

## Relation Between $B$ and $Q$

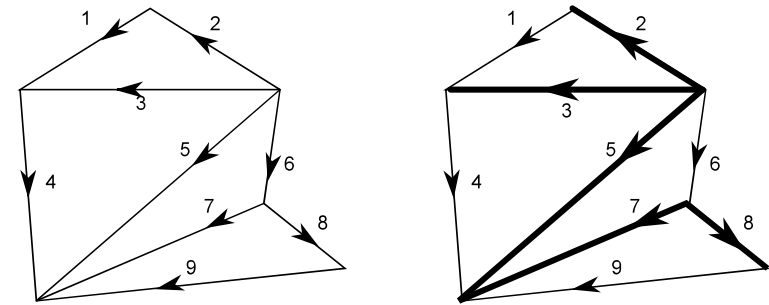


$$F = -E^T$$

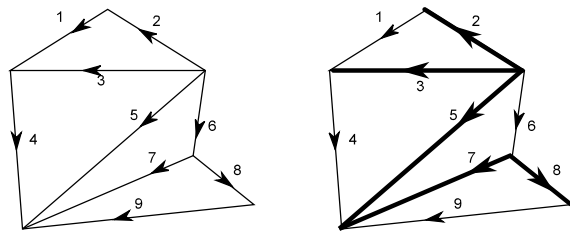
**Proof :** Since they are the tree-branch voltages of the tree, the branch voltages are given by

$$\begin{aligned} V &= Q^T V_n \\ BV &= BQ^T V_n = 0 \\ BQ^T V_n &= 0 \\ BQ^T &= 0 \\ IE^T + FI &= 0 \\ E^T + F &= 0 \\ E^T &= -F \end{aligned}$$

## Example



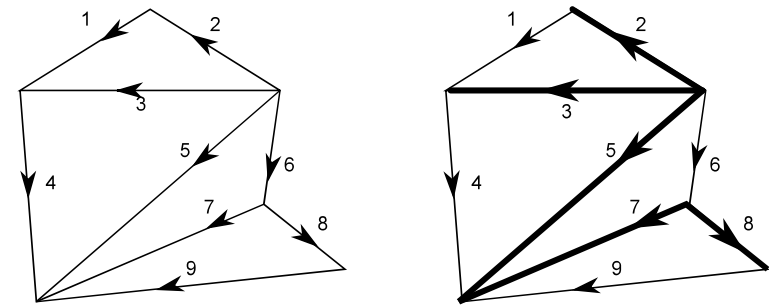
Fundamental cut sets of the tree  $G_T = \{2, 3, 5, 7, 8\}$  are  $G_{C2} = \{2, 1\}$ ,  $G_{C3} = \{3, 1, 4\}$ ,  $G_{C5} = \{5, 4, 6\}$ ,  $G_{C7} = \{7, 6, 9\}$ ,  $G_{C8} = \{8, 9\}$ .



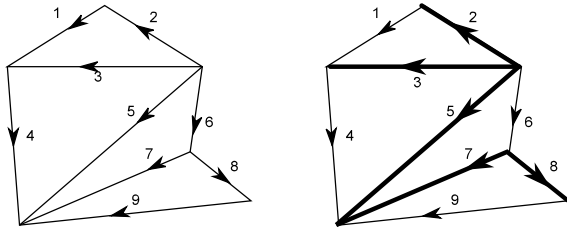
KCL equations based on Fundamental cut sets

$$\left[ \begin{array}{cccc|cccc} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{c} i_1 \\ i_4 \\ i_6 \\ i_9 \\ \hline i_2 \\ i_3 \\ i_5 \\ i_7 \\ i_8 \end{array} = 0$$

## Example

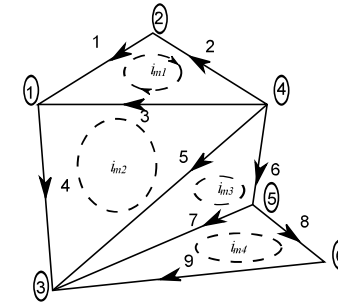


Fundamental Loop sets of the tree  $G_T = \{2, 3, 5, 7, 8\}$  are  $G_{L1} = \{1, 2, 3\}$ ,  $G_{L4} = \{4, 3, 4\}$ ,  $G_{L6} = \{6, 5, 6\}$ ,  $G_{L9} = \{9, 7, 8\}$ .



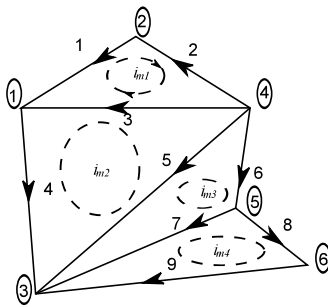
KVL equations based on the Fundamental loops

$$\left[ \begin{array}{cccc|cccccc} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \end{array} \right] \begin{array}{l} V_1 \\ V_4 \\ V_6 \\ V_9 \\ \text{---} \\ V_2 \\ V_3 \\ V_5 \\ V_7 \\ V_8 \end{array} = 0$$



KVL equations for the nodes

$$\begin{array}{l} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{array} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{array}{l} V_{n1} \\ V_{n2} \\ V_{n3} \\ V_{n4} \\ V_{n5} \\ V_{n6} \end{array}$$



Mesh equations

KCL equations for the nodes

$$\left[ \begin{array}{cccccccccc} -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \end{array} \right] \begin{array}{l} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \end{array} = 0$$

$$\begin{array}{l} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \end{array} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} i_{m1} \\ i_{m2} \\ i_{m3} \\ i_{m4} \end{array}$$