Circuit and System Analysis
EHB 232E

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Outline I

1. Laplace Transform in Circuit Analysis-Cont.
   - Impedance and Admintance concept in s-domain
   - Mesh currents method in s-domain
   - Node-voltage method in s-domain

Impedance and Admintance concept in s-domain

If no energy is stored in the inductor or capacitor, the relationship between the terminal voltage and current for each passive element takes the form:

$$V(s) = Z(s)I(s)$$

or

$$I(s) = Y(s)V(s)$$

$Z(s)$ is impedance and $Y(s)$ is admintance function.

Resistor has an impedance of RΩ, an inductor has an impedance of $Ls\Omega$, and a capacitor has an impedance of $1/sC\Omega$

KCL and KVL in s-domain

The algebraic sum of the currents/voltages at the node/loop is zero in time domain, the algebraic sum of the transformed currents/voltages is also zero.

Mesh currents method

Write mesh equations

$$BV(s) = 0$$

Laplace transform of the equ.

$$BV(s) = B_1 V_e(s) + B_2 V_k(s) = 0$$

where $V_k(s)$ and $V_e(s) = [V_R(s) \;\; V_C(s) \;\; V_L(s)]^T$ voltages of independent voltage sources and the others, respectively. The terminal equ.s

$$V_e(s) = \mathbf{Z(s)} I_e(s) + \begin{bmatrix} 0_{n_R \times n_R} & 0 & 0 \\ 0 & \frac{1}{s} I_{n_C \times n_C} & 0 \\ 0 & 0 & -\mathbf{L}_{n_L \times n_L} \end{bmatrix} \begin{bmatrix} i_R \\ v_C(0) \\ i_L(0) \end{bmatrix}$$

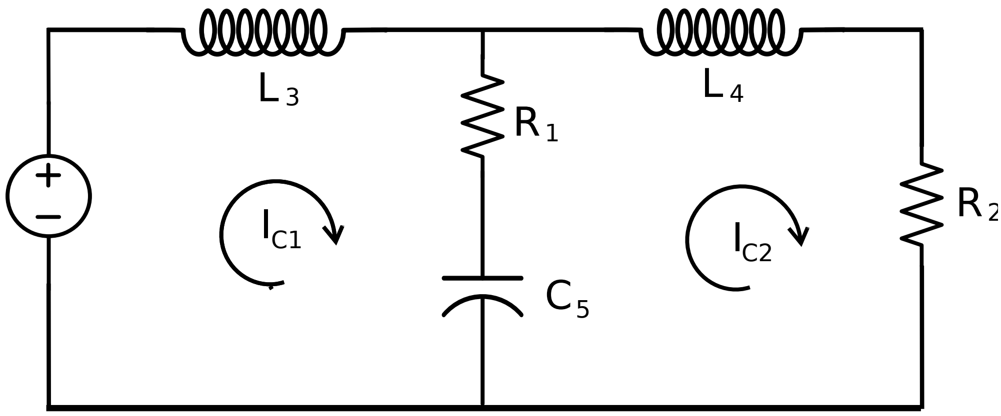
Substituting the above equ. into the mesh equ.

$$B_1 \mathbf{Z} I_e + B_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{s} I & 0 \\ 0 & 0 & -LI \end{bmatrix} \begin{bmatrix} i_R \\ v_C(0) \\ i_L(0) \end{bmatrix} + B_2 V_k = 0$$

Hence the mesh currents is obtained

$$B_1 \mathbf{Z} B_1^T I_c(s) + B_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{s} I & 0 \\ 0 & 0 & -LI \end{bmatrix} \begin{bmatrix} i_R \\ v_C(0) \\ i_L(0) \end{bmatrix} + B_2 V_k = 0$$

