10.4 TORQUES AND TORQUERS

The torques, arising from moments of forces about the center of mass, and couples, must be identified as being *external* or *inertial* to the spacecraft. The former will affect its total momentum, whereas the later will affect only the distribution between its moving parts. The case has already been made that it is necessary to include controllable external torques whereas internal ones.

The main source of torques and accruing either naturally or as disturbances are introduced bellow and summarized in Table 10.1. The magnitude of torques in space is small when compared with terrestrial standards. Even very small ones become significant when there is no friction to oppose them and when the orientation has to be very accurate.

Some of the phenomena listed as disturbances torques in Table 10.1 may be used as means of achieving the required orientation of the spacecraft. For that job, they will normally need to be controllable. A possible exception is the gravity gradient torque, which will establish an Earth-facing equilibrium orientation passively, with the axis of last inertia along the local vertical (seen 10.4.3)

Table 10.2 summarizes the main advantages and disadvantages of various types of torquer.

Type	Advantages	Disadvantages
EXTERNAL TYPES		
Gas jet	Insensitive to altitude Suit any orbit Can torque about any axis	Requires fuel On-off operation only Has min impulse Exhaust plume contaminants
Magnetic	No fuel required Torque magnitude is controllable	No torque about local field direction Torque is altitude and latitude sensitive Can cause magnetic interference
Gravity-gradient	No fuel or energy needed	No torque about the local vertical Low accuracy Low torque, altitude sensitive Libration mode need damping
Solar radiation	No fuel required	Need controllable panels Very low torque
INTERNAL TYPES	No fuel required Can store momentum Torque magnitude is controllable	Cannot control momentum build-up
Reaction wheels (RW)		Non linearly at zero speed
Momentum wheels (MW)	Provide momentum bias	
Control momentum gyroscope (CMG)	Suitable for three-axis control Provides momentum bias	Complicated Potential reliability problem

Table 10.2 Types of torquer

10.4.1 Thrusters (external type)

Orbit-changing thrusters provide potentially the largest source of force on spacecraft, and potentially the largest source of torque. Being external, the torque will affect the total momentum. Ideally the thrust vector passes through the center of mass, but inevitably there is a tolerance on this and consequently a disturbance torque arises.

The main means of countering the effects of this torque when large thrust levels are present are either to spin the vehicle about the intended thrust direction (it was explained in the spinning spacecraft) or to provide means of controlling the achieved thrust direction. At lift-off, for example, the latter method must clearly be used. This involves mounting some of the thrusters in gimbals, or using secondary fuel injection into the rocket nozzle, and controlling the thrust direction so as to achieve the required trajectory. Later stages of booster rockets may adopt the alternative method of spinning the vehicle in order to average out the effects of thrust offset upon the trajectory. Thruster firings used for changing the orbit, such as from LEO to Transfer Orbit and again from transfer Orbit to GEO, will normally be preceded by a spin-up maneuver, followed by de-spin after the orbit changes are complete.

Thrusters with very much lower levels of thrust are in common use in attitude-control systems for providing controllable external torquing, and hence controlling the total momentum of the spacecraft. For this purpose they will be mounted in clusters on the surface of the vehicle, pointing in different directions in order to provide three components of torque. They have a number of advantages and disadvantages compared with their main rival, the magnetic torquer.

Their main advantage is that their torque level is independent of altitude and there is potentially no limit to its magnitude. However, the magnitude is not controllable when installed; only the switch-on duration is. This torquing system integrates well with the strength of the Earth's field reduces with height. The field's strength and direction also vary with the position of the spacecraft in its orbit in general, and when using magnetic torquers it is common practice to carry a magnetometer to measure the local field.

10.4.2 Magnetic Torque (external type)

Magnetic torques acting on a spacecraft can result from the interaction of the spacecraft's residual magnetic field and the geomagnetic field. Thus, if $\vec{M}$ is the sum of all magnetic moments in the spacecraft the torque acting on the spacecraft is as bellow.

$$\vec{T}^{(m)} = \vec{M} \times \vec{B} \qquad (10.4.1)$$

where $\vec{B}$ is geomagnetic field vector. In general, $\vec{M}$ can be caused by permanent and induced magnetism or by spacecraft-generated current loops. The unit of M may be gausscm3, amper-m2, or pole-cm. For example, if M is in amper-m^2 and B in tesla or webers/m^2, then $T^{(m)}$ is in newton-meters; or if M is in pole-cm and B in gauss, then $T^{(m)}$ is in gyne-cm In general,

1 amper-m^2 = 100 pole-cm
1 pole-cm = 1 (dyne-cm/gauss)

The spinning motion of the spacecraft causes torques, which are induced by eddy currents, which then interact with any magnetized permeable spacecraft materials. Eddy current torques are of the form

$$\vec{T}^{(m)}_{eddy} = k_e\left(\vec{\omega}\times\vec{B}\right)\times\vec{B} \qquad (10.4.2)$$

where k_e is a constant and $\vec{\omega}$ is the spacecraft spin vector.

10.4.3 Gravity-gradient Torque (external type)

This source of torque occurs because of a gravitational field, which gets weaker with increase in height a body will only be in stable equilibrium if its axis of minimum inertia is aligned with the local vertical.

