Basic of Electrical Circuits
EHB 211E

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Lecture 6.

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1. Circuit Elements
   - Two-terminal Elements
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   - Multi-terminal Circuit Elements

Two-terminal Elements

Two-terminal elements play a major role in electric circuits!

Two-terminal circuit elements are defined by the between basic variables which are current ($i(t)$), voltage ($v(t)$), charge ($q(t)$) and flux ($\phi(t)$). The units of them are Amperes, Volts, Coulomb and Weber, respectively.

Two pairs of the basic variables

$$i(t) = \frac{dq}{dt},$$

and

$$v(t) = \frac{d\phi}{dt},$$

are the definition.

Two-terminal Elements

Controlled circuit element (Dependent element)

If the relation between the terminal variable is given by the equation $x = h(y,t)$, this two-terminal element is called as a y controlled element e.g. voltage controlled voltage sources,...

Time-invariant two-terminal element

A two-terminal element whose variables x and y fall on some fixed curve in the $x - y$ plane at any time t is called a time-invariant circuit element e.g. Linear resistor $Vv = Ri$.

$x - y$ characteristic

The curve on the $x - y$ plane at any time t is called $x - y$ characteristic e.g. $v - i$ characteristic of linear resistor.

Two-terminal Elements

Bilateral property

A element has a $x - y$ characteristics which is not symmetric with respect to the origin of the $x - y$ plane.

Linear element

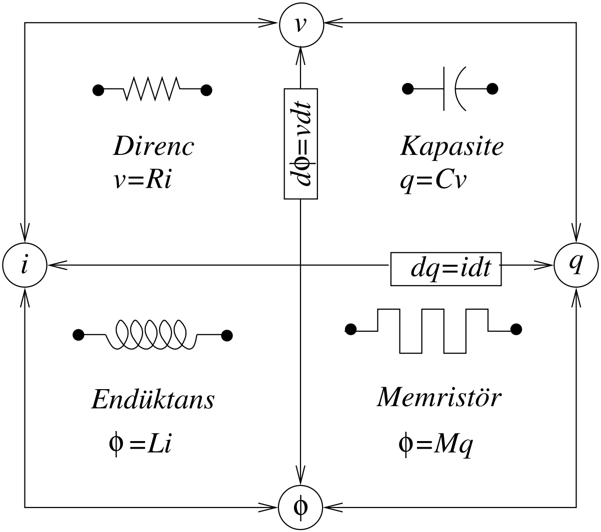
A linear element is an element with a linear relationship between its variables x and y.

Linear

$f(x)$ is a function which satisfies the following two properties:

- Additivity (superposition): $f(x + y) = f(x) + f(y)$.
- Homogeneity : $f(\alpha x) = \alpha f(x)$ for all α.

Basic circuit element diagram



Resistor

A two-terminal element will be called a resistor if its voltage v and current i satisfy the following relation:

$$\mathrm{R} = \{(v, i) | f(v, i) = 0\}$$

This relation is called the $v - i$ characteristic of the resistor and can be plotted graphically in the $v - i$ plane. The equation $f(v, i) = 0$ represents a curve in the $v - i$ plane and specifies completely the two-terminal resistor.

The linear resistor is a special case of a resistor and satisfies Ohm's law which is

$$f(v, i) = v - Ri \quad or \quad f(v, i) = Gv - i$$

It means that the voltage across resistor is proportional to the current flowing through it.

