Circuit and System Analysis
EHB 232E

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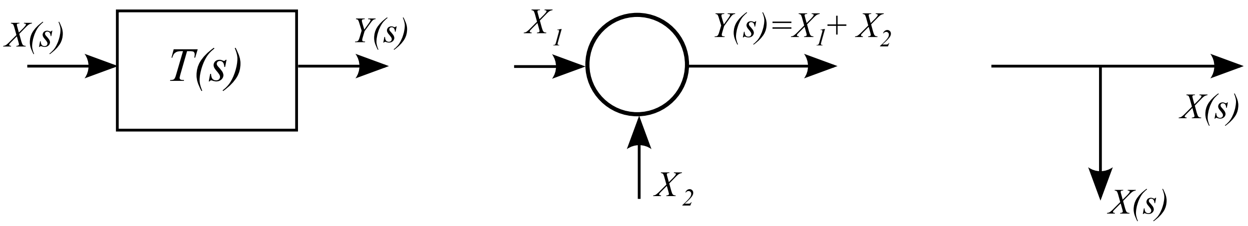
Outline I

The elements of block diagrams

Block diagrams are ways of representing relationships between signals in a system.

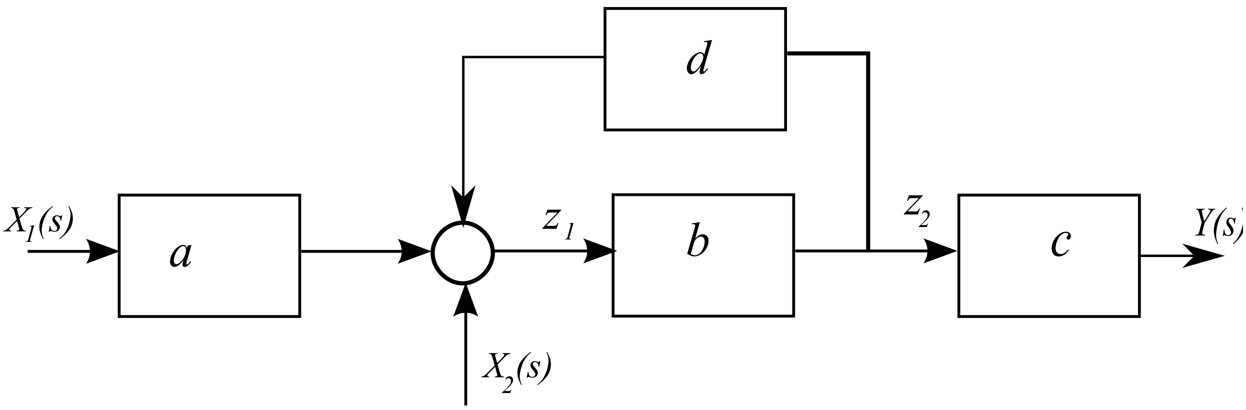
A block diagram is a representation of a system using blocks.