Gravitational gradient torque acting on a distributed mass body in orbit is given by the expression

$$\vec{T}^{(g)} = \int_m \vec{r}\times\vec{a}_g dm \approx -\frac{GM}{R_0^3}\int_m \vec{r}\times\left[\vec{R}_0 + \vec{r} - 3\frac{\vec{R}_0\cdot\vec{r}}{\vec{R}_0^2}\vec{R}_0\right]$$

$$\approx -\frac{GM}{R_0^3}\int_m \vec{r}\times\frac{\vec{R}_0}{\vec{R}_0^2}\vec{R}_0 dm\,. \qquad (10.4.3)$$

Equation (10.4.3) can be integrated as follows in equation (10.4.4). Let $\mu = GM$, and $\vec{R}_0 = R_0\hat{E}_1$,

$$\begin{bmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{bmatrix} = R\begin{bmatrix}\hat{E}_1\\ \hat{E}_2\\ \hat{E}_3\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}\hat{E}_1\\ \hat{E}_2\\ \hat{E}_3\end{bmatrix}$$

where $\hat{E}_1$ is the unit vector along the outward radius. Then equation (1) becomes as bellow:

$$\vec{T}^{(g)} \approx \frac{3\mu}{R_0^3}\int_m \frac{\left(\vec{R}_0\cdot\vec{r}\right)\vec{r}\times\vec{R}_0}{\vec{R}_0^2}dm = \frac{3\mu}{R_0^3}\hat{E}_1\cdot\int_m \vec{r}\vec{r}dm\times\hat{E}_1$$

$$= \frac{3\mu}{R_0^3} \hat{E}_1 \times \int_m \left(\overline{\overline{E}} r^2 - \vec{r}\vec{r} \right) dm \cdot \hat{E}_1 = \frac{3\mu}{R_0^3} \hat{E}_1 \times \overline{\overline{I}} \cdot \hat{E}_1 \tag{10.4.4}$$

where $\overline{\overline{E}} = \hat{E}_1\hat{E}_1 + \hat{E}_2\hat{E}_2 + \hat{E}_3\hat{E}_3$ is a unit dyadic, and where the inertia dyadic about the body 's center of mass (origin of reference frame) is

$$\overline{\overline{I}} = \int_m \left(\overline{\overline{E}} r^2 - \vec{r}\vec{r} \right) dm . \tag{10.4.5}$$

With respect to the satellite body axes $e_\alpha (\alpha = 1,2,3)$, the gravity gradient torque becomes

$$\vec{T}^{(g)} = K\hat{E}_1 \times \overline{\overline{I}} \cdot \hat{E}_1 = K a_{\alpha 1} \hat{e}_\alpha \times I_{\alpha\beta} \hat{e}_\alpha \hat{e}_\beta \cdot a_{\alpha 1} \hat{e}_\alpha \tag{10.4.6}$$

where

$$\begin{aligned} &\hat{E}_1 = a_{\alpha 1}\hat{e}_\alpha = a_{11}\hat{e}_1 + a_{22}\hat{e}_2 + a_{33}\hat{e}_3 \\ &\overline{\overline{I}} = I_{\alpha\beta}\hat{e}_\alpha\hat{e}_\beta \qquad\qquad (\alpha, \beta = 1,2,3) \\ &K = \frac{3\mu}{R_0^3} \end{aligned}$$

and $a_{\alpha 1}$ are the direction cosines between the $\hat{E}_1$ and $\hat{e}_\alpha$ unit vectors.

Equation (10.4) can be written in scalar form to yields the body components of torque as follows:

$$T_\lambda^{(g)} = \vec{T}^{(g)} \cdot \hat{e}_\lambda = K a_{\alpha 1} a_{\beta 1} I_{\gamma\beta} \varepsilon_{\alpha\gamma\lambda} \qquad (\lambda = 1,2,3) \tag{10.4.7}$$

where. $\varepsilon_{\alpha\beta\gamma}$ is the three-dimensional epsilon permutation symbol defined by

$$\varepsilon_{\alpha\beta\gamma} = \begin{cases} 1 \text{ for } \alpha, \beta, \gamma \text{ an even permutation of } 1, 2, 3 \\ -1 \text{ for } \alpha, \beta, \gamma \text{ an odd permutation of } 1, 2, 3 \\ 0 \text{ otherwise (i.e., if any repetitions occure)} \end{cases}$$

$$\varepsilon_{\alpha\beta\gamma} = \left(e_\alpha \times e_\beta \right) \cdot e_\gamma = \frac{1}{2} (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha) \tag{10.4.8}$$

The α, β, γ indices range from 1 to 3.