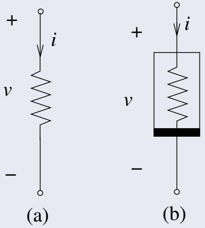
Linear and Nonlinear Resistor

Ohm's law states

$$v = Ri \quad or \quad i = Gv$$

where the constant R is the resistance of the linear resistor measured in the unit of ohms (Ω), and G is the conductance measured in the unit of Siemens (S). A resistor which is not linear is called nonlinear.

$$G = \frac{1}{R}$$

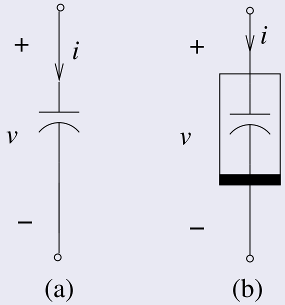


Capacitor

A two-terminal element whose charge $q(t)$ and voltage $v(t)$ fall on some fixed curve in the $q - v$ plane at any time t is called a time-invariant capacitor. Linear time- invariant capacitor is represented by the equations

$$q = Cv \text{ or } i = C\frac{dv}{dt}$$

Values of capacitors are specified in ranges of farads (F).



Time-varying and Nonlinear Capacitor

If the $q - v$ characteristic changes with time, the capacitor is said to be time-varying. Then the mathematical model becomes

$$q = C(t)v$$

and

$$i = \frac{dC}{dt}v + C(t)\frac{dv}{dt}$$

The most general case, a time-varying nonlinear capacitor is defined by a family of time-dependent and nonlinear $q - v$ characteristics

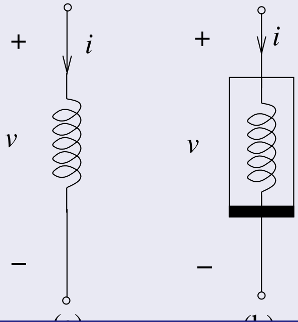
$$f(q, v, t) = 0$$

Inductor

A two-terminal element whose flux $\phi(t)$ and current $i(t)$ fall on some fixed curve in the $\phi - i$ plane at any time t is called a time-invariant inductor. The mathematical model of LTI inductor is

$$\phi = Li \text{ veya } \mathrm{v} = \mathrm{L}\frac{\mathrm{di}}{\mathrm{dt}}$$

Values of inductors are specified in ranges of Henry (H).



Inductor

If the $\phi - i$ characteristic changes with time, the inductor is said to be time-varying. Then the mathematical model becomes

$$v = L(t)i$$

and

$$v = \frac{dL}{dt}i + L(t)\frac{di}{dt}$$

The most general case, a time-varying nonlinear capacitor is defined by a family of time-dependent and nonlinear $\phi - v$ characteristics

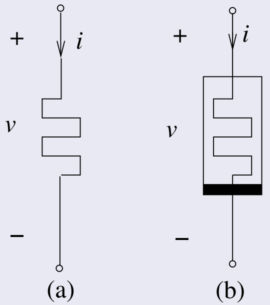
$$f(\phi, v, t) = 0$$

.

A two-terminal element whose flux $\phi(t)$ and current $q(t)$ fall on some fixed curve in the $\phi - q$ plane at any time t is called a time-invariant memristor. The mathematical model of LTI memristor is

$$\phi = Mq.$$

Using the definitions, one can obtain $v = Mi$ which is a ....



Memristor

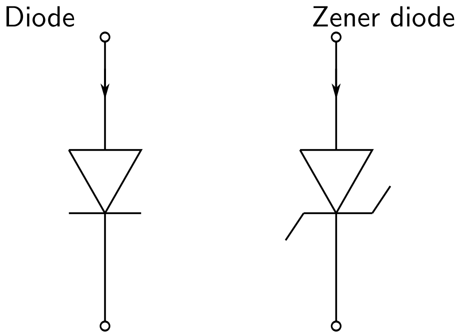
Nonlinear memristor is defined by a family of nonlinear $\phi - v$ characteristics

$$\phi = f(q)$$

It can be written in the form

$$v = \frac{df(q)}{dt} i = \frac{df(q(t_0 + \int_{t_0}^{t} i(\tau)d\tau))}{dt} i$$

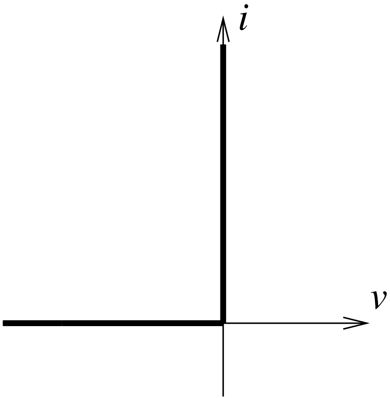
Nonlinear Resistors



Nonlinear Resistors: Diode

$$i = I_0 e^{(v/v_T - 1)}$$

where $v_T = 0.026V$ ve I_0 μA.