Block $Y(s) = T(s)X(s)$ summing junction pickoff point



Output $Y(s)$, block gain $T(s)$, input $X(s)$

Block diagrams



$$z_1 = aX_1(s) + X_2(s) + dz_2$$

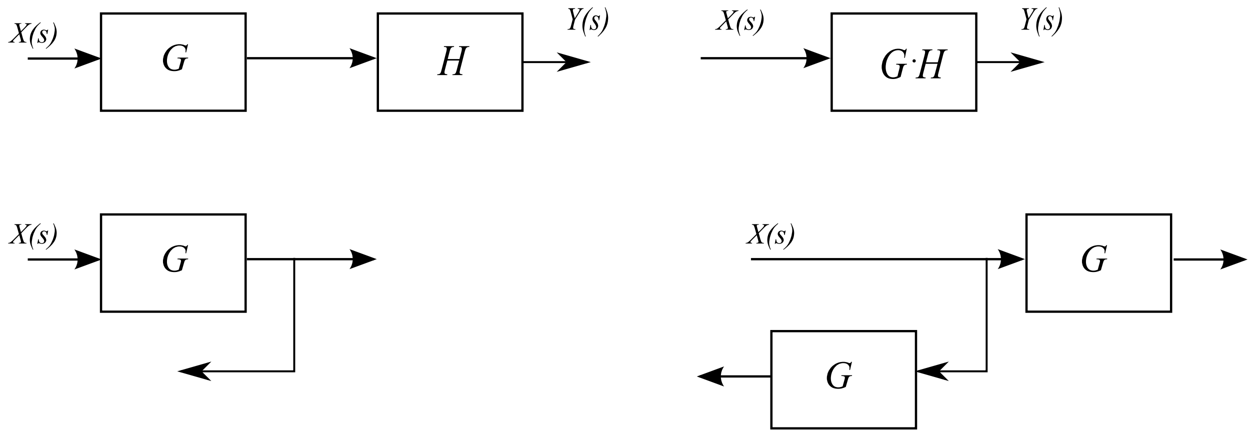
$$z_2 = bz_1(s)$$

and

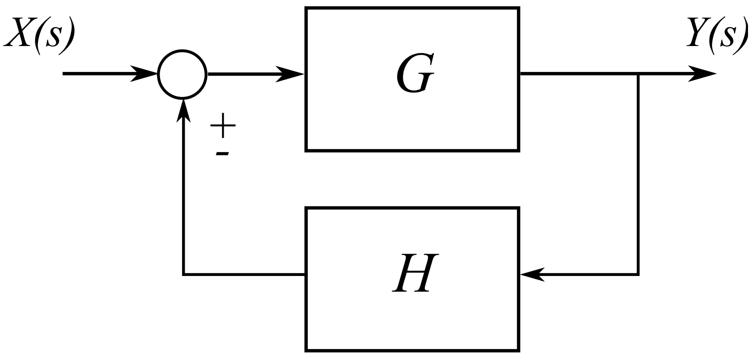
$$Y(s) = cz_2(s)$$

Block diagram reduction

We now consider the forms in which blocks are typically connected and how these forms can be reduced to single blocks.
Cascade Form & Parallel Form



Feedback block



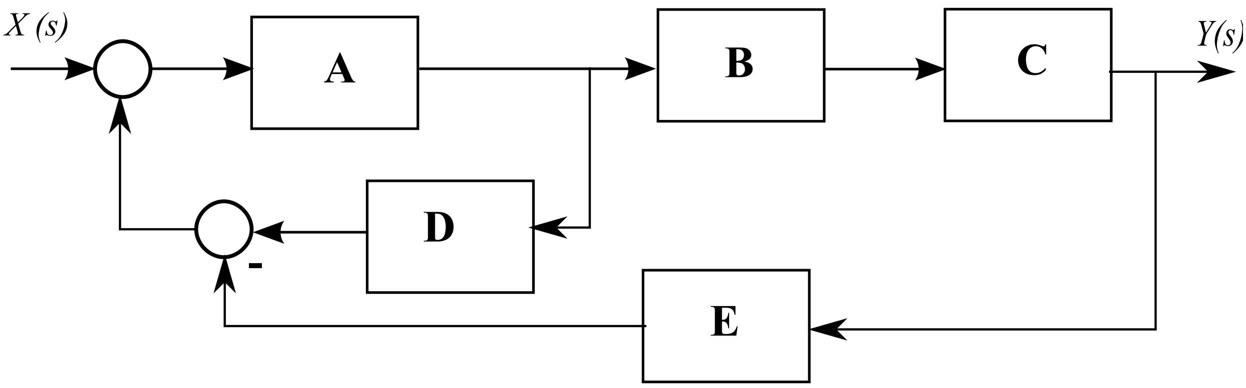
$$u = X(s) \pm H(s)Y(s)$$

$$Y(s) = G(s)u$$

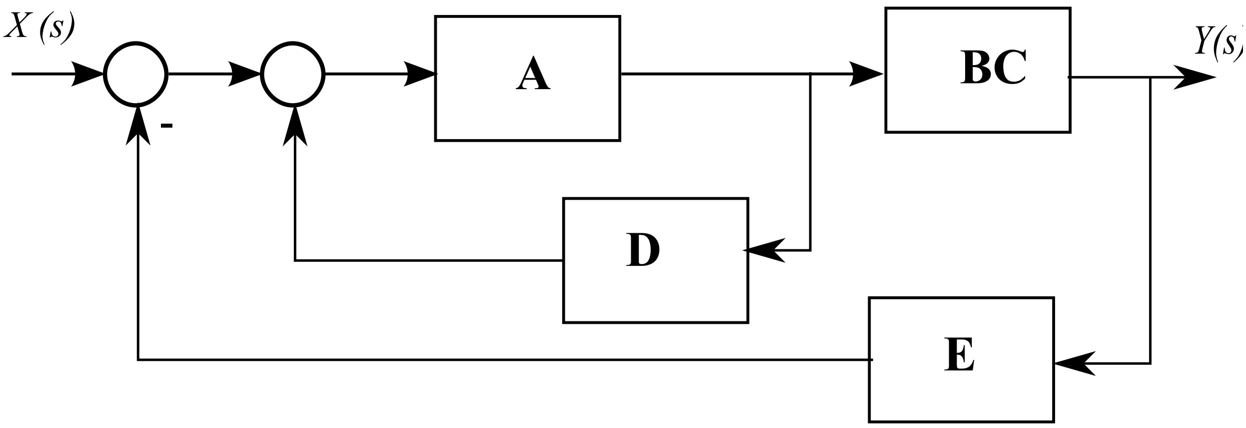
$$Y(s) = \frac{G(s)}{1 \mp G(s)H(s)} X(s)$$

Example

Reduce the following system to a single block:



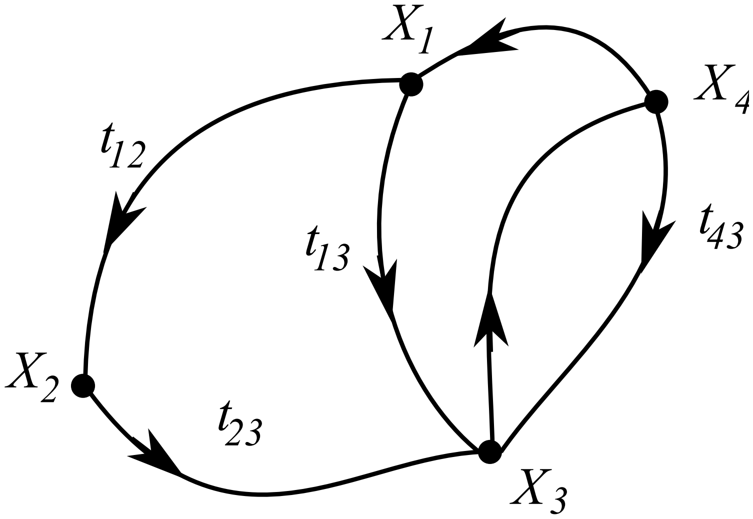
Example



$$\frac{Y(s)}{X(s)} = \frac{\frac{ABC}{1-AD}}{1+\frac{ABCE}{1-AD}}$$

When reducing subsystems in cascade form adjacent subsystems are assumed that do not load each other. Hence, a subsystem's output remains the same no matter what the output is connected to. If another subsystem connected to the output modifies that output, we say that it loads the first system.

Signal-Flow Graphs

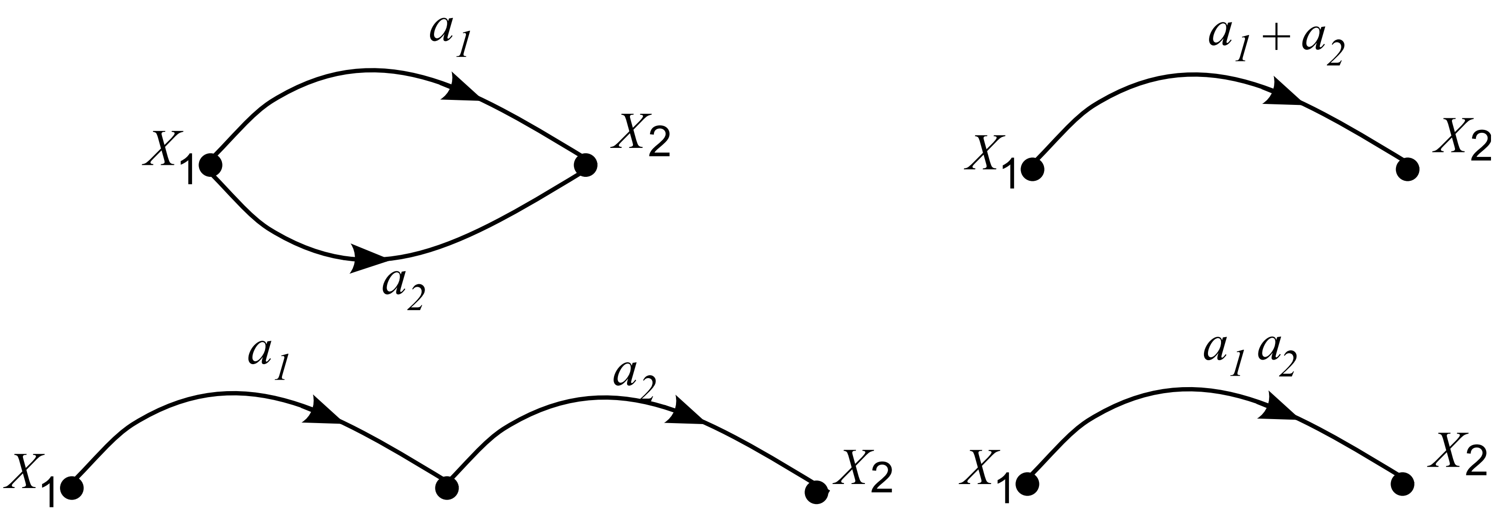


Signal-flow graphs are an alternative to block diagrams. They consist of edges which represent systems and nodes which represent signals. Multiple edges converging on a node implies summation.