For the principal body axes, $I_{\gamma\beta} = 0$ for $\gamma \neq \beta$, and the torque components become

$$T_1^{(g)} = K(I_{33} - I_{22})a_{21}a_{31}$$
$$T_2^{(g)} = K(I_{11} - I_{33})a_{11}a_{31} \qquad (10.4.9)$$
$$T_3^{(g)} = K(I_{22} - I_{11})a_{11}a_{21}$$

where the $\alpha_{\alpha\beta}$ terms are elements of the transformation matrix. That is $\hat{e}_\alpha = a_{\alpha\beta}\hat{E}_\beta$. The gravitational torques about the principle axes are of the form as bellow.

$$T_1^{(g)} = -K(I_{33} - I_{22})\cos\theta_2 \sin\theta_3 \sin\theta_2 \qquad = 0 \text{ for } \theta_2 = \theta_3 = 0$$
$$T_2^{(g)} = K(I_{11} - I_{22})\cos\theta_3 \cos\theta_2 \sin\theta_2$$
$$T_3^{(g)} = -K(I_{22} - I_{11})\cos\theta_3 (\cos\theta_2)^2 \sin\theta_3 \qquad (10.4.10)$$

These torques contribute to the total disturbance torque in general, but an oscillatory 'libration' mode will occur if they govern the motion about equilibrium state (see section 3.5.1 [3]). For small oscillations of an axisymmetric spacecraft $(I_{yy} = I_{xx})$ the motion is like a conical pendulum, which frequency is as bellow.

$$\omega_{lib} = \sqrt{\left[\left(\frac{3\mu}{r^3}\right)\left(1 - \frac{I_{zz}}{I_{xx}}\right)\right]} \qquad \text{rad/s} \qquad (10.4.11)$$

Gravity-gradient torque provides a passive self-aligning torque, but the libration does need damping to be incorporated. The torque levels will be low unless a long thin configuration is used, or in the case of tethered satellites.

10.4.4 Aerodynamic Torques (external type)

Aerodynamic torques are the drag force dominated, which is dependent on frontal area A. Considering the projection in the direction of travel may assess their total moment about the center of mass C.

A spacecraft, in general, pass through an atmosphere of density ρ, with a velocity of $\vec{V}$. The magnitude of aerodynamic force $F^{(a)}$ [1] is given as bellow.

$$F^{(a)} = \frac{1}{2}\rho\vec{V}\cdot\vec{V}AC_D \qquad (10.4.12)$$

where the A is reference area of a spacecraft (such as the cross section along $\vec{V}$). C_D is total drag coefficient. The torque contribution about the z-axis will be as bellow.

$$T_z^{(a)} = \frac{1}{2}\rho C_D \int yV^2 dA \qquad (10.4.13)$$

If the equation is integrated the new form will be as bellow.

$$T^{(a)} = \frac{1}{2}\rho V^2 yAC_D \qquad (10.4.14)$$

where y is the length of the perpendicular from the mass center to the force line of action. At spacecraft altitudes, ρ is highly dependent on the time of day and the level of solar activity. See the Figure-10.1 bellow.

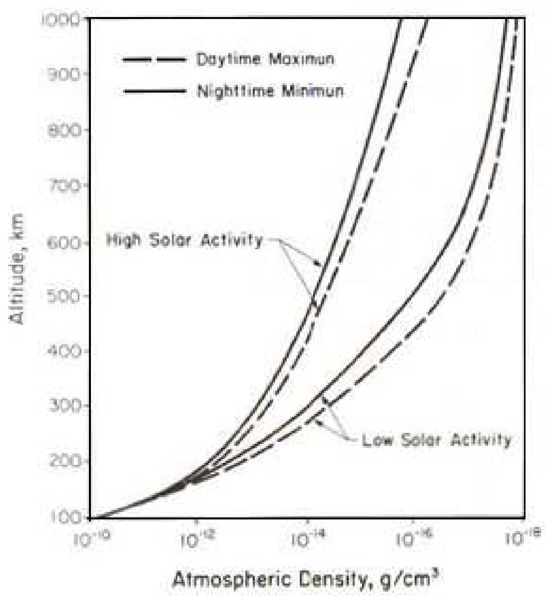

Figure 10.1 Atmospheric density [2]

Height (km)	Density (kg/m^3)
200	4×10^{-10}
300	5×10^{-11}
400	$1,5 \times 10^{-11}$
500	5×10^{-12}
600	2×10^{-12}
700	8×10^{-13}

Table 10.3 Atmospheric density [1].

For a spacecraft having a spherical shape, an average value of C_D = 2.2 and for a cylindrical shape, an average value of C_D = 3 [1].

For zero torque spacecraft designers will of course aim to locate the center of mass close to the center of area, but tolerances, shifts of the center of the mass and thermal distortion will affect the balance.

10.4.5 Solar Radiation Pressure (external type)

Solar radiation produces a force on a surface, which depends upon its distance from the Sun. It is independent of the height above the Earth. Large flat surfaces with a significant moment arm about the center of mass, such as solar arrays, may be produced a significant torque.

