Basic of Electrical Circuits
EHB 211E

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Lecture 8

Contents I

Branch Voltages Method

$$MV_{b'} + Qi_s + Mv_s = 0$$

where $V_{b'}$ branch voltage vector which does not include the voltages of the voltage sources in tree (which are the unknown), i_s current sources vector, v_s voltage source vector.

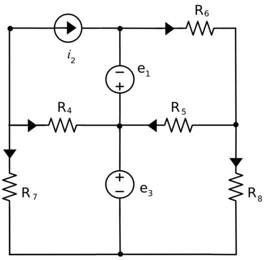
In this method, unknown variables will be the branch voltages, therefore voltage sources should be placed in tree in order to have minimum unknown.
Current sources must place in co-tree. Why ?

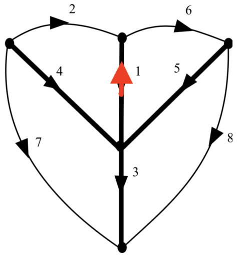
Branch Voltages Method

1. Select a proper tree: Place voltage sources in tree, current sources in co-tree, complete the tree with the resistors
2. Write the fundamental cut-set equations for the branches. (No need for the branches which correspond to voltage sources!)
3. Write the $v - i$ relations of the resistors in the form $i_k = G_k V_k$
4. Substitute the currents (Step 3) into fundamental cut-set equation (Step 2)
5. Write the fundamental loop equation (No need for the links which correspond to current sources!)
6. Substitute the fundamental loop equation (Step 5) into the equation in Step 4.
7. Present the equation in the form

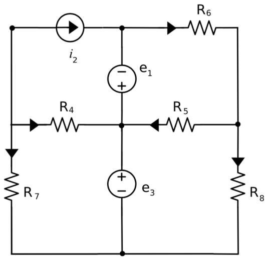
$$MV_{b'} + Qi_s + Mv_s = 0.$$

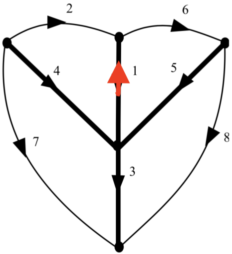
Branch Voltages Method





1. Proper tree $G_A = \{1, 3, 4, 5\}$.





2. Write the fundamental cut-set equations for the branches 4 and 5

$$\begin{aligned} i_4 + i_2 + i_7 &= 0 \\ i_5 - i_6 + i_8 &= 0 \end{aligned}$$

Branch Voltages Method

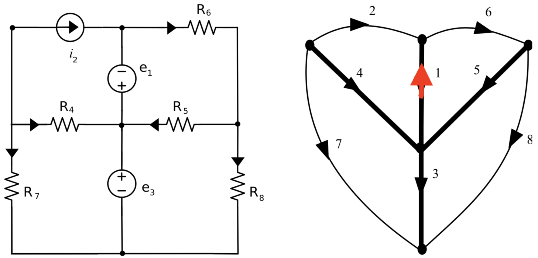
3. Write the $v - i$ relations of the resistors

$$i_k = G_k V_k \quad k = \{4, 5, 6, 7, 8\}$$

4. Substitute the equations in Step 3 into the equations in Step 2.

$$\begin{aligned} G_4 V_4 + i_2 + G_7 V_7 &= 0 \\ G_5 V_5 - G_6 V_6 + G_8 V_8 &= 0 \end{aligned}$$

We have branch and link voltages in the above equations.



5. Write the fundamental loop equations which do not correspond the currents sources :

$$
\begin{aligned}
V_6 + V_5 + V_1 &= 0 \\
V_7 - V_4 - V_3 &= 0 \\
V_8 - V_3 - V_5 &= 0
\end{aligned}
$$

Branch Voltages Method

Subtitute

$$\begin{aligned} V_6 &= -V_5 - V_1 \\ V_7 &= V_4 + V_3 \\ V_8 &= V_3 + V_5 \end{aligned}$$

into

$$\begin{aligned} G_4 V_4 + i_2 - G_7 V_7 &= 0 \\ G_5 V_5 - G_6 V_6 + G_8 V_8 &= 0 \end{aligned}$$

we have

$$\begin{aligned} G_4 V_4 + i_2 - G_7(V_4 + V_3) &= 0 \\ G_5 V_5 - G_6(-V_5 - V_1) + G_8(V_3 + V_5) &= 0 \end{aligned}$$

Branch Voltages Method

7. In matrix form

$$\begin{bmatrix} G_4 - G_7 & 0 \\ 0 & G_5 + G_6 + G_8 \end{bmatrix} \begin{bmatrix} V_4 \\ V_5 \end{bmatrix} + \begin{bmatrix} 0 & -G_7 \\ G_6 & G_8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i_2 = 0$$

The link voltages which are obtained from the branches:

$$\begin{bmatrix} V_2 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}$$

Generalized Branch Voltages Method

- If the circuit includes dependent sources and / or multi-terminal component, they are thought to be an independent source. The type of the source is decided by its $v - i$ relation. Then a proper tree is picked.

- At the last step, compare the number of the unknowns and the number of equations ! Currents and/or voltages of dependent sources and / or multi-terminal component, will be the additional unknowns.
- Using $v - i$ relations of the dependent sources, new unknown variables are written in the terms of the branch voltages, voltage sources and current sources.