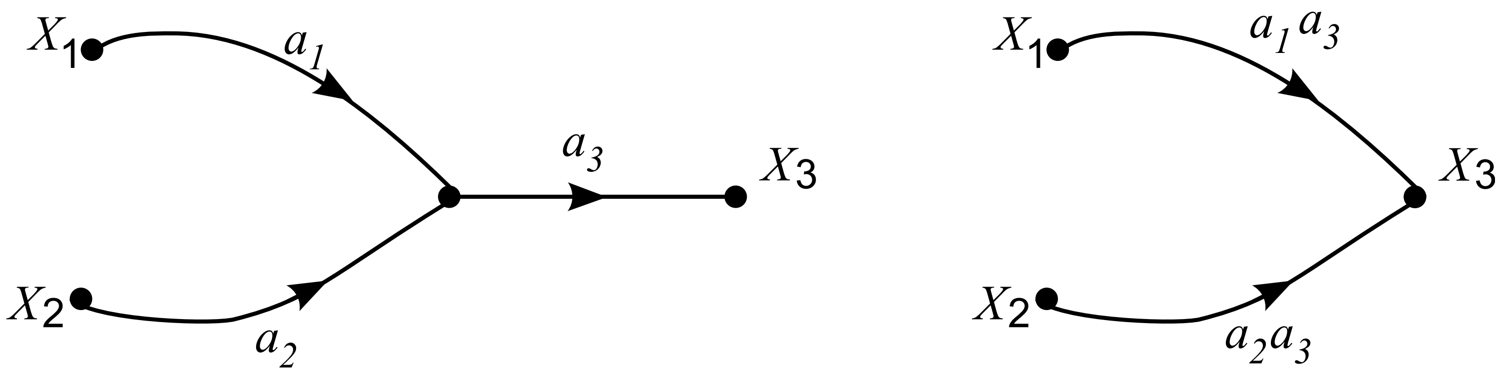
$$X_3 = t_{23}X_2 + t_{13}X_1 + t_{43}X_4$$

Signal-Flow Graphs

We can convert the cascaded and parallel forms into signal-flow graphs:

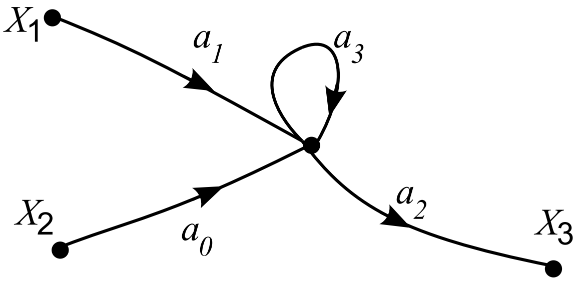


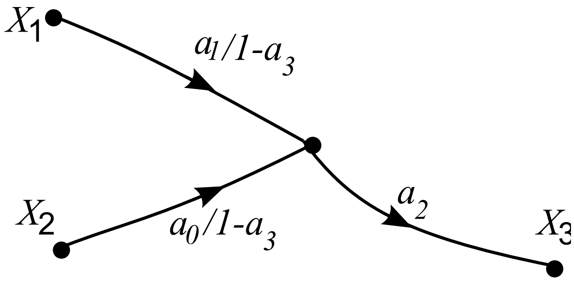
Signal-Flow Graphs



Signal-Flow Graphs

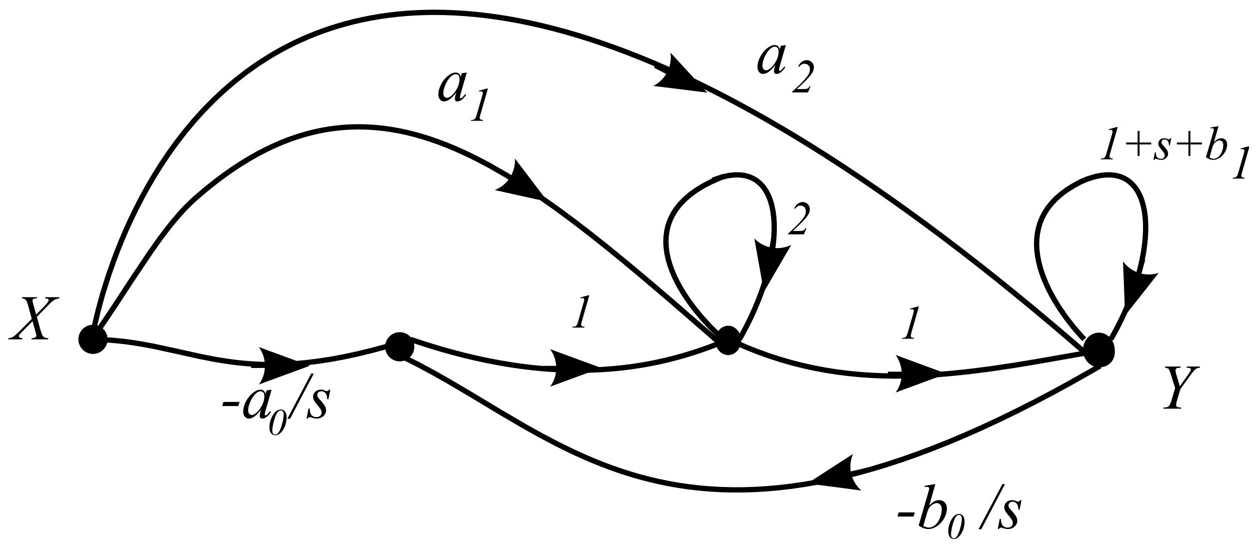
We can convert the feedback form into signal-flow graph:



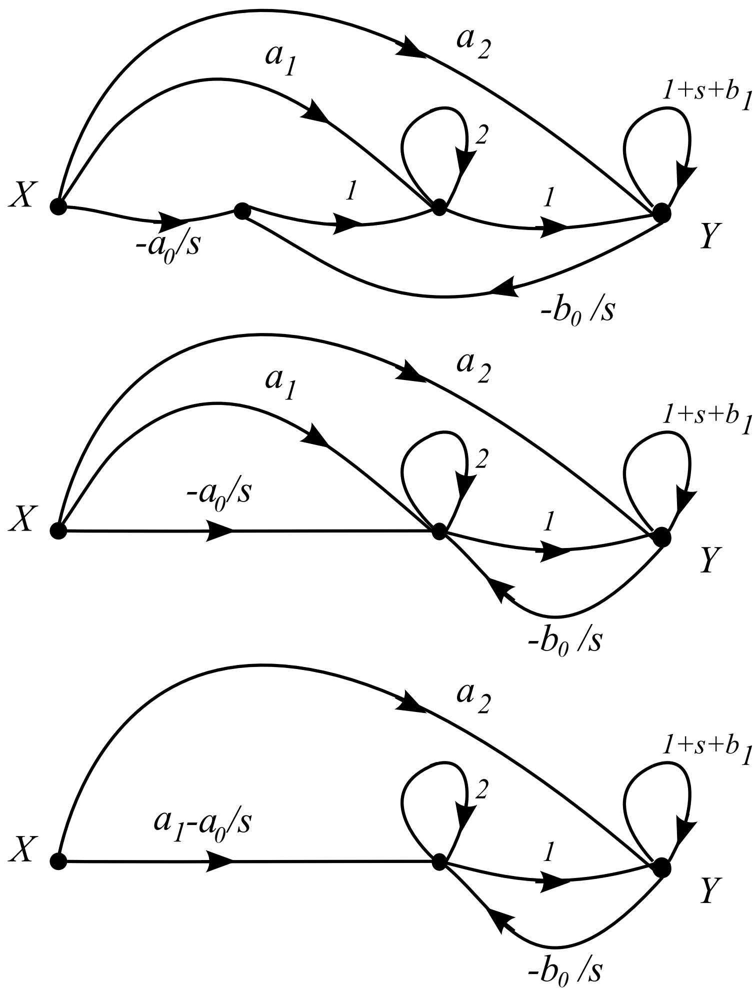


Signal-Flow Graphs

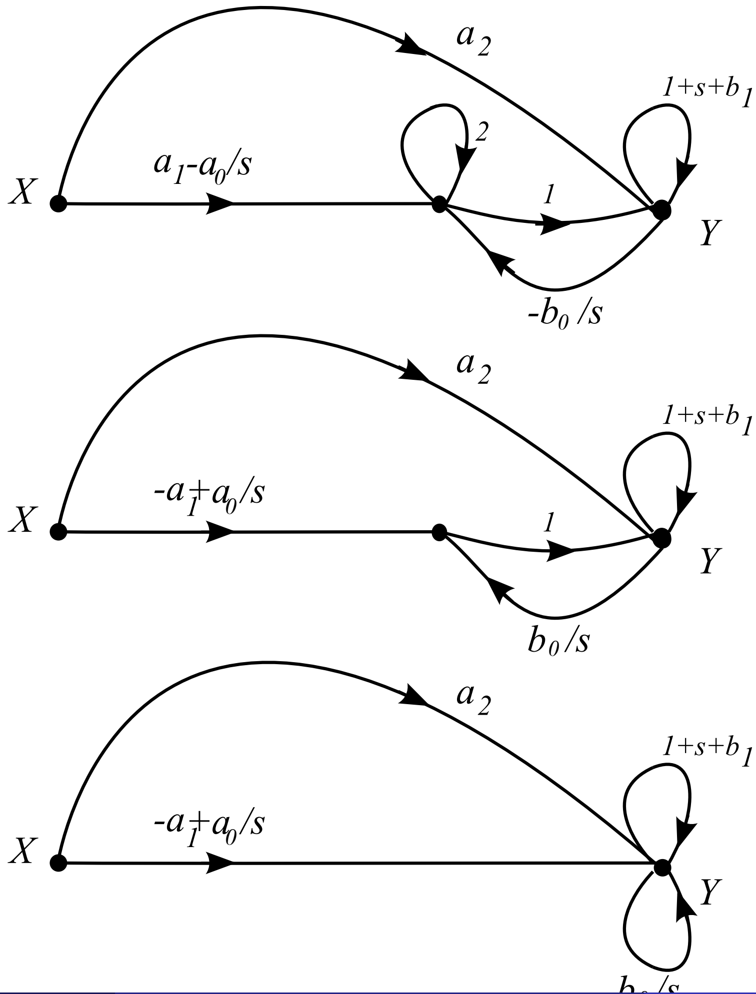
Reducing to a single edge:



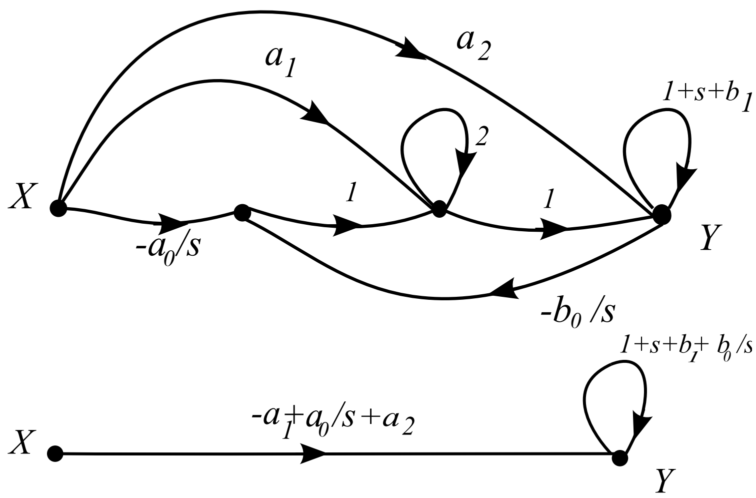
Signal-Flow Graphs



Signal-Flow Graphs



Signal-Flow Graphs



$$\frac{Y}{X} = \frac{-a_1 + a_2 + \frac{a_0}{s}}{1 - (1 + s + b_1 + \frac{b_0}{s})}$$

Mason's gain formula

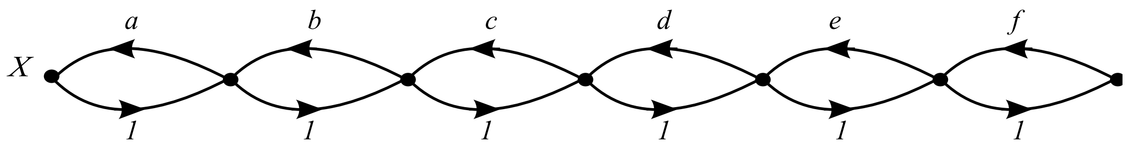
Mason's gain formula (MGF) is a method for finding the transfer function of a linear signal-flow graph (SFG) The gain formula is as follows:

$$\frac{X_N}{X_1} = \frac{\sum T_k \Delta_k}{\Delta}$$

$$\Delta = 1 - \sum P_{j1} + \sum P_{j2} - \sum P_{j2} + \cdots + (-1)^k \sum P_{jk}$$

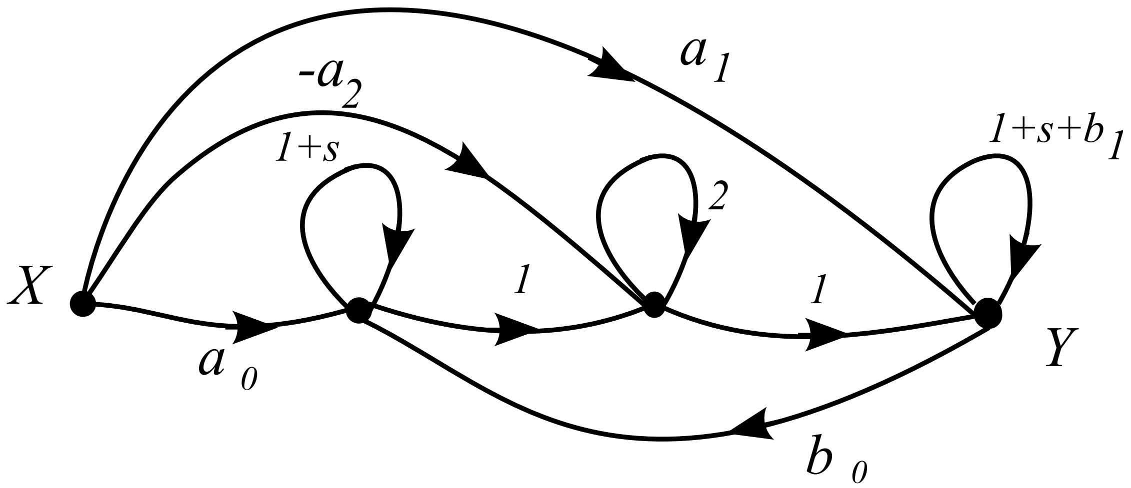
where Δ = the determinant of the graph.
X_1 = input-node variable
X_N = output-node variable
T_k = gain of the kth forward path between input and output
P_{j1} = loop gain of each closed loop in the system
P_{j2} = product of the loop gains of any two non-touching loops (no common nodes)
P_{j3} = product of the loop gains of any three pairwise nontouching loops
Δ_k = the cofactor value of Δ for the kth forward path, with the loops touching the kth forward path removed.