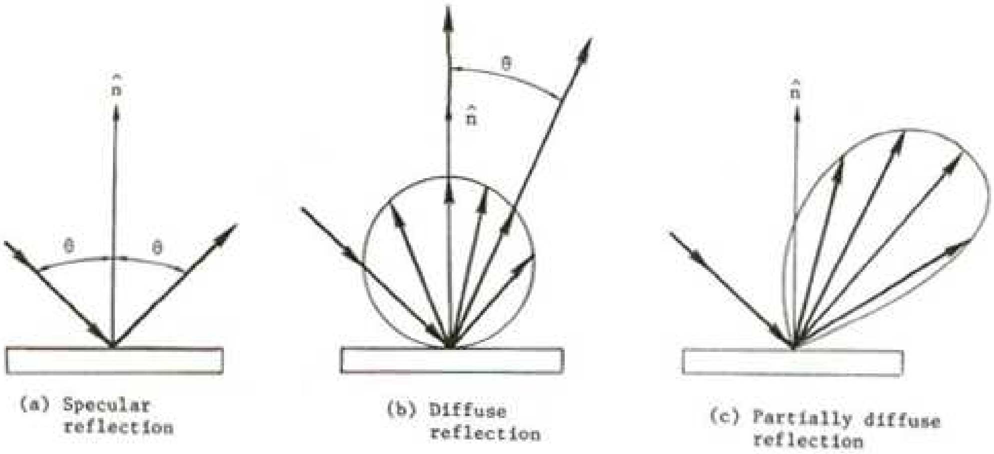

Figure 10.2 Reflection types [1,2]

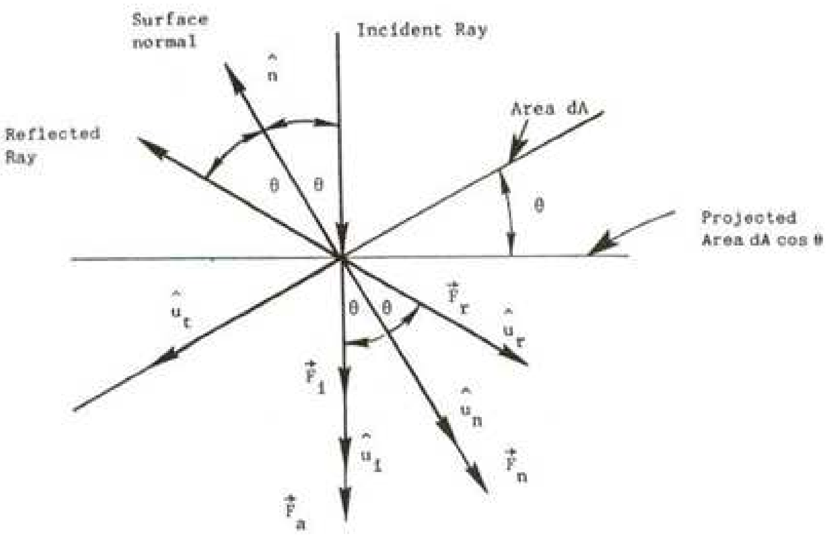

Figure 10.3 Solar force geometry [1]

Specular Reflection: Consider an element of area *dA* oriented at an angle relative to the incoming radiation, as shown in Figure 3 the solar radiation differential force components in terms of coefficient of reflection β are as bellow.

$$d\vec{F}_i = \beta\, PdA\cos\theta\, \hat{u}_i \tag{10.4.15}$$

= force due to incident ray, which is specular reflected.

$$d\vec{F}_a = (1-\beta)PdA\cos\theta\, \hat{u}_i \tag{10.4.16}$$

= force due to absorbed incident ray

$$d\vec{F}_r = \beta\, PdA\cos\theta\, \hat{u}_r \tag{10.4.17}$$

= force due to specularly reflected ray

The coefficient of reflection is the reflected fraction of the solar radiation constant *I(0≤β≤1)*. Total force on an element of area *dA* for a specular reflection is as bellow.

$$d\vec{F}_{sp} = d\vec{F}_a + d\vec{F}_i + d\vec{F}_r = d\vec{F}_a + d\vec{F}_n \qquad (10.4.18)$$

where

$$\begin{aligned} d\vec{F}_n &= \left(d\vec{F}_i + d\vec{F}_r\right)\cos\theta \\ &= 2\beta\, PdA\cos^2\theta\,\hat{u}_n \end{aligned} \qquad (10.4.19)$$

<u>*Diffuse Reflection*</u>*:* If only a subfraction $s\beta$ of the reflected ray is reflected specularly, then $(1 - s)\beta$ is reflected diffusely. Here $0 \le s \le 1$. For pure specular reflection $s = 1$. For completely diffuse reflection $s = 0$. The force due only to the specular reflected ray is then as bellow.

$$d\vec{F}_s = s d\vec{F}_n$$

The force due to the stopping of incoming ray is as bellow.

$$d\vec{F}_{di} = (1 - s)\beta\, PdA\cos\theta\,\hat{u}_i \qquad (10.4.20)$$

The force due to diffuse reflection of the $(1 - s)\beta$ fraction is as bellow.

$$d\vec{F}_{dr} = (1 - s)\beta\,\frac{2P}{3}\,dA\cos\theta\,\hat{u}_n \qquad (10.4.21)$$

Total force is as bellow.

$$d\vec{f} = d\vec{F}_s + d\vec{F}_{di} + d\vec{F}_{dr} + d\vec{F}_a \qquad (10.4.22)$$

or

$$d\vec{f} = \left\{\left[\frac{2\beta}{3}(1 - s)\cos\theta + (1 + s\beta)\cos^2\theta\right]\hat{u}_n + \left[(1 - s\beta)\cos\theta\sin\theta\right]\hat{u}_t\right\} PdA \qquad (10.4.23)$$

