Basic of Electrical Circuits
EHB 211E

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Lecture 9

Contents I

Nodal Analysis

Method will first study for circuits which includes resistors and independent sources.

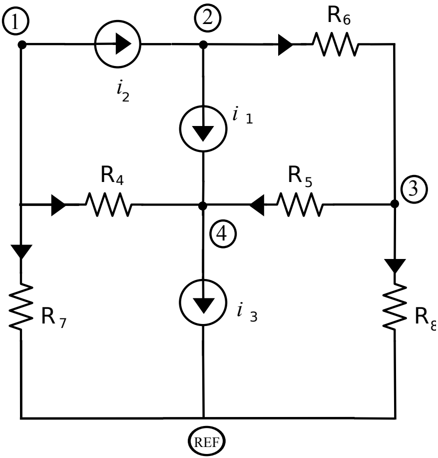
Which node will be the reference node ? choose a node which is connected to the maximum number of voltage sources.

If a voltage source is connected between a node and the reference node, the voltage is already known and it is not necessary to assign a variable. If there is a voltage source between two nodes, the difference between the node voltages equals to the voltage of the source.

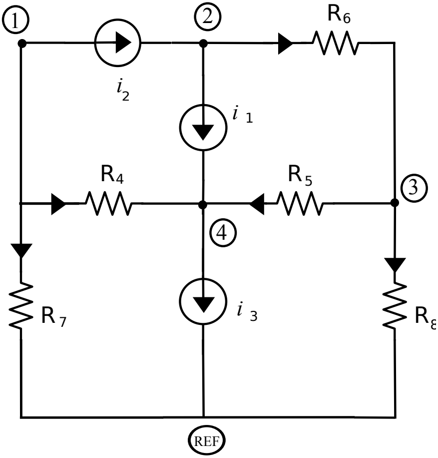
Nodal Analysis

1. Draw the circuit graph and one node is chosen as the reference node whose voltage is assumed as zero.
2. Write the fundamental cut-set equations for the nodes which do not correspond to node of a voltage sources: $A_d i = 0$
$$A_d[i_R \ i_k]^T = A_{11} i_R + A_{12} i_k = 0$$
3. Substitute $i_R = GV_R$ into the previous equation
$$A_d i = A_{11} G_R V_R + A_{12} i_k = 0$$
4. V_R is written in terms of the node voltages
$$V_R = A_{11}^T V_d$$
5. Equations are presented in the form:
$$A_{11} G_R A_{11}^T V_d + A_{12} i_k = 0$$

Nodal Analysis



1. Nodes are labels and reference node is chosen.



2. The fundamental cut-set equations for the nodes:

$$
\begin{aligned}
i_4 + i_2 + i_7 &= 0 \\
i_1 + i_6 - i_2 &= 0 \\
i_5 - i_6 + i_8 &= 0 \\
i_3 - i_1 - i_5 - i_4 &= 0
\end{aligned}
$$

3. Resistors in the circuit:

$$i_k = G_k V_k \quad k = \{4, 5, 6, 7, 8\}$$

4. Substitute terminal equations of the resistors into the previous equation

$$\begin{aligned}
G_4 V_4 + i_2 + G_7 V_7 &= 0 \\
i_1 - G_6 V_6 - i_2 &= 0 \\
G_5 V_5 - G_6 V_6 + G_8 V_8 &= 0 \\
i_3 - i_1 - G_5 V_5 - G_4 V_4 &= 0
\end{aligned}$$

5. Terminal voltage is written in terms of the node voltages:

$$\begin{aligned}
V_1 &= -V_{d2} \\
V_2 &= V_{d1} - V_{d2} \\
V_3 &= -V_{d4} \\
V_4 &= V_{d1} \\
V_5 &= V_{d3} \\
V_6 &= V_{d2} - V_{d3} \\
V_7 &= V_{d4} - V_{d1} \\
V_8 &= V_{d3} - V_{d4}
\end{aligned}$$