Mason's gain formula



$$\Delta = 1 - (a + b + c + d + e + f) + (ac + ad + ae + af + bd + be + bf + ce + cf + df) - (ace + acf + bdf + adf)$$

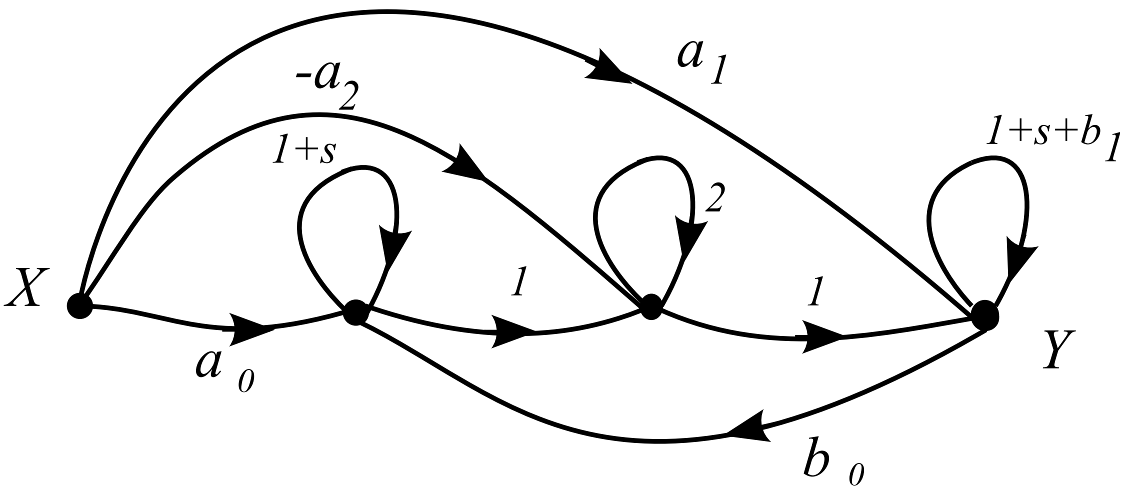
Mason's gain formula



$\frac{X_4}{X_1}$
gain of the kth forward path between input X_1 and output X_4

$$T_1 = a_0$$
$$T_2 = -a_2$$
$$T_3 = a_1$$

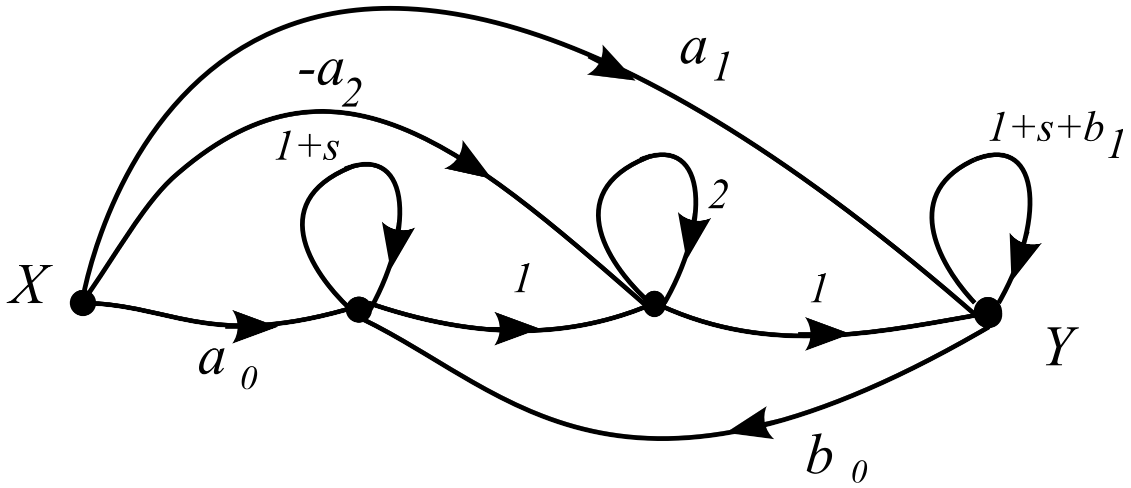
Mason's gain formula



Δ for the kth forward path

$$\Delta_1 = 1$$
$$\Delta_2 = 1 - (1 + s)$$
$$\Delta_3 = 1 - (1 + s + 2) + (1 + s)2$$

