Circuit and System Analysis
EHB 232E

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Outline I

1. Sinusoidal Steady-State Analysis-Cont.
   - The Passive Circuit Elements in the Frequency Domain
   - The Concept of Impedance and Admittance
   - Resonance
   - Phasor diagram

The Passive Circuit Elements in the Frequency Domain

Resistors: From Ohm's law, if the current in a resistor varies sinusoidally with time, the voltage at the terminals of the resistor

$$v(t) = R\ \mathrm{Re}\{I_R e^{jwt}\}$$

$$v(t) = \mathrm{Re}\{RI_R e^{jwt}\}$$

$$\mathrm{Re}\{V_R e^{jwt}\} = \mathrm{Re}\{RI_R e^{jwt}\}$$

from the properties of phasor

$$V_R = RI_R$$

or

$$I_G = GV_G$$

There is no phase shift between the current and voltage of resistor. The signals of voltage and current are said to be in phase.

The Passive Circuit Elements in the Frequency Domain

Capacitor: Substituting the phasor representation of the current and phasor voltage at the terminals of a capacitor into $i = C\frac{dv}{dt})$

$$\mathrm{Re}\{I_C e^{jwt}\} = C\frac{d\mathrm{Re}\{V_C e^{jwt}\}}{dt}$$

using the properties of phasor

$$\mathrm{Re}\{I_C e^{jwt}\} = \mathrm{Re}\{CV_C\frac{de^{jwt}}{dt}\} = \mathrm{Re}\{CV_C jwe^{jwt}\}$$

we get

$$I_C = jwCV_C$$

The current leads the voltage across the terminals of a capacitor by 90°.

The Passive Circuit Elements in the Frequency Domain

Inductor:

$$V_L = jwLI_L$$

The current lags the voltage by 90°.

Independent current and voltage sources

$$I_k = I_{mk}e^{j\theta_k}$$

and

$$V_k = V_{mk}e^{j\theta_k}$$

The Concept of Impedance and Admittance

The driving-point impedance of the one-port N (is formed by an arbitrary interconnection of linear time-invariant elements) at the frequency w to be the ratio of the port-voltage phasor V and the input-current phasor I that is,

$$Z(jw) = \frac{V}{I}$$

Thus the amplitude of the port voltage is the product of the current amplitude times the magnitude of the impedance. Z represents the impedence of the circuit element

$$Z = \frac{V}{I} = R + jL$$

R, is called resistance and L, is called reactance.
Y represents the admittance of the circuit element

$$Y = \frac{I}{V} = G + jB$$

G, is called conductance and B, is called susceptance.

Element	Impedance	Reactance	Admintance	Susceptance
Resistor	R	-	G	-
Capacitor	-j/wC	-1/wC	jwC	wC
Inductor	jwL	wL	-j/wL	-1/wL

Combining Impedance in Series and Parallel

Impedances in series can be combined into a single impedance by simply adding the individual impedances.

$$Z = Z_1 + Z_2 + ... + Z_n$$

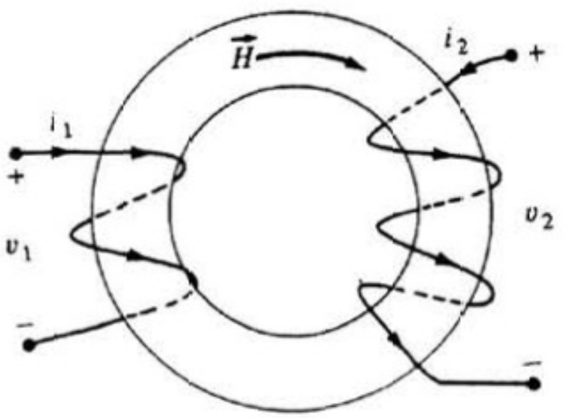
when they in parallel

$$Z = \left\{ \frac{1}{Z_1} + \frac{1}{Z_2} + ... + \frac{1}{Z_n} \right\}^{-1}$$

admittances in parallel:

$$Y = Y_1 + Y_2 + ... + Y_n$$

Mutual Inductance

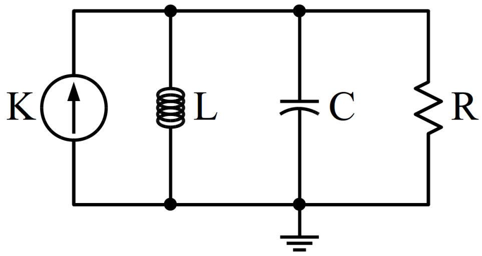


$$\phi_1 = L_1 i_1 + M i_2$$
$$\phi_2 = L_2 i_2 + M i_1$$

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$
$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Example