6. Substitute the equation in step 5 into step 4

$$
\begin{aligned}
G_4(V_{d1} - V_{d4}) + i_2 + G_7 V_{d1} &= 0 \\
i_1 - G_6(V_{d2} - V_{d3}) - i_2 &= 0 \\
G_5(V_{d3} - V_{d4}) - G_6(V_{d2} - V_{d3}) + G_8 V_{d3} &= 0 \\
i_3 - i_1 - G_5(V_{d3} - V_{d4}) - G_4(V_{d1} - V_{d4}) &= 0
\end{aligned}
$$

Equations are presented in the matrix form

$$
\begin{bmatrix} G_4 + G_7 & 0 & 0 & -G_4 \\ 0 & G_6 & -G_6 & 0 \\ 0 & -G_6 & G_6 + G_5 + G_8 & -G_5 \\ -G_4 & 0 & -G_5 & G_4 + G_5 \end{bmatrix}
\begin{bmatrix} V_{d1} \\ V_{d2} \\ V_{d3} \\ V_{d4} \end{bmatrix}
+ \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}
\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = 0
$$

Generalized Nodal Analysis

This method is modified version of Nodal Analysis for circuits which include also dependent sources and/or multi-terminal circuit elements.

We will have additional unknown variables because of dependent sources and/or multi-terminal circuit elements.

If there are dependent sources in the circuit, write down equations that express their values in terms of node voltages.

1. Draw the circuit graph and one node is chosen as the reference node.
2. Write the fundamental cut-set equations for the nodes:

$$Ai = [A_1 \; A_2 \; A_3][i_1 \;\; i_2 \;\; i_3]^T = A_1 i_1 + A_2 i_2 + A_3 i_3 = 0$$

   i_1 vector of resistor currents, i_2 vector of current source currents and i_3 vector of voltage source currents.

3. $i_1 = G_R V_R$
4. $A_1 G_R V_R + A_2 i_2 + A_3 i_3 = 0$
5. Using $V = A^T V_d$, we have $V_1 = A_1^T V_d$.
6. $v - i$ relation of the dependent sources

$$M_3 i_3 + N_3 V_3 = TV_3$$

$$T = \begin{cases} T_{i,i} = 1 & if\, V_{3_i} isavoltagesource \\ T_{i,j} = 0 & other \end{cases}$$

Generalized Nodal Analysis

$$TV_3 = [V_{k1} \;\; V_{k2} \;\; ... \;\; V_{kl} \; 0 \; 0 \; ... \; 0]^T$$

Substitute the node equation

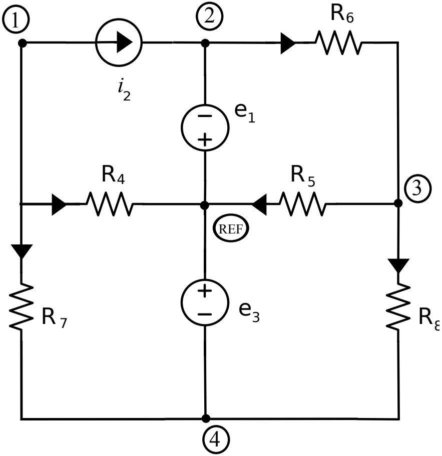
$$M_3 i_3 + N_3 A_3^T V_d = TV_3$$

Using the equ. in step 4

$$A_1 G_R A_1^T V_d + A_2 i_2 + A_3 i_3 = 0$$

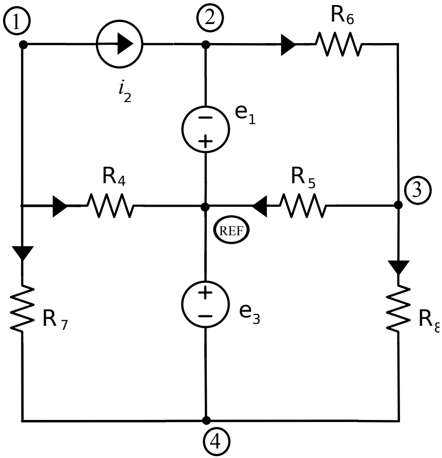
$$\begin{bmatrix} A_1 G_R A_1^T & A_3 \\ N_3 A_3^T & M_3 \end{bmatrix} \begin{bmatrix} V_d \\ i_3 \end{bmatrix} = - \begin{bmatrix} A_2 \\ 0 \end{bmatrix} i_2 - \begin{bmatrix} TV_3 \\ 0 \end{bmatrix}$$