<u>*Limiting Cases:*</u> Elemental solar force expressions for the totally absorbing, specularly reflecting, and diffusely reflecting surfaces shown are as bellow.

a. Absorbing surface only

$$d\vec{f}_a = \left[\cos\theta\,\hat{u}_n + \sin\theta\,\hat{u}_t\right]\cos\theta\, PdA \qquad \beta = 0 \qquad (10.4.24)$$

b. Specularly reflecting surface only

$$d\vec{f}_{rs} = \left[(1 + \beta)\cos\theta\,\hat{u}_n + (1 - \beta)\sin\theta\,\hat{u}_t\right]\cos\theta\, PdA \qquad (10.4.25)$$

β: coefficient of specular reflection, for this case s = 1.

c. Diffusely reflecting surface only

$$d\vec{f}_{rd} = \left[\left(\frac{2\beta_d}{3} + \cos\theta\right)\hat{u}_n + \sin\theta\,\hat{u}_t\right]\cos\theta\, PdA \tag{10.4.26}$$

β: coefficient of diffuse reflection, for this case s = 0.

In general, the solar radiation torque on a spacecraft is of the form as bellow.

$$\vec{T}^{(s)} = \int \vec{r} \times d\vec{f} \tag{10.4.27}$$

Example, radiation torque on a geosynchronous satellite. Typical torques about e_1, e_2, and e_3 body axes can be expressed as bellow.

$$\begin{aligned} T_1^{(s)} &= a\cos S' + b\sin S'\cos\mu \\ T_2^{(s)} &= c\sin S'\cos\mu \\ T_3^{(s)} &= d\cos S' - e\sin S'\cos\mu \end{aligned} \tag{10.4.28}$$

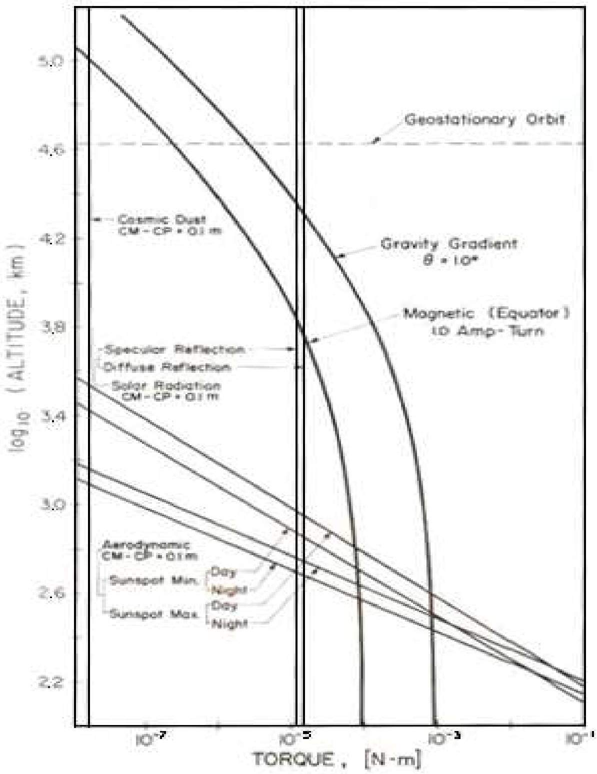

Figure 10.4 Typical torques [2].

10.4.6 Mass Movement (internal type)

The movements of masses within a spacecraft may directly exert torques upon the main structure. These are classified as internal torques and do not affect the total momentum. The movements may also alter the location of the center of mass within the spacecraft, and change the inertia matrix.

The center of mass C has been identified as a key reference point for establishing the dynamic behavior (Chapter3 [3]). Moving the point affect balance of the vehicle in dynamic ways .It also affects the torques due to forces on the vehicle, but not the couple of the magnetic torquer. In principle the center of mass location could be controlled in order to balance out the disturbance torques.

A major source of mass movement is that of the fuel. The tanks are normally located in such a way that as their contents are used up the center of mass does not shift. Fuel movement within the tanks causes a different sort of problem in that it moves in a dynamic way in response to the motion of the spacecraft-fuel slosh-affecting its modal characteristics.

Mass movements from one position to another, such as the erection of solar arrays and other appendages, and movement of astronauts, etc., have an effect upon attitude which is best assessed by using the fact that angular momentum is conserved.

10.4.7 Momentum Storage Torquers (internal type)

Torquers associated with momentum storage such as reaction wheels (RWs) and momentum wheels (MWs) are essentially internal torquers, suitable for attitude control but not for controlling the total momentum.

These devices are purpose-built precision-engineered wheels, which rotate about a fixed axis, with a built-in torque motor. In this, 3-phase coils in the stator are controlled by the drive electronics, which may be seen under the wheel. The resulting rotating magnetic field interacts with the permanently magnetized wheel to produce a torque on the stator, and hence on the structure of the spacecraft. The equal and opposite torque on the wheel changes its speed and its momentum.

