Basic of Electrical Circuits
EHB 211E

Prof. Dr. Müştak E. Yalçın

Istanbul Technical University
Faculty of Electrical and Electronic Engineering

mustak.yalcin@itu.edu.tr

Lecture 5.

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Fundamental Loop Analysis

Fundamental Loop

Every link of G_T and the unique tree path between its nodes constitute a unique loop. This loop is called the Fundamental loop associated with the link.

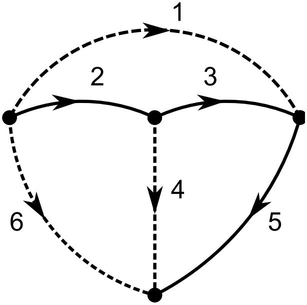
Consider a link l which connects nodes 1 and 2. There is a unique tree path between 1 and 2. This path, together with the link l, constitutes a loop. There cannot be any other loop.

Fundamental Loop Equation

The linear algebraic equations obtained by applying KVL to each Fundamental loop constitute a set of $n_e - n_n - 1$ linearly independent equations.

Reference direction for the loop which agrees with that of the link defining the loop.

Fundamental Loop Analysis



The links $G_L = \{1, 4, 6\}$ for the chosen tree $G_T = \{2, 3, 5\}$. The Fundamental loop sets are $G_{L1} = \{1, 2, 3\}$ $G_{L4} = \{4, 5, 3\}$ $G_{L6} = \{6, 2, 3, 5\}$.

If we apply KVL to the Fundamental loops, we obtain:

$$
\begin{aligned}
V_1 - V_3 - V_2 &= 0 \\
V_4 - V_5 - V_3 &= 0 \\
V \quad V \quad V \quad V &= 0
\end{aligned}
$$

Fundamental Loop Analysis

$$V = \begin{bmatrix} V_l \\ -- \\ V_b \end{bmatrix} = \begin{bmatrix} V_1 \\ V_4 \\ V_6 \\ -- \\ V_2 \\ V_3 \\ V_5 \end{bmatrix}$$

where V_l is link voltage vector, V_b is tree branch voltage vector. In matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & | & -1 & -1 & 0 \\ 0 & 1 & 0 & | & 0 & -1 & -1 \\ 0 & 0 & 1 & | & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_4 \\ V_6 \\ -- \\ V_2 \\ V_3 \\ V \end{bmatrix} = 0$$

Fundamental Loop Analysis

$$B = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 & -1 & -1 \end{array} \right]$$

B is an $n_b - n_n + 1 \times n_b$ matrix called the Fundamental loop matrix.

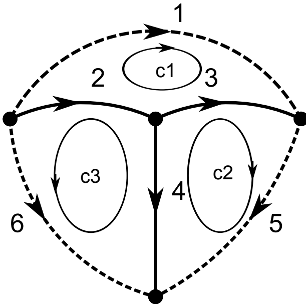
$$BV = \left[\; I|F \; \right] \left[\begin{array}{c} V_l \\ -- \\ V_b \end{array} \right] = 0$$

$$V_l = -FV_b$$

The number of Fundamental loop equations is $n_e - n_n + 1$ (=number of links).

Mesh Equation

Meshes are special case of the Fundamental loops i.e., there exists a tree such that the meshes are Fundamental loops****.



There are 3 meshes. Corresponding loop sets and mesh currents (loop currents) $G_{M1} = \{1, 2, 3\}$ and i_{m1}: $G_{M2} = \{3, 4, 5\}$ and i_{m2}; $G_{M3} = \{2, 4, 6\}$ and i_{m3}.

Mesh Analysis

$$\begin{aligned}
i_1 &= i_{m1} \\
i_2 &= -i_{m3} - i_{m1} \\
i_3 &= -i_{m2} + i_{m1} \\
i_4 &= -i_{m3} - i_{m1} \\
i_5 &= i_{m2} \\
i_6 &= i_{m3}
\end{aligned}$$

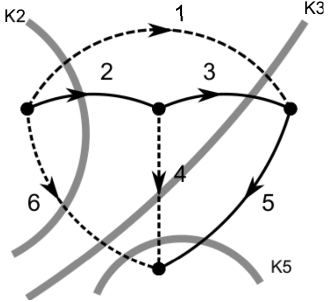
$$i = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{bmatrix}^T = B_f^T i_c$$

where $i = [i_1 \;\; i_5 \;\; i_6 \;\; i_2 \;\; i_3 \;\; i_4]^T$ is branch current vector and $i_m = [i_{m1} \;\; i_{m2} \;\; i_{m3}]^T$ is mesh current vector.