Example



1. Nodes are labeled and the reference node is chosen

Example



2.the fundamental cut-set equations for the nodes

$$\begin{aligned} i_4 + i_2 - i_7 &= 0 \\ i_5 - i_6 + i_8 &= 0 \end{aligned}$$

3. Write the $v - i$ relations of the resistors:

$$i_k = G_k V_k \quad k = \{4, 5, 6, 7, 8\}$$

4. Substitute the equations in Step 3 into the equations in Step 2.

$$\begin{aligned} G_4 V_4 + i_2 - G_7 V_7 &= 0 \\ G_5 V_5 - G_6 V_6 + G_8 V_8 &= 0 \end{aligned}$$

5. Terminal voltage is written in terms of the node voltages:

$$\begin{aligned} V_1 &= -V_{d2} \\ V_2 &= V_{d1} - V_{d2} \\ V_3 &= -V_{d4} \\ V_4 &= V_{d1} \\ V_5 &= V_{d3} \\ V_6 &= V_{d2} - V_{d3} \\ V_7 &= V_{d4} - V_{d1} \\ V_8 &= V_{d3} - V_{d4} \end{aligned}$$

6. Substitute the equation in step 5 into step 4:

$$\begin{aligned} G_4 V_{d1} + i_2 - G_7(V_{d4} - V_{d1}) &= 0 \\ G_5 V_{d3} - G_6(V_{d2} - V_{d3}) + G_8(V_{d3} - V_{d4}) &= 0 \end{aligned}$$

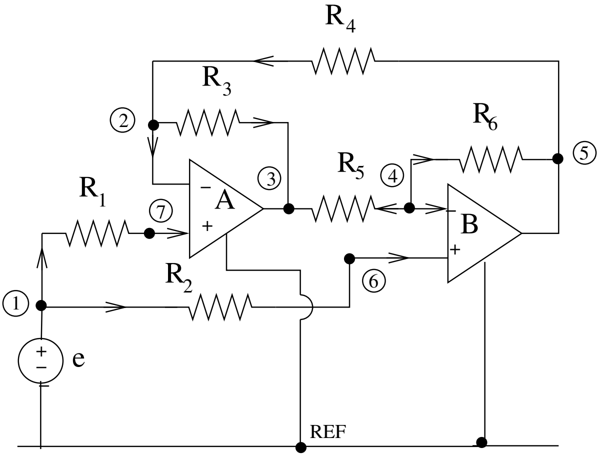
Voltage sources are written in the terms of the node valtages

$$\begin{aligned} V_{d2} &= -e_1 \\ V_{d4} &= -e_3 \end{aligned}$$

Equations are presented in the matrix form:

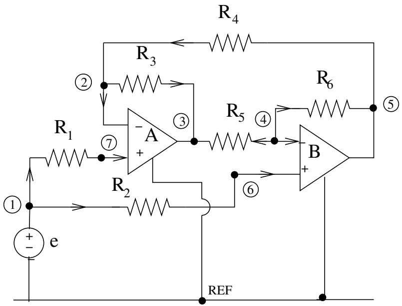
$$\begin{bmatrix} G_4 + G_7 & 0 \\ 0 & G_5 + G_6 + G_8 \end{bmatrix} \begin{bmatrix} V_{d1} \\ V_{d3} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i_2 + \begin{bmatrix} 0 & G_7 \\ G_6 & G_8 \end{bmatrix} \begin{bmatrix} e_1 \\ e_3 \end{bmatrix} = 0$$