Reaction wheels have a nominally zero speed, and may be rotated in either direction in response to the control torques for called for by the spacecraft's ACS.

Momentum wheels on the other hand have a high mean speed of perhaps 6000 r.p.m. in order to provide momentum bias. The control torques will then slow or increase the wheel speed, the permissible amount being about 10% of the mean value.

Both types of wheel provide momentum storage, and need to be used in conjunction with external torquers, as desirable in Section 10.2.2. See also Section 16.3.1.

For three-axis control, three orthogonal wheels will be the minimum requirement. A redundant fourth is normally added at an equal angle to the other three, in order to avoid a single-point failure. When more than one momentum wheel is used the total bias is the *vector* sum of contributions from the separate wheels.

The principle of momentum wheels has been extended by the development of more advanced forms, such as control moment gyroscopes(CMGs).By mounting the wheel in gimbals fitted with torque motors all three components of torque may be developed from a single wheel. This can be done to a limited extent with sophisticated wheels mounted on five-degree-of-freedom magnetic bearings (see Section 16.3.1). There is potential for incorporating attitude-sensing with momentum-storage and momentum bias in sophisticated devices of this type.

10.5 ATTITUDE MEASURMENT

10.5.1 Attitude: Its Meaning and Measurement

The meaning of 'attitude' or 'orientation' usually represent no conceptual difficulties. There must be some datum frame of reference, and once this has been chosen then the attitude of a spacecraft refers to its angular departure from this datum. A right-handed set of axes is normally used in order to define a frame of reference, and if both a datum set and a set of spacecraft axes are chosen, then the attitude may be defined in a way that may be quantified.

Specifying attitude may be done in a number of ways such as Euler angles, direction cosines, quaternions, etc. Three pieces of information are need. A common way is to use the three Euler angles, which are defined in the same way, as is standard practice for aircraft. These are the angles of yaw ψ, pitch θ, and roll ϕ, as measures of the rotation about the *z*-, *y*-, and *x*-axes respectively, in that sequence, which are needed to bring the datum axes into alignment with those of the spacecraft.

It is worth noting that angles, and consequently attitude, are not vector quantities. The combination (ψ,θ,ϕ) should not be thought of as three components of a vector. On the other hand the rates of change $\dot{\psi},\dot{\theta},\dot{\phi}$ can be interpreted as vector quantities, which directions are along the (non-orthogonal) axes about, which the rotation take place. Resolving $\dot{\psi},\dot{\theta},\dot{\phi}$ along spacecraft axes enables the components of the spacecraft's angular velocity ω relative to the datum axes to be expresses as bellow.

$$\begin{aligned} \omega_x &= \dot{\phi} - \dot{\psi}\sin\theta \\ \omega_y &= \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi \\ \omega_z &= \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \end{aligned} \qquad (10.5.1)$$

the inverse relation is

$$\begin{aligned}
\dot{\psi} &= \left(\omega_y \sin\phi + \omega_z \cos\phi\right)/\cos\theta \\
\dot{\theta} &= \omega_y \cos\phi - \omega_z \sin\phi \\
\dot{\phi} &= \omega_x + \left(\omega_y \sin\phi + \omega_z \cos\phi\right)\tan\theta
\end{aligned} \qquad (10.5.2)$$

When the angles are small, then $\dot{\psi} \approx \omega_z, \dot{\theta} \approx \omega_y, \dot{\phi} \approx \omega_x$ [3].

Equation indicates how, by integration, the attitude in the form of the Euler angles (ψ, θ, ϕ) may be obtained from measured components of angular velocity. The singularity at $\theta = 90^\circ$ shows up in the form of $\tan\theta$ and will lead to problems with the integration as θ approaches this value.

10.5.2 Measurement System Fundamentals

Fundamentally, measurement of attitude requires the determination of three pieces of information, which relate the spacecraft axes to some datum set, whether they are in the form of Euler angles or in other forms. The measurement subsystem must include sufficient sensors to enable the information to be extracted with the necessary accuracy, and with reasonable simplicity. This must be done at all phase of the mission.

There are two categories of sensor, and they are commonly used to compliment each other in a measurement system:

- *Te reference sensor* gives a definite 'fix' by measuring the direction of an object such as the Sun or a star, etc., but there are normally periods of eclipse during which its information is not available.
- *Inertial sensor*, but they measure only changes

Using reference and inertial sensors to complement each other may form a measurement system. In a simple combination the reference sensor will calibrate the inertial sensor at discrete times and the latter will then effectively 'remember' the reference object's direction until the next calibration. This allows a period in eclipse to be covered.

Complete attitude information requires three pieces of information as explained above. Reference sensors that are based upon detecting the direction of a single vector are incapable of providing all three pieces. A sun sensor cannot detect any rotation of a spacecraft about the Sun vector for example. Two vector directions, and an angle is needed to complete attitude information to be obtained from simultaneous measurements.

Reference Object	Potential accurancy
Stars	1 arc second
Sun	1 arc minute
Earth	6 arc minutes
RF beacon	1 arc minute
Magnetometer	30 arc minutes

Table 10.4 Potential accuracies of reference sensors. This table gives only a guideline.

