Basic of Electrical Circuits
EHB 211E

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Lecture 7

Contents I

Chord (Link) Current Method

$$Bi_{l'} + Qi_s + Mv_s = 0$$

where $i_{l'}$ is link voltage vector which does not include current sources in co-tree and i_s is current sources vector and v_s is voltage sources vector.

Currents in the co-tree (except the currents of current sources) are the unknown variables. Therefore independent sources must be placed to links and independent voltage sources must be places to branches.

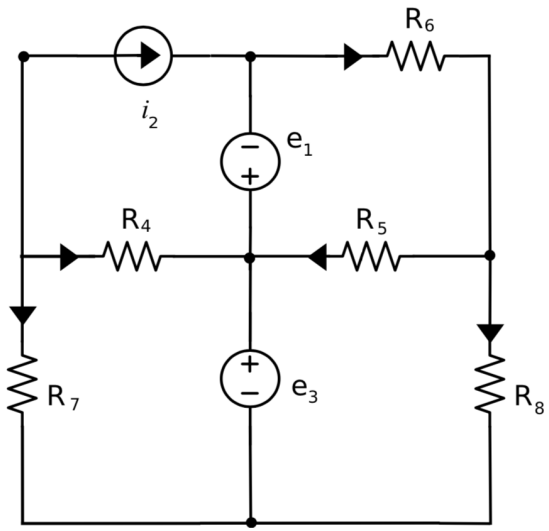
Chord Current Method

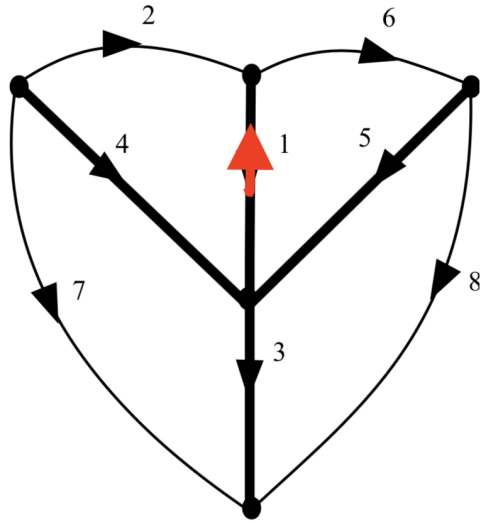
Linear Circuits Containing Two-Terminal Resistors and Independent Sources

1. Pick a proper tree of the graph of circuit which includes all the voltage sources. Current sources are placed in co-tree. Complete the tree with the resistors.
2. Write the fundamental loop equations which do not correspond to the current sources in co-tree.
3. Write the $v - i$ relations of the resistors in the form $V_k = R_k i_k$
4. Substitute the voltages in Step 3 into the equations in Step 2.
5. Write the fundamental cut-set equations which do not correspond voltage sources.
6. Substitute the fundamental loop equations in Step 4 into the equations in Step 5.
7. Present the equation in the form

