1}{16}\left(429x^7-693x^5+315x^3-35x\right)$$

$$P_8(x)=\frac{1}{128}\left(6435x^8-12012x^6+6930x^4-1260x^2+35\right)$$

$\vdots$

e-mail: abdulhamit@itu.edu.tr , web: www.students.itu.edu.tr/~abdulhamit

The plot of the polynomials between $-1 < x < 2$ and $-1 < y < 2$ is illustrated as below:

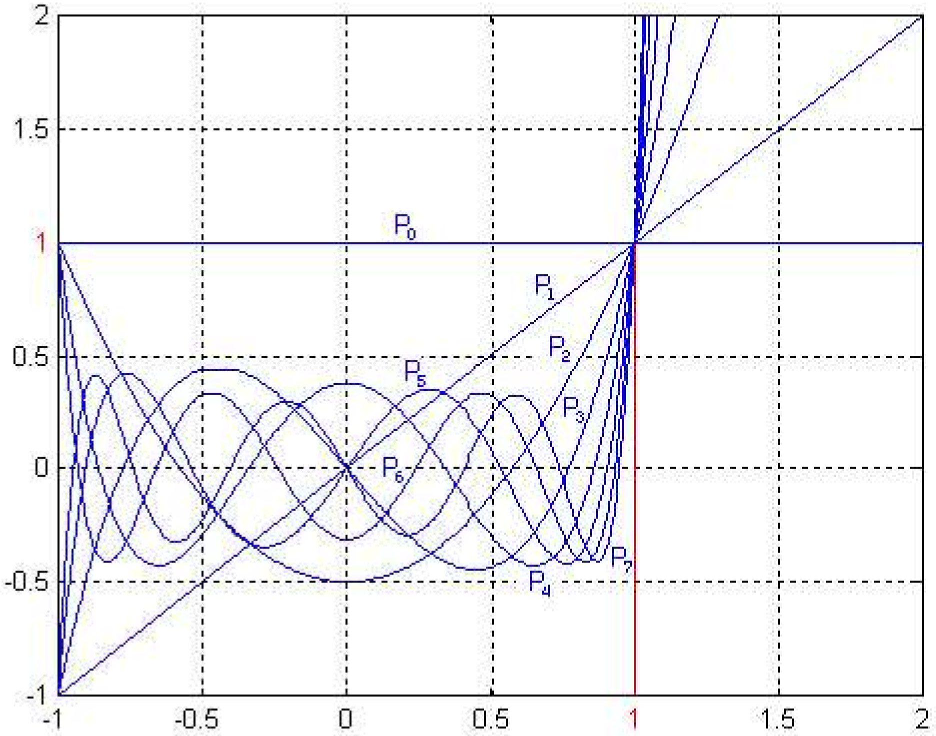

Figure 1. Plot obtained by MATLAB for $P_n(x)$, $n = 0,1,2,...,7$.

Also, relation between three terms of Legendre's polynomials is as below:

$(n+1)P_{n+1}(x) = x(2n+1)P_n(x) - (n+1)P_{n-1}(x)$.

REFERENCE:

[1] Cevdet KOÇAK, Yüksek Matematik. İTÜ Vakfı, 1996.
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