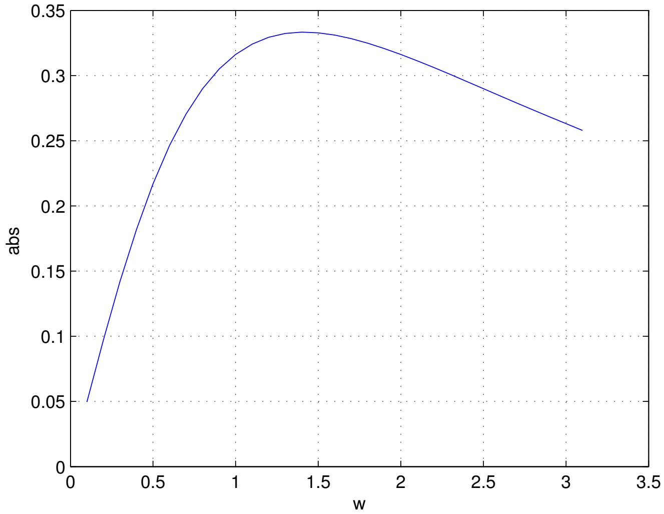
Find the complete solution for $R = 1/3\Omega$, $C = 1F$, $L = 1/2H$, $V_C(0) = 1V$ ve $i_L(0) = 1A$ and $i(t) = \cos(wt)$.



$$Y_{eq}(j\omega) = \frac{1}{j\omega L} + \frac{1}{R} + j\omega C = \frac{R + j\omega L - \omega^2 RLC}{j\omega RL}$$

$$V_C = Z_{eq}(j\omega) \cdot I_K = \frac{I_K}{Y_{eq}(j\omega)} = \frac{1}{Y_{eq}(j\omega)} = \frac{j\omega RL}{R - \omega^2 RLC + j\omega L}$$

Magnitude of $V_C(j\omega)$ is maximun when $\omega = \frac{1}{\sqrt{LC}}$!

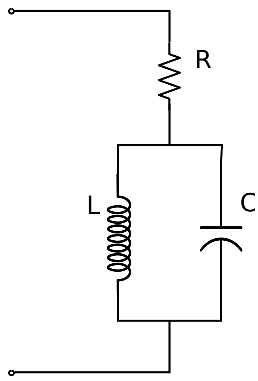


$$Z(j\frac{1}{\sqrt{LC}}) = R$$

Resonance

Resonance occurs at a particular resonance frequency when the imaginary parts of impedances or admittances of circuit elements cancel each other.

Example



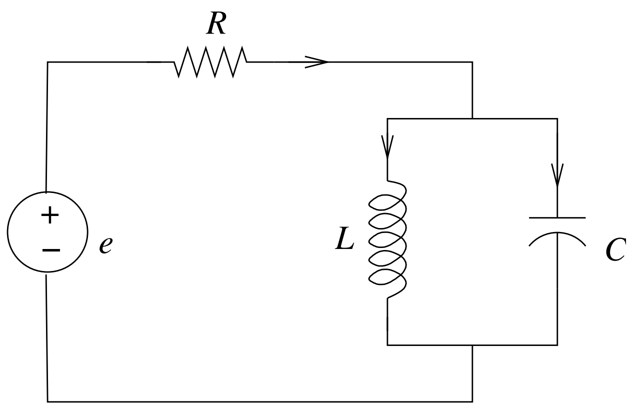
Find the impedance Z between the terminals:

$$Y = Cjw + \frac{1}{Ljw}$$

$$Z = R + \frac{Ljw}{1 - LCw^2}$$

$$Z = \frac{R - RLCw^2 + Ljw}{1 - LCw^2}$$

Example



State equation for $R = 1/5$, $C = 1F$ ve $L = 1/6$

$$\frac{d}{dt}\begin{bmatrix} V_{C1} \\ i_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}\begin{bmatrix} V_{C1} \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{RC} \\ 0 \end{bmatrix} e$$

which is

$$\frac{d}{dt}\begin{bmatrix} V_{C1} \\ i_L \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} V_{C1} \\ i_L \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix} e$$

The roots of the function

$$\det\left\{\lambda I - \begin{bmatrix} -5 & -1 \\ 6 & 0 \end{bmatrix}\right\} = \lambda(\lambda+5)+6$$

are the eigenvalues of A which $\lambda_1 = -3$ and $\lambda_2 = -2$. Corresponding eigenvalues are $[1 \ -2]^T$ and $[1 \ -3]^T$. Fundamental matrix

$$M = \begin{bmatrix} e^{-3t} & e^{-2t} \\ -2e^{-3t} & -3e^{-2t} \end{bmatrix}$$

The homogeneous solution

$$x_h(t) = \begin{bmatrix} e^{-3t} & e^{-2t} \\ -2e^{-3t} & -3e^{-2t} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