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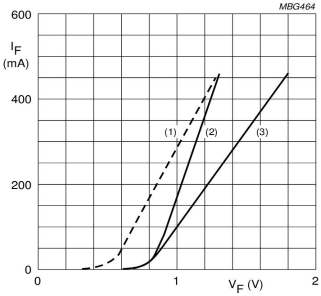
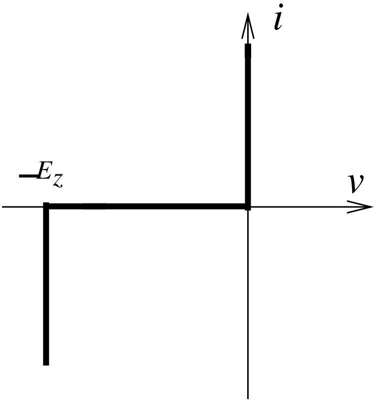


(1) T_j = 175 °C; typical values.
(2) T_j = 25 °C; typical values.
(3) T_j = 25 °C; maximum values.

(b)



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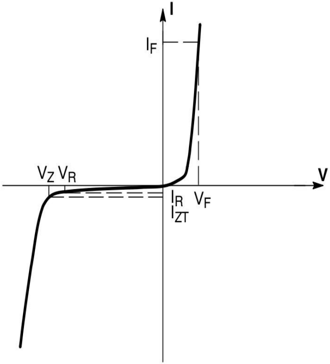

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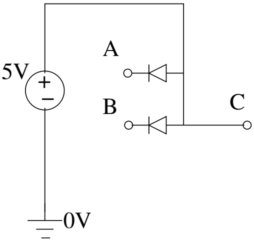

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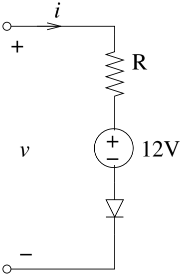

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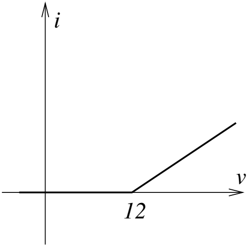

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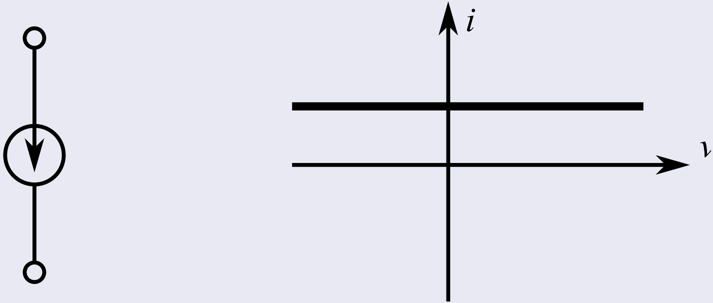
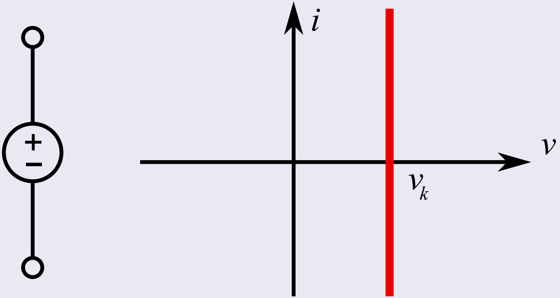
Independent Sources

Independent current source: An independent currrent source is defined as a two-terminal circuit element whose current is a specified waveform $i_s(\cdot)$ irrespective of the voltage across it.