$$Bi_{l'} + Qi_s + Mv_s = 0$$

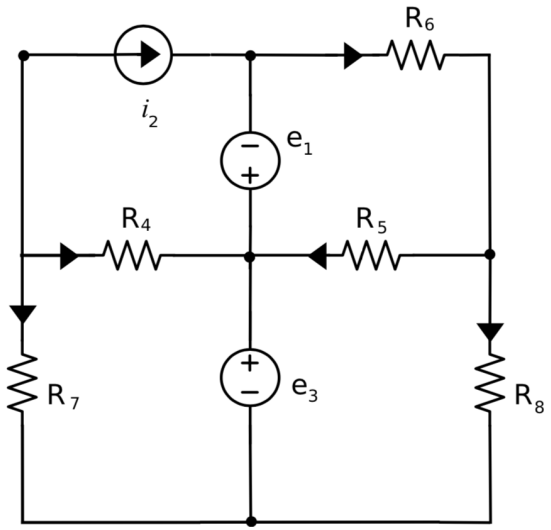
Chord Current Method

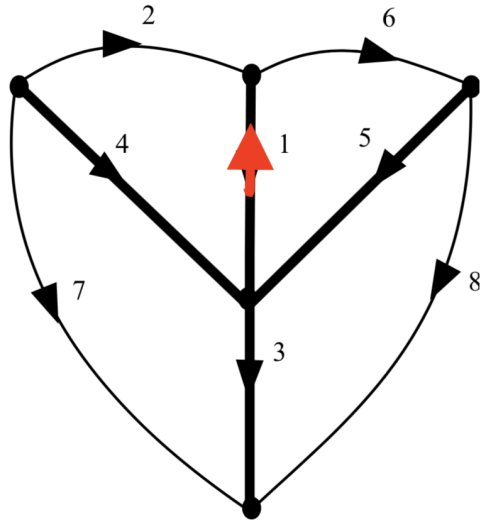




1. Proper tree $G_A = \{1, 3, 4, 5\}$.

Chord Current Method

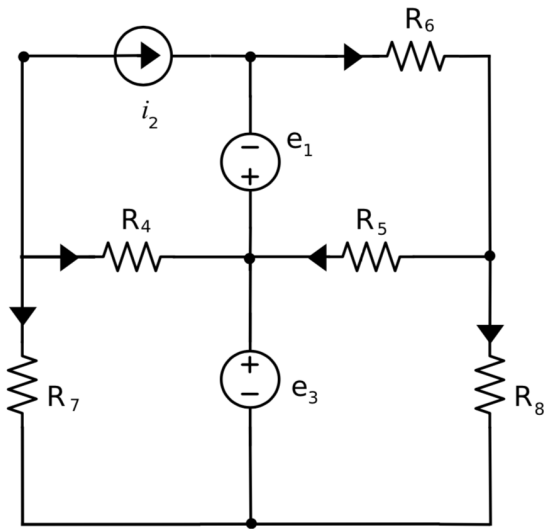


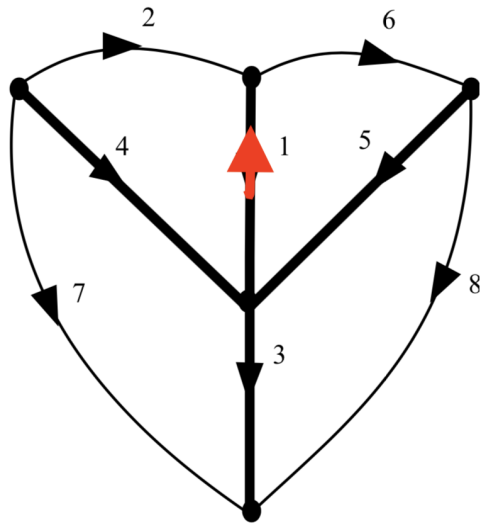


2. The fundamental loop equations for the links 6, 7 and 8:

$$\begin{aligned} V_6 + V_5 + V_1 &= 0 \\ V_7 - V_4 - V_3 &= 0 \\ V_8 - V_3 - V_5 &= 0 \end{aligned}$$

Chord Current Method





3. Write the $v - i$ relations of the resistors:

$$V_k = R_k i_k \quad k = \{4, 5, 6, 7, 8\}$$

Chord Current Method

4. Substitute the $v - i$ relations into

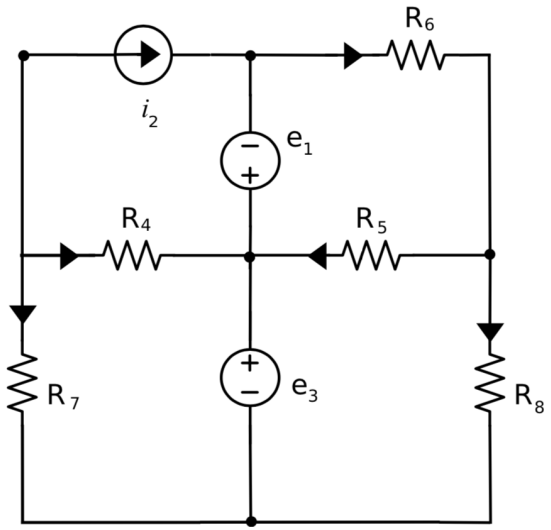
$$\begin{aligned} V_6 + V_5 + V_1 &= 0 \\ V_7 - V_4 - V_3 &= 0 \\ V_8 - V_3 - V_5 &= 0 \end{aligned}$$

