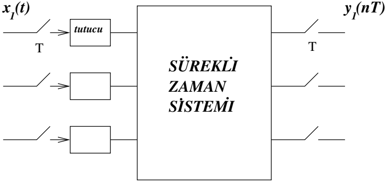
Sayısal Filtreler ve Sistemler
EHB 433

Prof. Dr. Müştak E. Yalçın

Istanbul Technical University
Faculty of Electrical and Electronic Engineering

mustak.yalcin@itu.edu.tr

Outline I



$$\begin{aligned} \dot{v} &= A_c v + B_c x \\ y &= C_c v + D_c x \end{aligned}$$

Burda $v \in R^n$, $x \in R^r$ ve $y \in R^m$ alınmıştır (Çoğunlukla durum olarak x giriş olarak u kullanılır.)
Yukardaki sistemin çözümü

$$v(t) = e^{A_c(t-t_0)} v(t_0) + \int_{t_0}^{t} e^{A_c(t-\tau)} B_c x(\tau) d\tau$$

Bu çözüm geçiçi hal ve kalıcı hal çözümlerinin toplamıdır !

u nun parça parça sürekli olduğunu düşünerek $nT \leq t \leq (n+1)T$ aralığında $x(t) = x(nT)$ alınmakta ayrıca $t_0 = nT$ ve $t = (n+1)T$ alındığında

$$v((n+1)T) = e^{A_c T} v(nT) + \int_{nT}^{(n+1)T} e^{A_c[(n+1)T-\tau]} B_c x(nT) d\tau$$

$t = (n+1)T - \tau$ alıp integralde değişken dönüşümü yapıldığında

$$v((n+1)T) = e^{A_c T} v(nT) + \int_{T}^{0} e^{A_c t} B_c x(nT)(-dt)$$

burdan

$$\begin{aligned} v(k+1) &= Av(k) + Bx(k) \\ y(k) &= Cv(k) + Dx(k) \end{aligned}$$

$A = e^{A_c T}$ ve $B = \int_0^T e^{A_c t} B_c dt$ olmak üzere bulunur.

Laplace dönüşümü yardımıyla durum denklemlerinin bulunması

$$sV(s) - v(nT) = A_c V(s) + \frac{1}{s} B_c x(nT)$$

$$V(s) = (sI - A_c)^{-1} v(nT) + \frac{1}{s}(sI - A_c)^{-1} B_c x(nT)$$

$$v((n+1)T) = \Phi(\tau) v(nT) + \int_0^{\tau} \Phi(\tau - \lambda) B_c x(nT) d\lambda$$

$$v((n+1)T) = \Phi(T) v(nT) + \int_0^{T} \Phi(T - \lambda) B_c x(nT) d\lambda$$

burdan A ve B bulunabilir.

$$A = \{\mathscr{L}^{-1} \left\{(sI - A_c)^{-1}\right\}, B = \{\mathscr{L}^{-1} \left\{\frac{1}{s}(sI - A_c)^{-1} B_c\right\}$$

Katsuhiko Ogata, Discrete-Time Control Systems, Pearson Education (1994) Chapter 5

Transfer fonksiyonun (matrisinin) durum denklemlerinden bulunması

$$\begin{aligned} v(k+1) &= Av(k) + Bx(k) \\ y(k) &= Cv(k) + Dx(k) \end{aligned}$$

$$Y(z) = H(z)X(z)$$

z-dönüşümünü kullanarak $(x(0) = 0)$

$$\begin{aligned} zV(z) &= AV(z) + BX(z) \\ Y(z) &= CV(z) + DX(z) \end{aligned}$$

burdanda

$$Y(z) = [C(zI - A)^{-1}B + D]X(z)$$

Örnek :

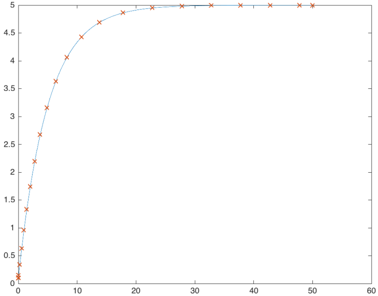
Örnek : Aşağıda A_c ve B_c matrisleri verilen sürekli zaman sistemi ayrıklaştırıldığı durumda elde edilen ayrık zaman sisteminin A ve B matrislerini bulun.