Independent Sources

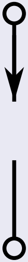
Independent voltage source: A two-terminal element is called an independent voltage source if the voltage across it is a given waveform $v_s(\cdot)$ irrespective of the current flowing through it.

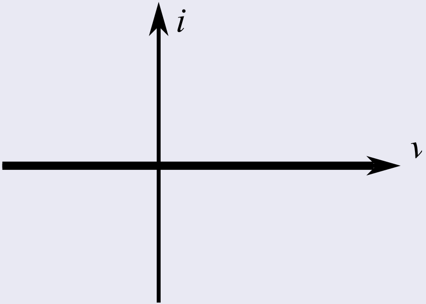


Nonlinear resistor

Open circuit

$$i(t) = 0$$

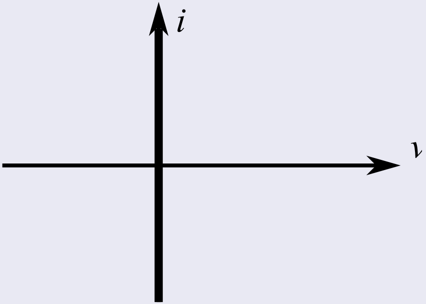




Nonlinear resistor

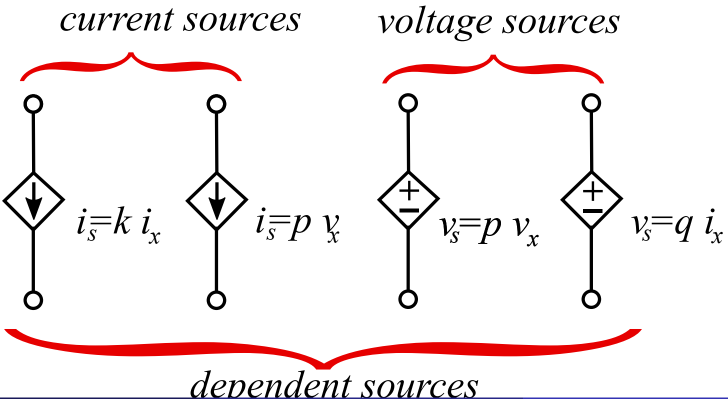
Short circuit

$$v(t) = 0$$



Linear Controlled Sources

Dependent Source: A controlled sources is a resistive two-port element consisting of two branches: a primary branch which is either an open circuit or a short circuit and a secondary branch which is either a voltage source or a current source. Diamond-shaped symbol to denote controlled sources.



Linear Controlled Sources

Current-Controlled Current Source (CCCS)

CCCS is characterized by two linear equations

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

where α is called the current transfer ratio.

Current-Controlled Voltage Source (CCVS)

CCVS is characterized by two linear equations

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

where r is called the transresistance.

Linear Controlled Sources

Voltage-Controlled Current Source (VCCS)

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

where g is called the transconductance.

Voltage-Controlled Voltage Source (VCVS)

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ k & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

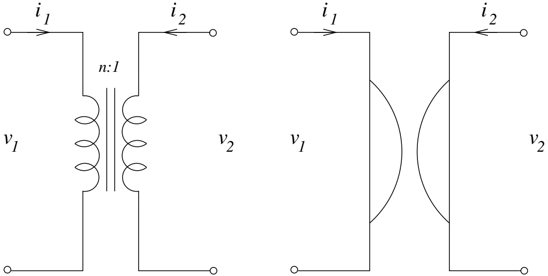
where k is called the voltage transfer ratio.

Two-port Elements

The ideal transformer is an ideal two-port resistive circuit element which is characterized by the following two equations:

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

where n is a real number called the turns ratio.