Fundamental Cut-set

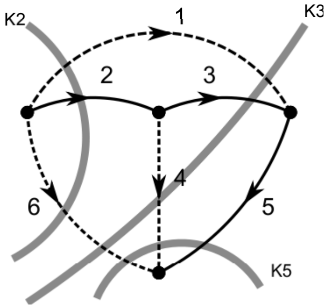
Cut-set

is made up of links and of one tree branch, namely the tree branch which defines the cut set. Every tree branch defines a unique Fundamental cut set.



Cut sets of the tree of $G_T = \{2, 3, 5\}$ are $G_{C2} = \{2, 1, 6\}$ $G_{C3} = \{3, 1, 4, 5\}$ $G_{C5} = \{5, 4, 6\}$.

Fundamental Cut-set



If we apply KCL to the three cut sets, we obtain

$$
\begin{aligned}
i_2 + i_1 + i_6 &= 0 \\
i_3 + i_1 + i_4 + i_6 &= 0 \\
i_4 + i_5 + i_6 &= 0
\end{aligned}
$$

which are called Fundamental cut-set equations. Reference direction for the cut set which agrees with that of the tree branch defining the cut set.

Fundamental Cut-set

$$i = \begin{bmatrix} i_l \\ -- \\ i_b \end{bmatrix} = \begin{bmatrix} i_1 \\ i_4 \\ i_6 \\ -- \\ i_2 \\ i_3 \\ i_5 \end{bmatrix}$$

where i_l is link current vector and i_b is tree branch current vector. In matrix form:

$$\begin{bmatrix} 1 & 0 & 1 & | & 1 & 0 & 0 \\ 1 & 1 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_4 \\ i_6 \\ -- \\ i_2 \\ i_3 \\ i_5 \end{bmatrix} = 0$$

Fundamental Cut-set

$$Qi = \left[\ E|I\ \right] \begin{bmatrix} i_l \\ -- \\ i_b \end{bmatrix} = 0$$

Q is called the Fundamental cut-set matrix. Q is an $n_b - n_n + 1 \times n_n - 1$

$$i_b = -Ei_l$$

Q has a rank $n_n - 1$ it includes the unit matrix. Hence the linear algebraic equations obtained by applying KCL to each Fundamental cut set constitute a set of $n_n - 1$ linearly independent equations.

Relation Between B and Q

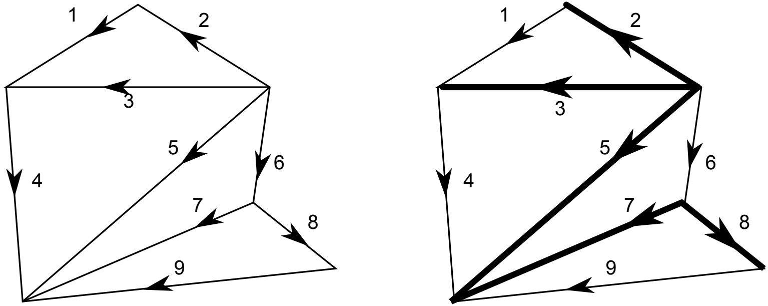
$F = -E^T$

Proof : Since they are the tree-branch voltages of the tree, the branch voltages are given by

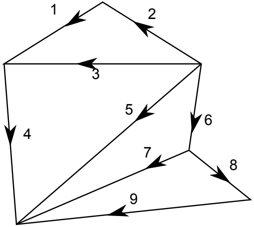
$$V = Q^T V_n$$
$$BV = BQ^T V_n = 0$$
$$BQ^T V_n = 0$$

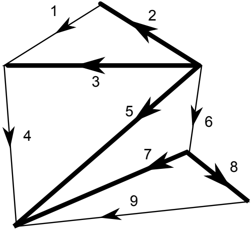
$$BQ^T = 0$$
$$IE^T + FI = 0$$
$$E^T + F = 0$$
$$E^T = -F$$

Example



Fundamental cut sets of the tree $G_T = \{2, 3, 5, 7, 8\}$ are $G_{C2} = \{2, 1\}$, $G_{C3} = \{3, 1, 4\}$, $G_{C5} = \{5, 4, 6\}$, $G_{C7} = \{7, 6, 9\}$, $G_{C8} = \{8, 9\}$.