10.5.3 Types of Reference Sensors

There are so many different sensors types. Only a few will be explained.

Sun Sensor:

The vector Sun sensor works on a different principle. In fact, a vector Sun sensor consists of two sensors, each consisting essentially of a rectangular chamber with a thin slit at the top. Light entering the chamber through the slit casts an image of a thin line on the bottom of a chamber. The bottom of chamber is lined with a network of light-sensitive cells that effectively measure the distance *d* of the image from a centerline. If *h* is the height of the chamber, then the angle of incidence to the sensor, α, is given as bellow.

$$\tan\alpha = \frac{d}{h} \tag{10.5.3}$$

By placing two sensors perpendicular to each other (Figure 10.5), one can measure the complete direction of the son with respect to the sensor axes. The geometry for interpreting these two measurements is shown in Figure 10.5.

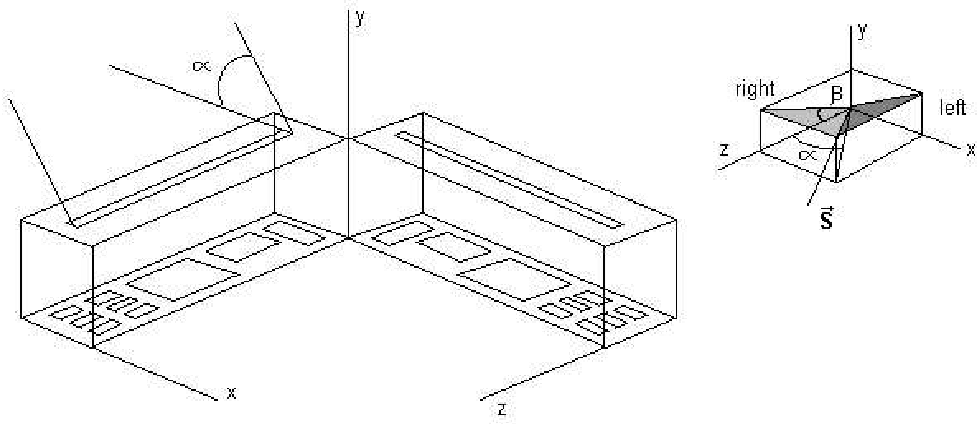

Figure 10.5 Two-slit Sun sensors.

To determine the Sun direction in the sensor frame from the two angle measurements α and β, we note that these are related to the Sun direction by where $\hat{x}, \hat{y}, \hat{z}$

$$\tan\alpha = \frac{\hat{x}\cdot\hat{S}}{\hat{z}\cdot\hat{S}}, \qquad \tan\beta = \frac{\hat{y}\cdot\hat{S}}{\hat{z}\cdot\hat{S}} \tag{10.5.4}$$

are the axes of the sensor coordinate system. The z axis of the sensor is usually defined as the outwardly directed normal. From these measurements we construct the unit vector $\hat{S}_{Sun}$ in sensor coordinates as bellow.

$$\hat{S}_{Sun} = \frac{1}{\sqrt{1+\tan^2\alpha+\tan^2\beta}}\begin{bmatrix}\tan\alpha \\ \tan\beta \\ 1\end{bmatrix} \tag{10.5.5}$$

This measurement is of the Sun direction in the Sun sensor frame, $\hat{S}_{Sun}$. The measurement in the body frame is obtained from

$$\hat{S}_B = S_{Sun}\hat{S}_{Sun} \tag{10.5.6}$$

and allowing for instrument noise

$$\hat{S}_B = C\hat{S}_1 + \Delta\hat{S}_B \tag{10.5.7}$$

in analogy with similar relations for the vector magnetometer.

DSAD, Digital Solar Aspect Detectors determine the angles of the Sun by determining which of the light – sensitive cells in the sensor is the most strongly illuminated. The accuracy of this sensor is limited by the angular diameter of the Sun, which is approximately 0.5 degrees, as seen from the Earth.

Vector Sun sensors generally have fields of view of ±60 degrees. Therefore if the spacecraft is not inertially stabilized at one throughout the mission, it will be necessary to use more than one sensor head to ensure that the Sun is always visible in one head.

Earth Sensor:

The Earth, radius R, subtends the angle 2 arcsin(1/(1+h/R)), at a satellite at a height h. At 500 km it is about 135°, falling to 17.5° at geostationary altitude. Sensing the direction of the local vertical entails bisecting the directions to the horizons at the ends of a diameter of its disc. And horizon sensors provide the mean of doing this…

Star Sensor:

Star sensors are the most accurate reference sources in common use for measuring attitude. Accuracies of 1 arc second or better may be obtained. But the large number of stars means that sophisticated techniques are needed in instrument and its associated computer in order to identify any particular star in its field of view (FOV). The sensors are heavy, power-hungry, and expensive, although considerable improvements of these characteristics are taking place.