$$A_c = \begin{bmatrix} -a & 0 \\ 1 & 0 \end{bmatrix} \quad B_c = [1 \;\; 0]^T \quad C_c = [0 \;\; 1]$$

$$\begin{aligned} A &= \mathscr{L}^{-1}\left\{\begin{bmatrix} s+a & 0 \\ -1 & s \end{bmatrix}^{-1}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s(s+a)}\begin{bmatrix} s & 0 \\ 1 & s+a \end{bmatrix}\right\} \\ &= \begin{bmatrix} e^{-aT} & 0 \\ \frac{1}{a}(1-e^{-aT}) & 1 \end{bmatrix}, t \geq 0 \end{aligned}$$

$$B = \int_0^T e^{A_c t} dt B_c = \left[\frac{1-e^{-aT}}{a} \quad \frac{1}{a}(T + \frac{e^{-aT}-1}{a})\right]^T$$

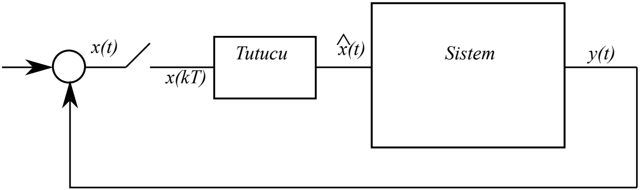
```
function dy = prime1(t,y)
a=.2;A=[-a 0;1 0];B=[1;0];
dy= A*y+B*1;
tt,Y= ode23('prime1', [0, 50], [0.1;0.2]);
a=0.2;T=.001;
Ak=[exp(-a*T) 0; (1-exp(-a*T))/a 1];
Bk=[(1-exp(-a*T))/a;(T+(exp(-a*T)-1)/a)/a];
yd(:,1)=[0.1;0.2];
for i=1:50000,yd(:,i+1)=Ak*yd(:,i)+Bk*1;end
>> plot(T.*[1:1:50001],yd(1,:));hold on;plot(tt,Y(:,1),'x')
```



Örnek

Sürekli zaman sistemine ilişkin durum denklemi;

$$A_c = \begin{bmatrix} -a & 0 \\ 1 & 0 \end{bmatrix} \quad B_c = [1 \;\; 0]^T \quad C_c = [0 \;\; 1]$$



$$v(k+1) = Av(k) + Bx(k); y(t) = Cv(t); x(t) = r(t) - y(t)$$

$$v(k+1) = Av(k) + B(r(k) - Cv(k)); v(k+1) = (A - BC)v(k) + Br(k);$$

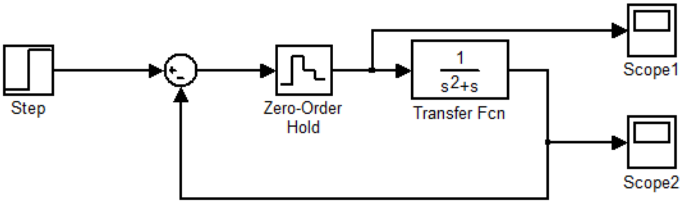
$$A = \begin{bmatrix} e^{-aT} & \frac{e^{-aT}-1}{a} \\ \frac{1}{a}(1 - e^{-aT}) & 1 - \frac{1}{a}(T + \frac{e^{-aT}-1}{a}) \end{bmatrix}, t \geq 0$$

```
A-BC=[exp(-a*T) (exp(-a*T)-1)/a; (1-exp(-a*T))/a
1-(T+(exp(-a*T)-1)/a)]
```

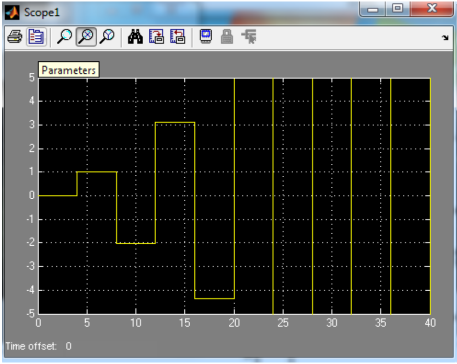
$$a = 1; T = 1; \lambda(A) = 0.5000 \pm 0.6182i$$

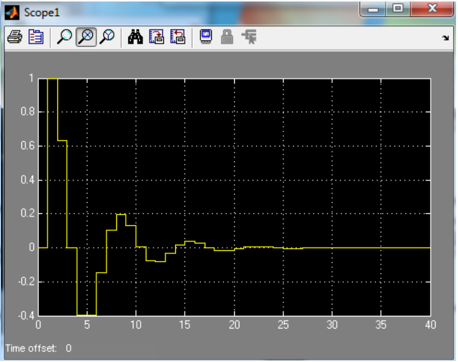
$$a = 1; T = 4; \lambda(A) = \{-0.7293, -1.2707\}$$

Geribesleme sistemi örnekleme frekansına bağlı olarak kararsız olmakta !

