Basic of Electrical Circuits
EHB 211E

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Lecture 12

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Analysis of Nonlinear Resistive Circuits

Linear approximation of the nonlinear element at the operating point Q can be obtained using Taylor series expansion

$$v_N(t) = f(i_N) = f(I_Q) + \frac{df(i)}{di_N}(i - I_Q)\Big|_Q + \mathrm{h.o.t}$$

- The first term $V_Q = f(I_Q)$ is obtained from DC analysis. The solutions to a circuit with dc input are called operating points. The term dc analysis refers to the determination of operating points.
- The second term $v(t) = \frac{df(i)}{di_N}(i - I_Q)\Big|_Q$ is obtain form ac analysis (small signal analysis). We assume that the applied signal (which are ac signal) has a sufficiently small voltage or current (in magnitude).

DC analysis

The nonlinear circuit consists of DC and AC type of independent sources. An Alternating Current (AC) source provides the alternating current and Direct Current (DC) is the unidirectional flow of electric charge.
The circuit will be analyzed using superposition theorem. First operating point is obtained when all the sources in the circuit expect DC sources are set to zero. Then circuit will be analyzed using the known method.

$$f(V_Q, I_Q) = 0$$

The solutions of the equation are the operating points we are looking for.

DC analysis

How to solve the nonlinear equation which is obtained from DC Analysis ?

- Analytic approach :

$$aV_Q^2 + bV_Q + c = 0$$

- Numerical method : The numerical method is very useful in solving nonlinear equations. The Newton-Raphson method is the most commonly used numerical method for finding dc operating points.

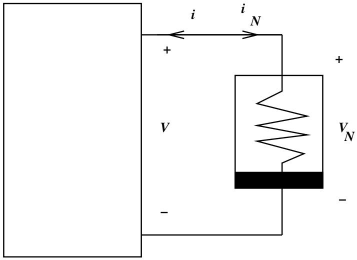
- Graphic Method (load line): Using Equivalent Circuit of the one-port, we have

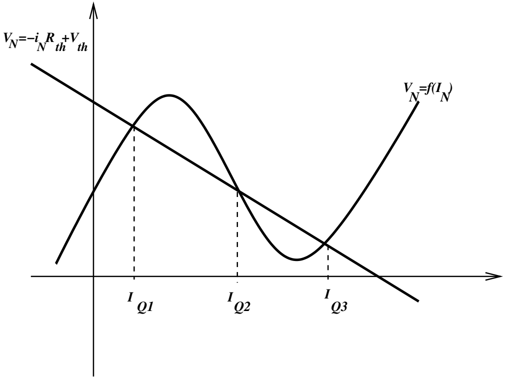
$$V = iR_{th} + V_{th}$$

from KCL: $i = -i_N$ and KVL: $V = V_N$ we will have

$$V_N = -i_N R_{th} + V_{th}$$

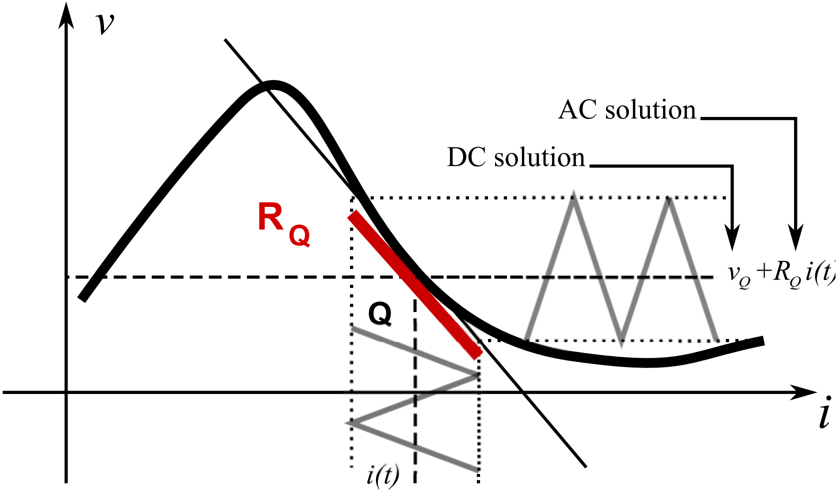
This is superimposed with the characteristic of the nonlinear one-port N, as shown





AC analysis

An operating point specifies a region in the $v - i$ plane in the neighborhood of which the actual voltage and current in the circuit vary as a function of time.