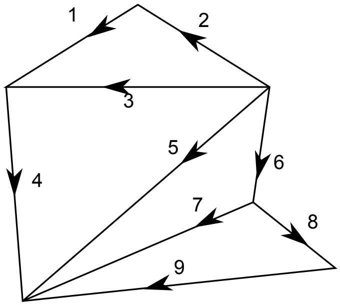


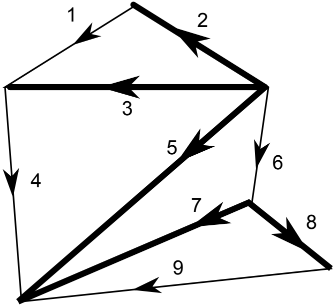


KCL equations based on Fundamental cut sets

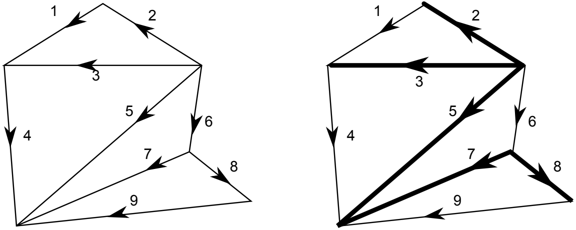
$$
\begin{bmatrix}
-1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & | & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & | & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 1 & | & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & | & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
i_1 \\ i_4 \\ i_6 \\ i_9 \\ -- \\ i_2 \\ i_3 \\ i_5 \\ i_7 \\ i_8
\end{bmatrix} = 0
$$

Example



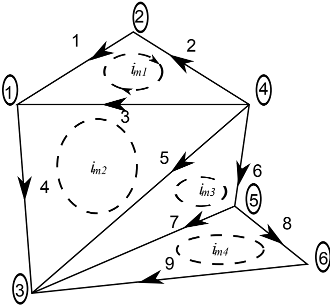


Fundamental Loop sets of the tree $G_T = \{2, 3, 5, 7, 8\}$ are $G_{L1} = \{1, 2, 3\}$, $G_{L4} = \{4, 3, 4\}$, $G_{L6} = \{6, 5, 6\}$, $G_{L9} = \{9, 7, 8\}$.



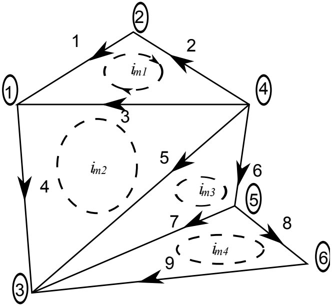
KVL equations based on the Fundamental loops

$$\left[\begin{array}{cccc|ccccc} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \end{array}\right] \begin{bmatrix} V_1 \\ V_4 \\ V_6 \\ V_9 \\ -- \\ V_2 \\ V_3 \\ V_5 \\ V_7 \\ V_8 \end{bmatrix} = 0$$



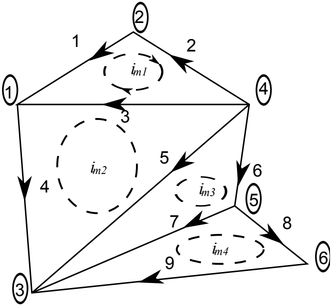
KVL equations for the nodes

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \\ V_{n4} \\ V_{n5} \\ V_{n6} \end{bmatrix}$$



KCL equations for the nodes

$$\begin{bmatrix} -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \end{bmatrix} = 0$$



Mesh equations

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{m1} \\ i_{m2} \\ i_{m3} \\ i_{m4} \end{bmatrix}$$