Basic of Electrical Circuits
EHB 211E

Prof. Dr. Müştak E. Yalçın

Istanbul Technical University
Faculty of Electrical and Electronic Engineering

mustak.yalcin@itu.edu.tr

Lecture 6.b.

Contents I

Elementary Function

Unit step function $u(t)$

It is defined by

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

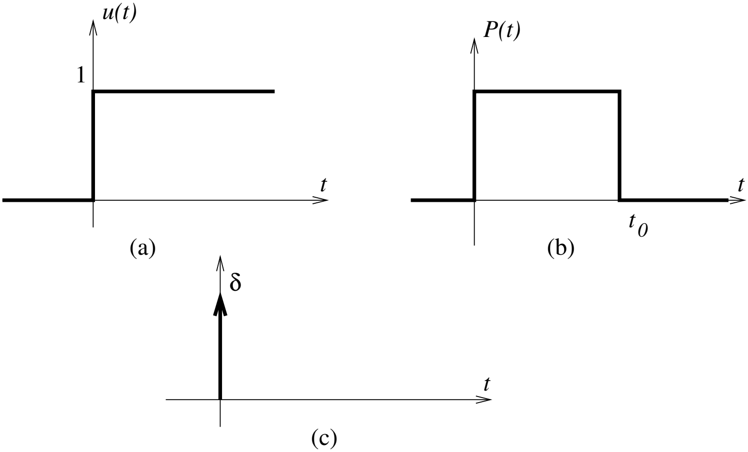
Rectangular pulse

$$P(t) = \begin{cases} 1 & 0 \leq t \leq t_0 \\ 0 & t < 0, t > t_0 \end{cases}$$

A unit impulse (or delta function)

$$\delta(t) = \lim_{\Delta \to \infty} P_\Delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

Elementary Function



Elementary Function

Exponential Function

$$x(t) = e^{\alpha t u(t)}$$

Periodic Function

A function f is said to be periodic with period T if we have $f(t) = f(t + T)$ for all values of t.

- T is called period. The SI unit for period is the second.
- f is Frequency

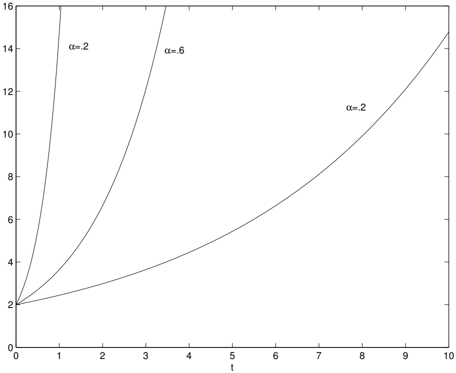
  $$f = \frac{1}{T}.$$

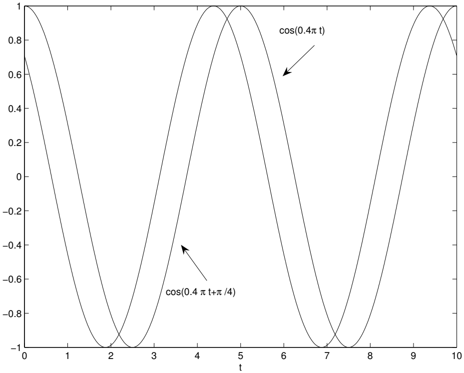
  Its unit is hertz (Hz).
- w is angular frequency.

  $$w = 2\pi/T$$

  .

Elementary Function





Instantaneous electrical power

P(t) is the instantaneous power, measured in watts (joules per second). The power delivered to the element from the outside to the n-terminal element at time t is

$$P(t) = v^T(t)i(t) = \sum_{i=1}^{n-1} v_i(t)i_i(t).$$

The average value of the power

The average value of the power over certain period of time T is given by

$$P_{\mathrm{ort}} = \frac{1}{T}\int_{t_0}^{t_0+T} P(t)dt$$

Electrical power and Energy

Root-mean-square (RMS)

The RMS value of any variable X(t) is generally defined by

$$X_{\mathrm{rms}} = \frac{1}{T}\left[\int_{t_0}^{t_0+T} X^2(t)dt\right]^{1/2}$$

The RMS value is the effective value of a varying voltage or current. AC voltmeters and ammeters show the RMS value of the voltage or current.

Apparent power

The product of RMS voltage and current $V_{\mathrm{rms}} \times I_{\mathrm{rms}}$ is called apparent power (or volt-amps) and measured in volt-amps.

Electrical power and Energy

Electrical Energy

Electrical Energy during $[t_0,\ t]$ is

$$E(t, t_0) = \int_{t_0}^{t} P(\tau) d\tau$$

şeklinde tanımlanır. Its unit is Joule.

Example

The power delivered to the resistor at time t for $V(t) = V_0\cos(wt)$ is

$$P = \frac{V^2}{R}\cos^2(wt)$$

and the average value of the power

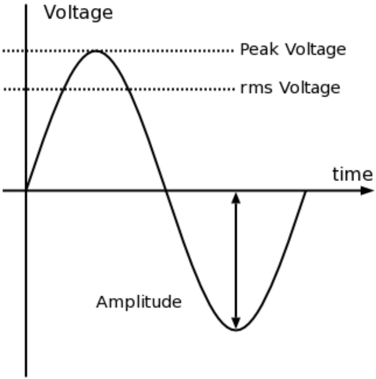
$$P_{\mathrm{ort}} = \frac{V^2}{2R}$$

Lets calculate rms value of $V = V_0\cos(wt)$ which is

$$V_{\mathrm{rms}} = V_{\mathrm{eff}} = \frac{V_o}{\sqrt{2}} = 0.7V_o$$

The Turkey mains supply is 230V AC, this means 230V RMS so the peak voltage of the mains is about 320V!

$$V(t) = 320cos(2\pi 50t + \phi)$$



Active and Passive Element

Active element: is capable of generating energy. Passive element: absorbs (dissipates) energy.

Passive element

if

$$\sup_{t,v,i} \left\{ -\int_0^t P(\tau) d\tau \right\} < \infty$$

for all t, n-terminal element is passive. An element is said to be active iff it is not passive.
(sup the supremum is referred to as the least upper bound).

If the resistor is linear having resistance R, we have

$$\sup_{t,v,i}\left\{-\int_0^t P(\tau)d\tau\right\} = \sup_{t,v,i}\left\{-\int_0^t Ri^2(\tau))d\tau\right\}$$

Inside if the integral term will be

$$-\int_0^t Ri^2(\tau)d\tau = -R\int_0^t i^2(\tau)d\tau$$

- if $R > 0$, resistor will be passive.
- if $R < 0$, resistor will be active.

A two-terminal resistor is said to be passive iff its $v - i$ characteristic lies in the closed first and third quadrants of the $v - i$ plane. A two-terminal resistor is said to be active iff it is not passive.