AC analysis

Amplitude of the AC signal is small compare to the operating point. to replacing the nonlinear characteristic by its linear approximation about the operating point Q.

$$v(t) = \frac{df(i)}{di_N}(i - i_Q)\Big|_Q$$

The term $\frac{df(i)}{di_N}\Big|_Q$ is the slope of the nonlinear characteristic at the operating point Q.

$$R_Q = \frac{df(i)}{di_N}\Big|_Q$$

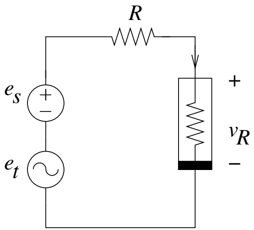
is called the "small-signal" resistance of the nonlinear element at the operating point Q.
Using R_Q in the circuit *small-signal equivalent circuit is obtained about operating point Q.*

The solution

From the superposition

$$v_N(t) = V_Q + R_Q i(t)$$

Example:



$R = 3.5\Omega,\ e_s = 9V,\ e_t(t) = 0.1\sin(10t)$ and $V_R = i_R^3 - 6i_R^2 + 9i_R$.

DC analysis:

$$e = i_R R + V_R$$

Substituting the nonlinear element's definition into above equation

$$e = i_R R + i_R^3 - 6i_R^2 + 9i_R.$$

Operating point: $I_Q = 2A$ and $V_Q = 2V$. Linear approximation about the operating point $(2V, 2A)$

$$R_Q = (3i_R^2 - 12i_R + 9)\Big|_{i_R=2} = -3\Omega$$

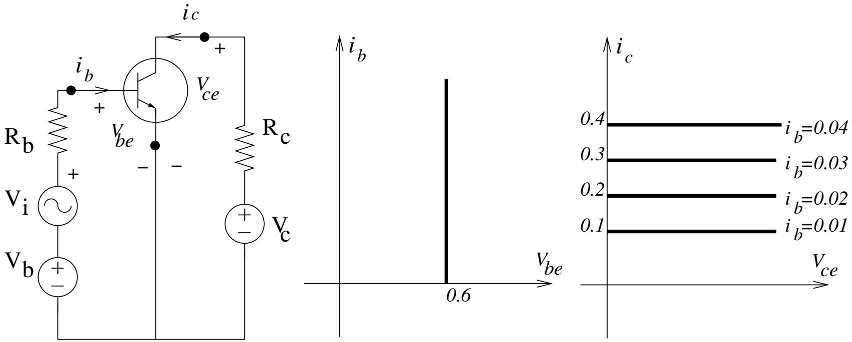
From the small-signal equivalent circuit

$$v_R = \frac{R_Q e(t)}{R_Q + R} = -0.6\sin(10t)$$

The solution

$$V_R = 2 - 0.6\sin(10t)$$

Example

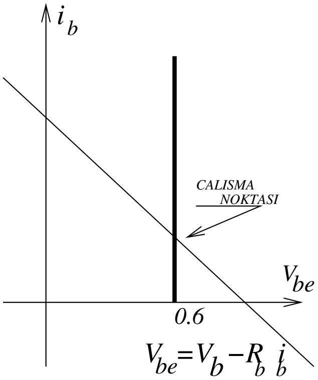


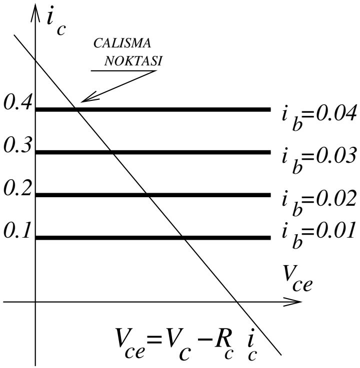
$R_b = 10\Omega,\ R_c = 1\Omega,\ V_b = 1V,\ V_c = 1V$ and $i_c = 10i_b$.

DC Analysis, From KVL:

$$V_{be} = V_i - R_b i_b \quad V_{ce} = V_c - R_c i_c$$

These are superimposed with (i_b, V_{be}) and (i_c, V_{ce}) characteristics





$V_{be} = 0.6$, $i_b = 0.04$ ve $i_c, = 0.4$ $V_{ce} = 0.6$.

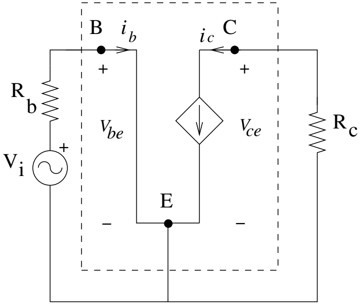
Around the operating point small signal conductance between the nodes (b) and (e)

$$G_i = \frac{di_b}{dV_{be}} = \infty$$

Linear approximation of $i_c = g(V_{ce}, i_b)$ (the terminal between (c) and (e))

$$i_c = \left.\frac{di_c}{dV_{ce}}\right|_Q V_{ce} + \left.\frac{di_c}{di_b}\right|_Q i_b = 0 + 10 i_b = 10 i_b$$

the small-signal equivalent circuit



Analyzing the small signal equivalent circuit, we obtain

$$i_b = \frac{v_i}{R_b} = 0.1\sin(wt)$$

and

$$v_{ce} = -R_C i_c = -0.1\sin(wt)$$

Then we have complete solution

$$V_{ce} = 0.6 - 0.1\sin(wt)$$