Star sensors may be classified as bellow:

- *Star Scanners* for mounting on a rotation base. They have one or more fan-shaped fields of view, which scan the heavens. The characteristics of stars passing through the fields of view can be compared with a star directory in order to determine the attitude of the spacecraft.
- *Star Trackers* for mounting on a three-axis-stabilized base. The field pf view is sufficient to include several stars, and their detector/controller enables them to select, locate and track one or more of these accurately. Each star has a different vector direction, and so the spacecraft's orientation about the sensor's axis may be determined.
- *Star Mappers* for mounting on a three-axis-stabilized base. They are basically similar to the tracker, but operate sequentially on the stars in their of view, locating and recording the position of one, and then moving on to the next.

Radio frequency beacons:

Direction-finding techniques may be used to detect the direction of an RF source, with an accuracy of other 1 arc minute. There are several techniques by which this can be done.

For instant, Ulysses, rotating at a nominal 5 rpm, carries an antenna, which axis is offset from spin axis. The intensity of the signal, which it receives from ground station, is thereby modulated at the spin frequency. The actual spin rate and its phase, and the angle between the spin axis and the ground station direction, can then be derived respectively from the frequency and the phase and the dept of modulation.

Magnetometers:

The magnetometer is a robust instrument but with accuracy, which is limited to about 0.5°. It measures the direction and possibility the strength of the local magnetic field. But the field is not well mapped and has abnormalities, which make the sensor of limited use for attitude sensing. It is used in conjunction with magnetic torquers as described in section 10.4.2

10.5.4 Inertial Sensor

Gyroscopes form the basis of the inertial sensing systems for attitude. The conventional, wheel, type of gyro has a rotor mounted in a single gimbal in an environment, which is very carefully controlled. In the rate– and rate–integrating types the gimbal is torqued so that it follows the motion of the spacecraft. The torque is then a measure of the angular rate about the instrument's sensitive axis.

A set of three orthogonal rate-gyros will measure the components ($\omega_x, \omega_y, \omega_z$) of the spacecraft's angular velocity; a fourth at a skew angle is normally carried to avoid a single–point failure. The output of a rate–integrating gyro (RIG) is the integral of the angular velocity component, such as $\int \omega_x dt$ etc. Only when the direction of a RIG axis remains fixed in space does its output represent the angular displacement about the axis.

Gyroscopes based upon the laser principle are later developments. In these the laser cavity is bent round the instrument's sensitive axis so that the path length of the light rays is lengthened or shortened when the instrument rotates. The beating of two frequencies resulting from beams in opposite directions gives a measure of instrument's angular rate. A set of ring laser qyro (RLG) for use on Ariane 4 has triangular cavities formed by mirrors at the apex of the triangle.

Fibre optic gyros (FOG), using fibre optic coils to guide the beams round the sensitive axis, with a bias stability of order 0.1 to 10 degree/h, can be expected to be in use in launch situations of the feature.

10.6 ACS COMPUTATION

10.6.1 The Computer

The development of digital computers for use in spacecraft has proceeded rapidly, and is still doing so. They must perform reliably in the radiation environment of space, and a number of space-qualified ones now exist. Further development is providing more power and speed, and capability of being programmed in higher-level languages. These on-board computers (OBCs) link with ground-control computers, which will normally host their software development tools (see Chapter 15). The availability of powerful computers means that spacecraft will be given greater autonomy, and many of the sophisticated control techniques, which find applications in ground-based systems may be used on spacecraft.

Robustness is a requirement for ACS and other on-board systems. For example the ACS must potentially operate with large flexible structures such as solar arrays, which natural frequencies cannot be established accurately before lunch. Fixed algorithms will tolerate only limited variation from their expected value. The ability to reprogram the OBC from Ground Control permits any necessary adjustment of the control algorithms to be made following calibration of the spacecraft's parameters after launch. For full autonomy or immediate response to any changes, which occur such as hardware failures, adaptive control techniques may be used.

Computer power will also benefit the attitude measurement subsystem. The mixing of sensor outputs to achieve maximum accuracy via the Kalman type of filter requires computer modeling. In addition they can provide the substantial data backup, which is needed when star mappers and scanners are used.

10.6.2 A simple Control Example

Control of the rotation of a rigid body about one axis can be achieved with a simple PID algorithm (proportional, integral, differential). This is the basis of many systems and is used in following example. It is important that cross coupling between the motions about the three axes is not sever, and any large reorientation may be implemented sequentially to avoid them.

The following example illustrates the type of motion, which may be expected when a simple three-term PID controller is used to control the roll attitude of a spacecraft.

When no moment bias is represent the roll error ϕ will respond to roll torque T_x as follows [3]

$$I_{xx}\ddot{\phi} = T_x \qquad (10.6.1)$$

A PID controller generates a demanded torque signal based upon measured values of roll error and roll rate, as in

$$T_{xD} = -K_p\phi_m - K_i\int\phi_m dt - K_d\left(\dot{\phi}\right)_m \qquad (10.6.2)$$

This can be made to represent a stable system by appropriate choice of the constants K_p, K_i, K_d due allowance being made for any delay in implementing the torque. The presence of any flexure mode, fuel movement, etc., will also impose constraints on the choice of these constants.

Pointing errors will result as a consequence of zero errors from the roll–error sensor and drift in the integration. Process, but constant disturbance torques and a constant error from the roll-rate sensor will be automatically compensated.

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