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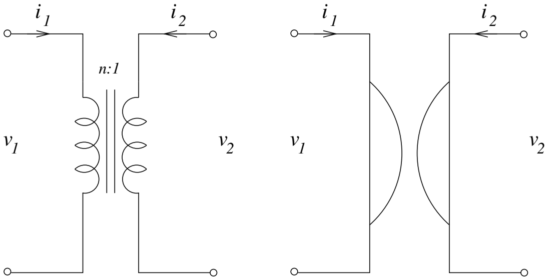
Jirator

A gyrator is an ideal two-port element defined by the following equations:

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

where the constant G is called the gyration conductance.



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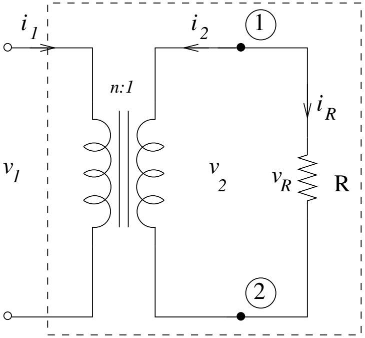
Examples



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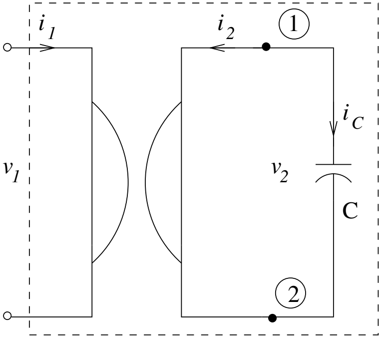

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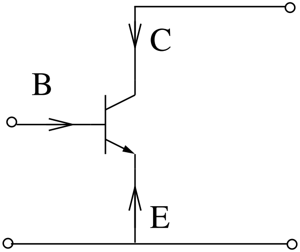
npn- Bipolar Transistor

The most commonly used three-terminal device is the transistor.
Low-frequency characterization is given by the one- dimensional diffusion model which yields the Ebers-Moll equations:

$$i_E = -I_{es}(e^{-v_{eb}/v_T - 1}) + \alpha_R I_{CS}(e^{-v_{cb}/v_T - 1})$$

$$i_C = \alpha_F I_{es}(e^{-v_{eb}/v_T - 1}) + I_{CS}(e^{-v_{cb}/v_T - 1}).$$



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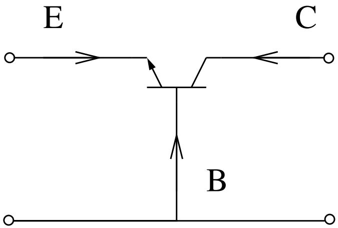

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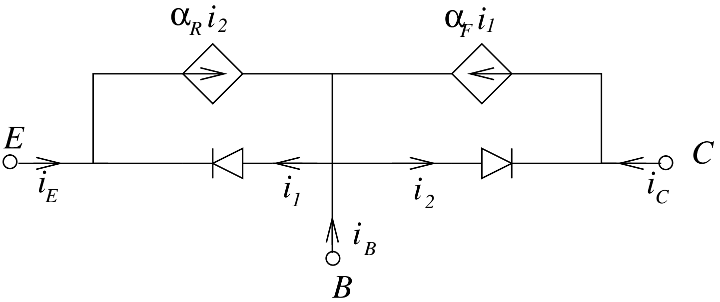
Modelling of the common emitter transistor configuration using diode and dependent sources. One can obtain

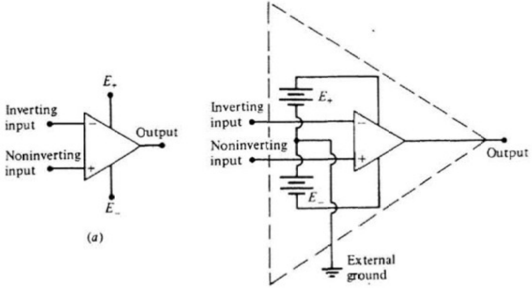
$$i_1 = I_{es}(e^{-v_{eb}/v_T - 1})$$

and

$$i_2 = I_{CS}(e^{-v_{cb}/v_T - 1})$$



Operational amplifiers (op amps) are multi-terminal devices. Terminals are labeled inverting input, non-inverting input, output, E_+ , E_- and external ground. A "biased" op amp can be considered as a 4-terminal device.