Mason's gain formula



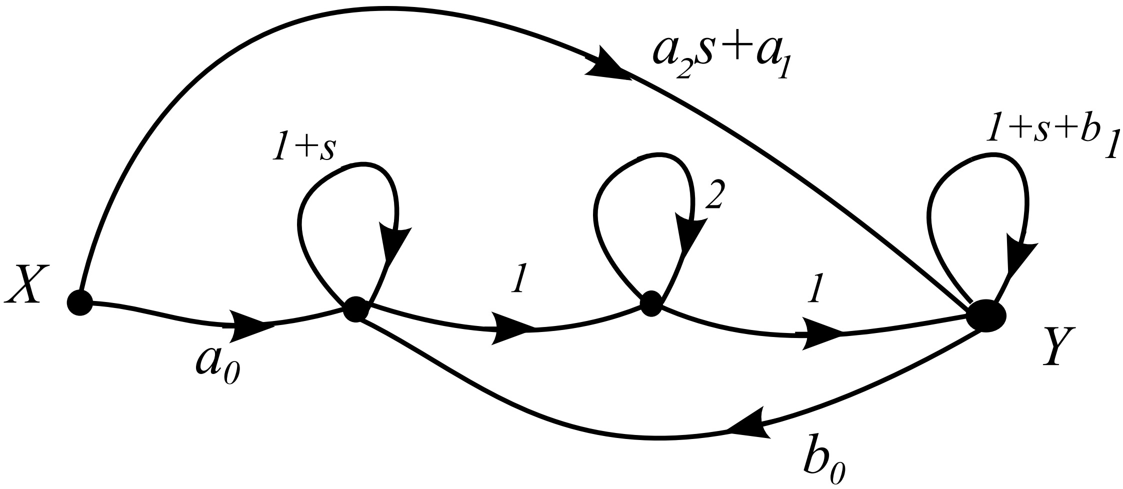
Δ for signal-flow

$$\Delta = 1 - (1 + s + 2 + 1 + s + b_1 + b_0) + 2(1 + s) + 2(1 + s + b_1) + (1 + s)(1 + s + b_1) - (1 + s)(1 + s + b_1)2.$$

Hence

$$\frac{X_4}{X_1} = \frac{\sum T_k \Delta_k}{\Delta} = \frac{a_0 + a_2 s + a_1 s}{-(s^2 + b_1 s + b_0)}$$

Mason's gain formula

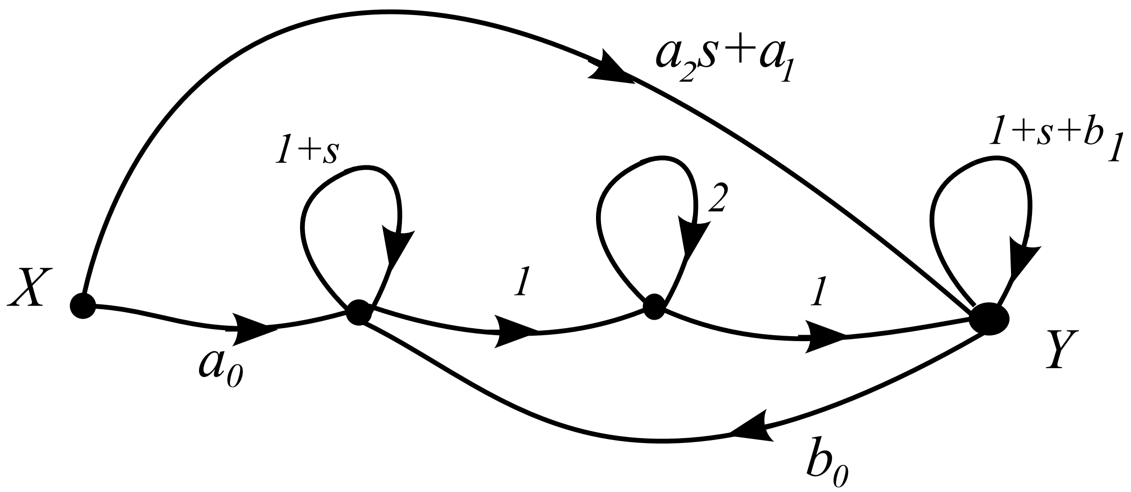


$\frac{Y}{X}$;

$$T_1 = a_0$$
$$T_2 = a_2 s + a_1$$

$$\Delta_1 = 1$$
$$\Delta_2 = 1 - (1 + s + 2) + 2(1 + s)$$

Mason's gain formula



$$\begin{aligned}\Delta =&1-(1+s+2+1+s+b_1+b_0)+2(1+s)+2(1+s+b_1)\\&+(1+s)(1+s+b_1)-2(1+s)(1+s+b_1)\\=&-s^2-b_1s-b_0\end{aligned}$$

$$\frac{Y}{X}=-\frac{a_2s^2+a_1s+a_0}{s^2+b_1s+b_0}$$