The state transition matrix of the circuit

$$\phi(t) = \begin{bmatrix} e^{-3t} & e^{-2t} \\ -2e^{-3t} & -3e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}^{-1}$$

Example

Using the The Concept of Impedance, lets find the V_R

$$V_R = R\frac{e}{Z}$$
$$= R\frac{(1 - LCw^2)}{Ljw + R(1 - LCw^2)}e$$

Using the state-equation

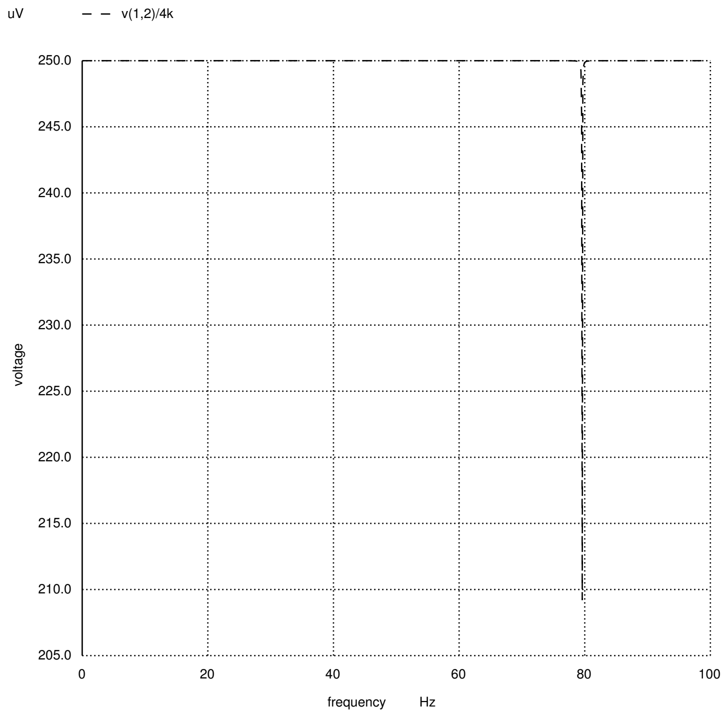
$$V_C = [1 \;\; 0] \begin{bmatrix} jw + \frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & jw \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{RC} \\ 0 \end{bmatrix} e$$
$$= \frac{\frac{jw}{RC}}{\frac{1}{LC} - w^2 + \frac{jw}{RC}}$$

$$V_R = e - V_C = e \left\{ 1 - \frac{\frac{jw}{RC}}{\frac{1}{LC} - w^2 + \frac{jw}{RC}} \right\}$$
$$= e \left\{ \frac{\frac{1}{LC} - w^2}{\frac{1}{LC} - w^2 + \frac{jw}{RC}} \right\}$$
$$= \frac{R(1 - LCw^2)e}{Ljw + R(1 - LCw^2)}$$

Example

```
Sekil 7.2
v1 1 0 sin(0 .1 10) dc 0 ac 1
r 1 2 4k
l 2 0 2m
c 2 0 2m
.control
ac lin 1000 .1 100
plot v(1,2)/4k
.endc
.end
```

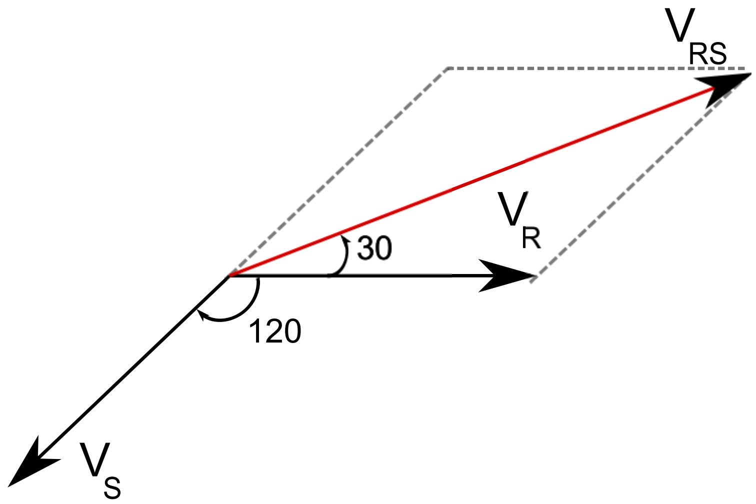
Example



YouTube Video: Example with two sources

Phasor diagram

A phasor diagram shows the magnitude and phase angle of each phasor quantity in the complex number plane.



$V_R = V,\ V_S = Ve^{-j120^\circ},\ V_{RS} = \sqrt{3}Ve^{j30^\circ},\ V_{RS}?f(V_S, V_R)$