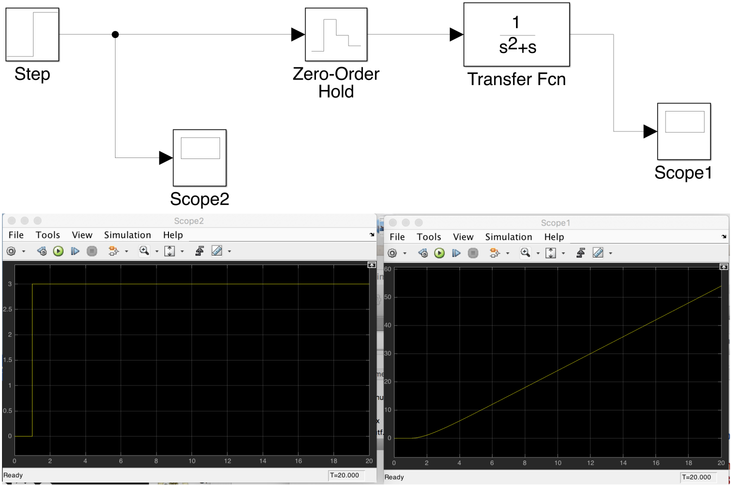


$T = 4$ ve $T = 1$ için Simulink sonuçları

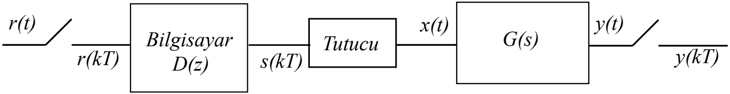




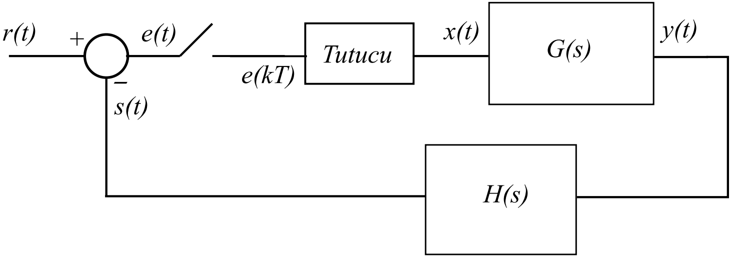
Bilgisayar Kontrollü sistem



Bilgisayar Kontrollü sistem



$$Y(z) = D(z)\mathscr{Z}\{G_m(s)G(s)\}R(z)$$



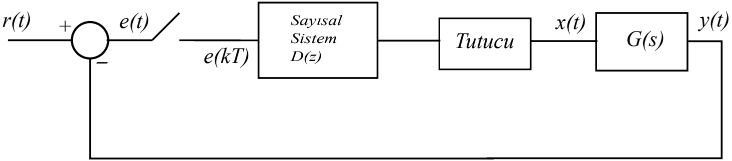
$$Y(z) = \mathscr{Z}\{G_m(s)G(s)\}E(z)$$

$$S(z) = \mathscr{Z}\{G_m(s)G(s)H(s)\}E(z)$$

$$e(kT) = r(kT) - s(kT); E(z) = R(z) - S(z)$$

$$\frac{Y(z)}{R(z)} = \frac{\mathscr{Z}\{G_m(s)G(s)\}}{1 + \mathscr{Z}\{G_m(s)G(s)H(s)\}}$$

Ayrık zaman sistemi tasarımı: Analitik



$$\frac{Y(z)}{R(z)} = K(z) = \frac{D(z)\mathscr{Z}\{G_m(s)G(s)\}}{1 + D(z)\mathscr{Z}\{G_m(s)G(s)\}}$$

Sistemi kontrol edecek $D(z)$

$$D(z) = \frac{K(z)}{\mathscr{Z}\{G_m(s)G(s)\}(1 - K(z))}$$

$$E(z) = R(z) - Y(z) = [1 - K(z)]R(z)$$

limit teoreminden

$$\begin{aligned} e(\infty) &= \lim_{z\to 1}(1 - z^{-1})[1 - K(z)]R(z) \\ &= \lim_{z\to 1}(1 - z^{-1})[1 - K(z)]\frac{P(z)}{Q(z)} \\ &= 0 \end{aligned}$$

Bu $(1 - z^{-1}) = 0$ ile sağlanmakta ancak belirsizliği ortadn kaldırmak için

$$[1 - K(z)] = Q(z)$$

şartı eklenir. Çıkışta birim basamak yanıtı istensin. Bu durumda $R(z) = \frac{1}{1-z^{-1}}$ için

$$[1 - K(z)] = 1 - z^{-1}$$

eşitliğinden

$$K(z) = z^{-1}$$

elde edilecektir.