Example

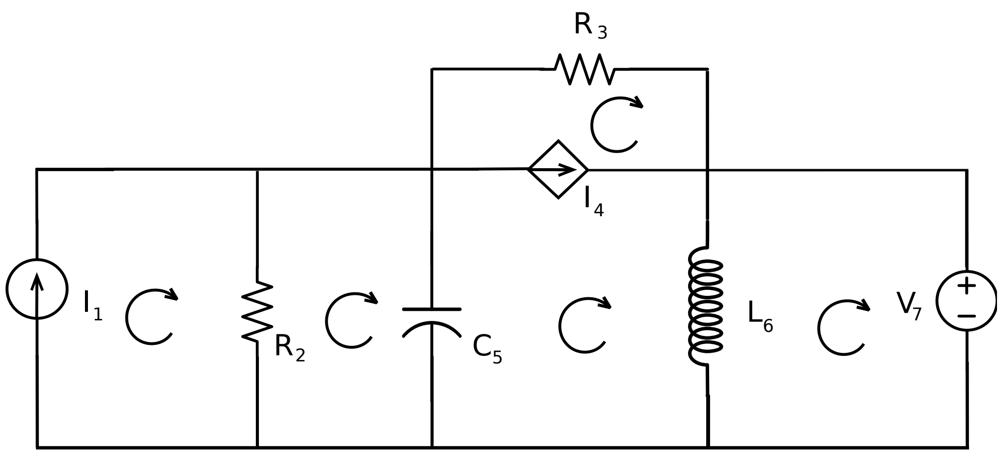


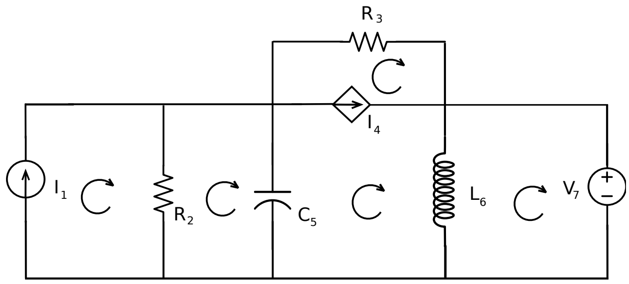
$$
\begin{array}{ll}
M1 & L_3 s I_{c1} - L_3 i_{L3}(0) + (\frac{1}{Cs} + R)(I_{c1} - I_{c2}) + \frac{1}{s} v_C(0) - V_G = 0 \\
M2 & L_4 s I_{c2} - L_4 i_{L4}(0) + R_2 I_{c2} + (\frac{1}{Cs} + R)(I_{c2} - I_{c1}) - \frac{1}{s} v_C(0) = 0
\end{array}
$$

In matrix form

$$
\begin{bmatrix} L_3 s & -\frac{1}{Cs} - R_1 \\ -\frac{1}{Cs} - R_1 & L_4 s + R_2 \end{bmatrix} \begin{bmatrix} I_{c1} \\ I_{c2} \end{bmatrix} = \begin{bmatrix} V_G \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{s} & L_3 & 0 \\ \frac{1}{s} & 0 & L_4 \end{bmatrix} \begin{bmatrix} v_C(0) \\ i_{L3}(0) \\ i_{L4}(0) \end{bmatrix}
$$

Example





$$\begin{bmatrix} R_2 & -R_2 & 0 & 0 & 0 \\ -R_2 & R_2+\frac{1}{C_5 s} & -\frac{1}{C_5 s} & 0 & 0 \\ 0 & -\frac{1}{C_5 s} & \frac{1}{C_5 s}+L_6 s & 0 & -L_6 s \\ 0 & 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & -L_6 s & L_6 s \end{bmatrix} \begin{bmatrix} I_{c1} \\ I_{c2} \\ I_{c3} \\ I_{c4} \\ I_{c5} \end{bmatrix}$$

$$= \begin{bmatrix} -V_1 \\ 0 \\ -V_4 \\ V_4 \\ -V_7 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1/s & 0 \\ 1/s & L_6 \\ 0 & 0 \\ 0 & L_6 \end{bmatrix} \begin{bmatrix} v_C(0) \\ i_L(0) \end{bmatrix}$$

$$I_1 = I_{c1}$$
$$I_4 = 2V_2$$
$$I_{c3} - I_{c4} = -2V_1$$

$$\begin{bmatrix} R_2 & -R_2 & 0 & 0 & 0 & 1 & 0 \\ -R_2 & R_2 + \frac{1}{C_5 s} & -\frac{1}{C_5 s} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_5 s} & \frac{1}{C_5 s} + L_6 s & 0 & -L_6 s & 0 & 1 \\ 0 & 0 & 0 & R_3 & 0 & 0 & -1 \\ 0 & 0 & -L_6 s & 0 & L_6 s & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} I_{c1} \\ I_{c2} \\ I_{c3} \\ I_{c4} \\ I_{c5} \\ V_1 \\ V_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -V_7 \\ I_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1/s & 0 \\ 1/s & L_6 \\ 0 & 0 \\ 0 & L_6 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_C(0) \\ i_L(0) \end{bmatrix}$$

Node-voltage Method

Write the fundamental cut-set equations for the nodes which do not correspond to node of a voltage sources:

$$AI(s) = 0$$

$$A_1 I_e(s) + A_2 I_k(s) = 0$$

where $I_k(s)$ currents of current sources and the currents of others $I_e = [I_R \;\; I_C \;\; I_L]$

$$I_e = \mathbf{Y(s)} V_e + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\mathbf{C} & 0 \\ 0 & 0 & \frac{1}{s} I \end{bmatrix} \begin{bmatrix} V_R \\ v_C(0) \\ i_L(0) \end{bmatrix}$$