Example



1. Nodes are labeled and the reference node is chosen

Example



2. the fundamental cut-set equations for the nodes

$$
\begin{aligned}
i_{A-} + i_3 - i_4 &= 0 \\
i_{Ao} - i_3 - i_5 &= 0 \\
i_{B-} - i_6 + i_5 &= 0 \\
i_{Bo} - i_6 + i_4 &= 0 \\
i_{B+} - i_2 &= 0 \\
i_{A+} - i_1 &= 0
\end{aligned}
$$

Example

3. Write the $v - i$ relations of the resistors: $i_k = G_k V_k \quad k = \{1, 2, .., 6\}$ 4. Substitute the equations in Step 3 into the equations in Step 2:

$$\begin{aligned}
i_{A-} + G_3 V_3 - G_4 V_4 &= 0 \\
i_{Ao} - G_3 V_3 - G_5 V_5 &= 0 \\
i_{B-} - G_6 V_6 + G_5 V_5 &= 0 \\
i_{Bo} - G_6 V_6 + G_4 V_4 &= 0 \\
i_{B+} - G_2 V_2 &= 0 \\
i_{A+} - G_1 V_1 &= 0
\end{aligned}$$

5.Terminal voltage is written in terms of the node voltages:

$$\begin{aligned}
V_1 &= V_{d1} - V_{d7} \\
V_2 &= V_{d1} - V_{d6} \\
V_3 &= V_{d2} - V_{d3} \\
V_4 &= V_{d5} - V_{d2} \\
V_5 &= V_{d4} - V_{d3} \\
V_6 &= V_{d4} - V_{d5}
\end{aligned}$$

Example

6. Substitute the equation in step 5 into step 4:

$$\begin{aligned}
i_{A-} + G_3(V_{d2} - V_{d3}) - G_4(V_{d5} - V_{d2}) &= 0 \\
i_{Ao} - G_3(V_{d2} - V_{d3}) - G_5(V_{d4} - V_{d3}) &= 0 \\
i_{B-} - G_6(V_{d4} - V_{d5}) + G_5(V_{d4} - V_{d3}) &= 0 \\
i_{Bo} - G_6(V_{d4} - V_{d5}) + G_4(V_{d5} - V_{d2}) &= 0 \\
i_{B+} - G_2(V_{d1} - V_{d6}) &= 0 \\
i_{A+} - G_1(V_{d1} - V_{d7}) &= 0
\end{aligned}$$

Additional equations

$$\begin{aligned}
V_{d1} &= e \\
i_{A+} = i_{A-} = i_{B+} = i_{B-} &= 0 \\
V_{A+} &= V_{A-} \\
V_{B+} &= V_{B-}
\end{aligned}$$

Additional equations is written in terms of the node voltages and independent sources:

$$\begin{aligned}
V_{d2} &= V_{d7} \\
V_{d4} &= V_{d6}
\end{aligned}$$

Example

$$
\begin{aligned}
G_3(V_{d2} - V_{d3}) - G_4(V_{d5} - V_{d2}) &= 0 \\
i_{Ao} - G_3(V_{d2} - V_{d3}) - G_5(V_{d4} - V_{d3}) &= 0 \\
-G_6(V_{d2} - V_{d5}) + G_5(V_{d4} - V_{d3}) &= 0 \\
i_{Bo} - G_6(V_{d4} - V_{d5}) + G_4(V_{d5} - V_{d2}) &= 0 \\
-G_2(e - V_{d4}) &= 0 \\
-G_1(e - V_{d2}) &= 0
\end{aligned}
$$

when the equations are given in matrix form

$$
\begin{bmatrix} G_3 + G_4 & -G_3 & 0 & -G_4 \\ -G_6 & -G_5 & G_5 & -G_6 \\ 0 & 0 & G_2 & 0 \\ G_1 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} V_{d2} \\ V_{d3} \\ V_{d4} \\ V_{d5} \end{bmatrix}
+ \begin{bmatrix} 0 \\ 0 \\ -G_2 \\ -G_1 \end{bmatrix} e = 0
$$