Örnek: $T = 1$

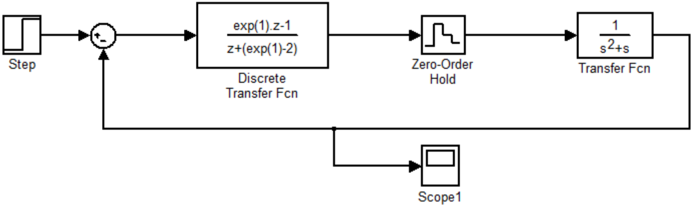
$$G(s) = \frac{1}{s(s+1)}$$

$$\mathscr{Z}\{G_m(s)G(s)\} = (1 - z^{-1})\mathscr{Z}\{\frac{1}{s^2(s+1)}\} = \frac{z^{-1}(1 + (e-2)z^{-1})}{(1 - z^{-1})(e - z^{-1})}$$

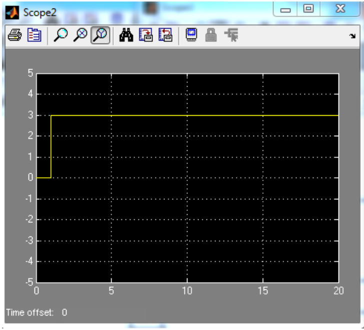
$$D(z) = \frac{K(z)}{\mathscr{Z}\{G_m(s)G(s)\}(1 - K(z))}$$

eşitliğinde $K(z) = z^{-1}$ alınarak

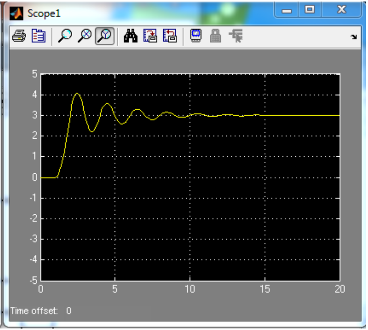
$$D(z) = \frac{e - z^{-1}}{1 + (e-2)z^{-1}}$$



Step
exp(1).z-1
z+(exp(1)-2)
Discrete
Transfer Fcn
Zero-Order
Hold
1
s2+s
Transfer Fcn
Scope1



Scope2
Time offset: 0



Scope1
Time offset: 0

Çıkışta rampa yanıtı istensin. Bu durumda $R(z) = \frac{\alpha T z^{-1}}{1-z^{-1}}$ için

$$[1 - K(z)] = (1 - z^{-1})^2$$

eşitliğinden

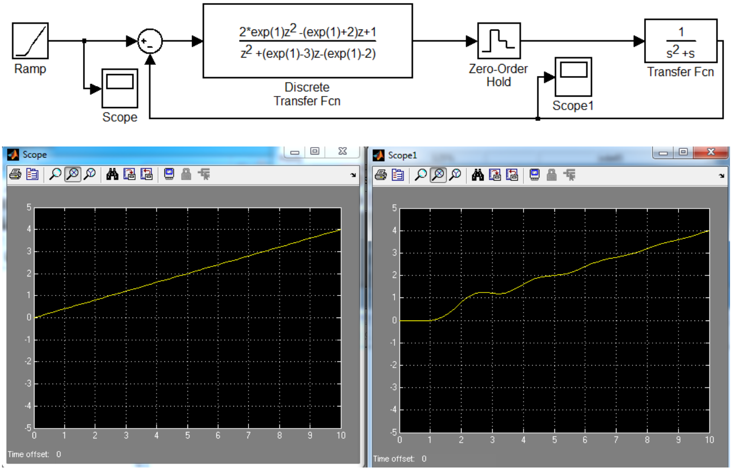
$$K(z) = 2z^{-1} - z^{-2}$$

elde edilecektir.

$$D(z) = \frac{K(z)}{\mathscr{Z}\{G_m(s)G(s)\}(1 - K(z))}$$

eşitliğinde $K(z) = 2z^{-1} - z^{-2}$ kullanılarak

$$D(z) = \frac{2e - (2+e)z^{-1} + z^{-2}}{1 + (e-3)z^{-1} + (2-e)z^{-2}}$$


Ramp
Scope
2*exp(1)z2 -(exp(1)+2)z+1
z2 +(exp(1)-3)z-(exp(1)-2)
Discrete
Transfer Fcn
Zero-Order
Hold
Scope1
1
s2 +s
Transfer Fcn
Scope
Time offset: 0
Scope1
Time offset: 0