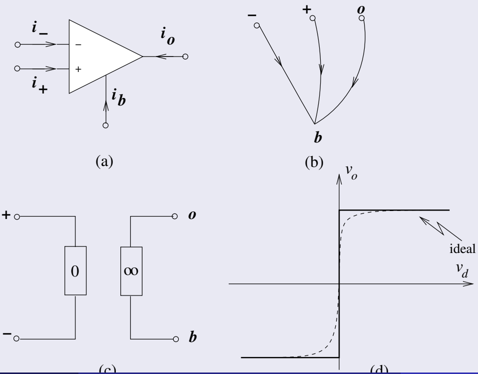
The variable $v_d = v_+ - v_-$ is called the differential input voltage and will play an important role in op-amp circuit analysis.

Operational amplifier

The op-amp terminal currents and voltages obey the following approximate relation- ships:

$$i_+ \approx 0,\ i_- \approx 0 \text{ and } v_o = G(v_+ - v_-)$$

where G called the open-loop voltage gain. v_o saturates at $v_o = E_{\text{sat}}$ where E_{sat} is less than the power supply voltage.



In a small interval $-\epsilon < v_d < \epsilon$, we have

$$G(v_+ - v_-) = v_o$$

which is called linear region.
In ideal op amp

$$i_+ = i_- = \epsilon = 0$$

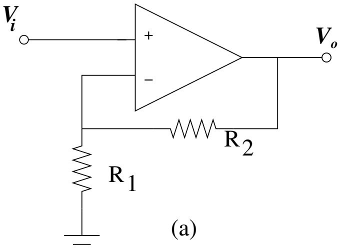
and

$$G = \infty$$

Using the gain formula, we will have

$$\begin{aligned} (v_+ - v_-) &= \tfrac{v_o}{G} \\ (v_+ - v_-) &\approx 0 \end{aligned}$$

Examples



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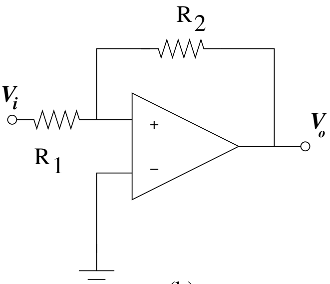

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Examples

Applying KCL at the node where the inverting input is connected and using the definition of op amp, we obtain

$$\begin{aligned} -G_1 V_- + G_2(V_o - V_-) &= 0 \\ G(V_i + V_-) &= V_o \end{aligned}$$

Substituting the first eqn. into the second eqn.

$$V_o = \frac{G}{1+KG} V_i$$

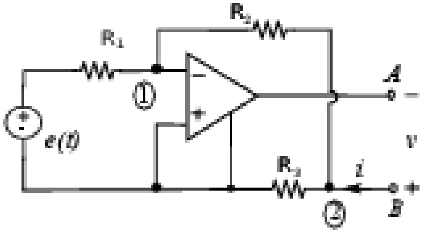
where $K = \frac{R_1}{R_1+R_2}$. $\frac{G}{1+KG}$ is the gain. G is too big therefore

$$V_o = \frac{1}{K} V_i$$

!

The same result can be obtained using the relation $V_+ = V_-$.

Examples



What is the $v - i$ characteristic ?

$$\frac{e}{R_1} = -\frac{V_{d2}}{R_2}$$

$$i_3 = -\frac{V_{d2}}{R_3} = -\frac{R_2 e}{R_3 R_1}$$

$$i = i_2 + i_3 = -\frac{e}{R_1} + \frac{R_2 e}{R_3 R_1} = -\frac{e}{R_1}(1 + \frac{R_2}{R_3})$$