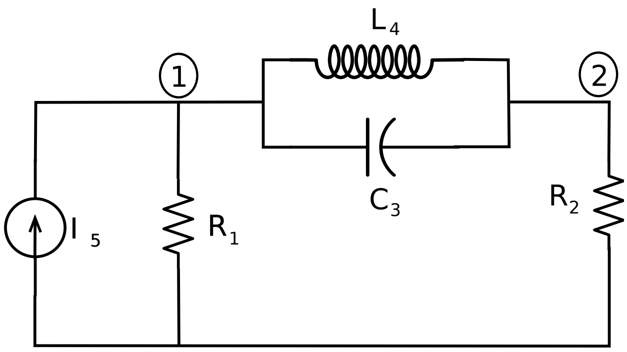
Substituting

$$A_1 \mathbf{Y(s)} V_e + A_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\mathbf{C} & 0 \\ 0 & 0 & \frac{1}{s} I \end{bmatrix} \begin{bmatrix} V_R \\ v_C(0) \\ i_L(0) \end{bmatrix} + A_2 i_k = 0$$

Using $V_e = A_1^T V_d$, we obtain the node voltage

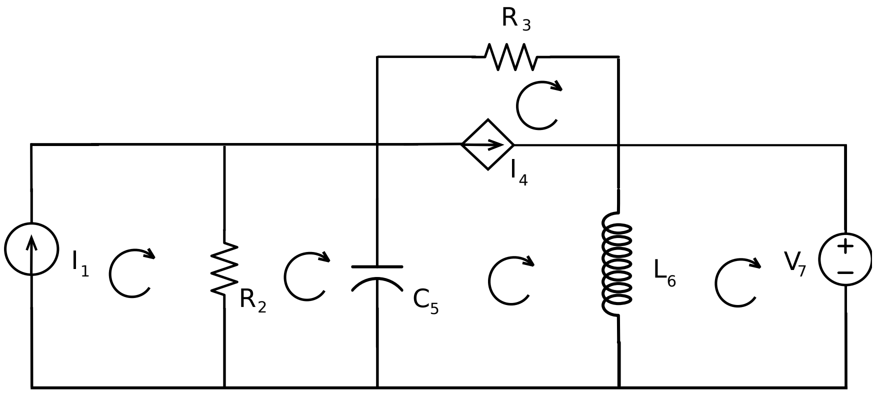
$$A_1 \mathbf{Y}(\mathbf{s}) A_1^T V_d + A_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\mathbf{C} & 0 \\ 0 & 0 & \frac{1}{s} I \end{bmatrix} \begin{bmatrix} V_R \\ v_C(0) \\ i_L(0) \end{bmatrix} + A_2 i_k = 0$$

Example



$$\begin{bmatrix} G_1 + C_3 s + \frac{1}{L_3 s} & -C_3 s - \frac{1}{L_3 s} \\ -C_3 s - \frac{1}{L_3 s} & G_2 + C_3 s + \frac{1}{L_3 s} \end{bmatrix} \begin{bmatrix} V_{d1} \\ V_{d2} \end{bmatrix} = \begin{bmatrix} I_5 \\ 0 \end{bmatrix} + \begin{bmatrix} C_3 & -\frac{1}{s} \\ -C_3 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} v_3(0) \\ i_4(0) \end{bmatrix}$$

Örnek



$$\begin{bmatrix} G_2 + G_3 + C_5 s & -G_3 \\ -G_3 & G_3 + \frac{1}{L_6 s} \end{bmatrix} \begin{bmatrix} V_{d1} \\ V_{d2} \end{bmatrix} = \begin{bmatrix} I_1 - I_4 \\ I_4 - I_7 \end{bmatrix} + \begin{bmatrix} 1/s & 0 \\ 0 & -L_6 \end{bmatrix} \begin{bmatrix} v_5(0) \\ i_6(0) \end{bmatrix}$$

$$V_7 = V_{d1}$$
$$I_4 = 2V_2 = 2V_{d1}$$

$$\begin{bmatrix} G_2 + 2 + G_3 + C_5 s & -G_3 & 0 \\ -G_3 - 2 & G_3 + \frac{1}{L_6 s} & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{d1} \\ V_{d2} \\ I_7 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ V_7 \end{bmatrix} + \begin{bmatrix} 1/s & 0 \\ 0 & -L_6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_5(0) \\ i_6(0) \end{bmatrix}$$