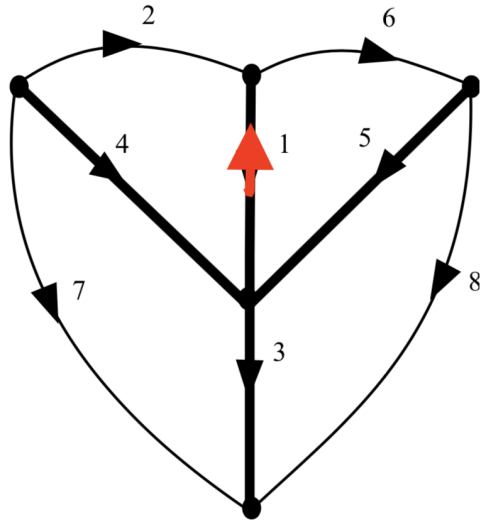
obtain

$$\begin{aligned} R_6 i_6 + R_5 i_5 + V_1 &= 0 \\ R_7 i_7 - R_4 i_4 - V_3 &= 0 \\ R_8 i_8 - V_3 - R_5 i_5 &= 0 \end{aligned}$$

which include the currents of links and branches.

Chord Current Method





5. fundamental cut-set equations for the links 4 and 5:

$$\begin{aligned} i_4 + i_2 + i_7 &= 0 \\ i_5 + i_8 - i_6 &= 0 \end{aligned}$$

Chord Current Method

6. Substitute

$$\begin{aligned} i_4 &= -i_2 - i_7 \\ i_5 &= -i_8 + i_6 \end{aligned}$$

into

$$\begin{aligned} R_6 i_6 + R_5 i_5 + V_1 &= 0 \\ R_7 i_7 - R_4 i_4 - V_3 &= 0 \\ R_8 i_8 - V_3 - R_5 i_5 &= 0 \end{aligned}$$

we will have

$$\begin{aligned} R_6 i_6 + R_5(-i_8 + i_6) + V_1 &= 0 \\ R_7 i_7 - R_4(-i_2 + i_7) - V_3 &= 0 \\ R_8 i_8 - V_3 - R_5(-i_8 + i_6) &= 0. \end{aligned}$$

Chord Current Method

The unknown variables $i_{l'} = [i_6 \ i_7 \ i_8]^T$ which are the currents in co-tree (except the link 2 which is a current source.)

$$\begin{bmatrix} R_6 + R_5 & 0 & -R_5 \\ 0 & R_7 - R_4 & 0 \\ -R_5 & 0 & R_8 + R_5 \end{bmatrix} \begin{bmatrix} i_6 \\ i_7 \\ i_8 \end{bmatrix} + \begin{bmatrix} 0 \\ R_4 \\ 0 \end{bmatrix} i_2 + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix} = 0$$

Chord Current Method

The branches currents

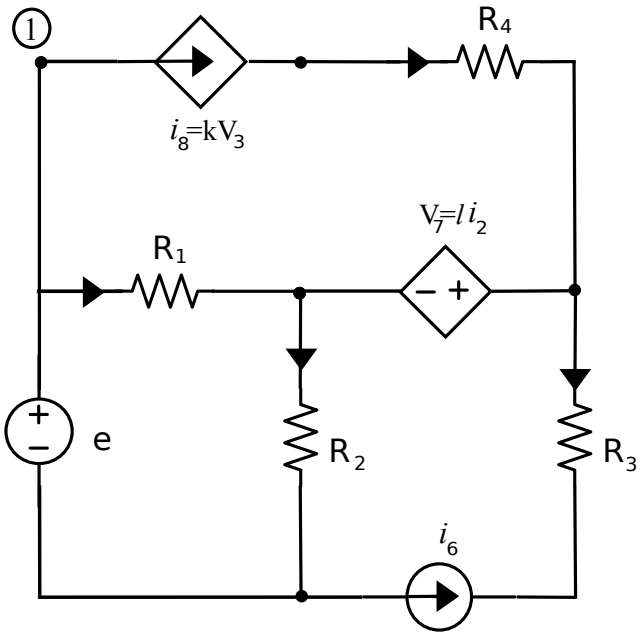
$$\begin{bmatrix} i_1 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_2 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix}$$

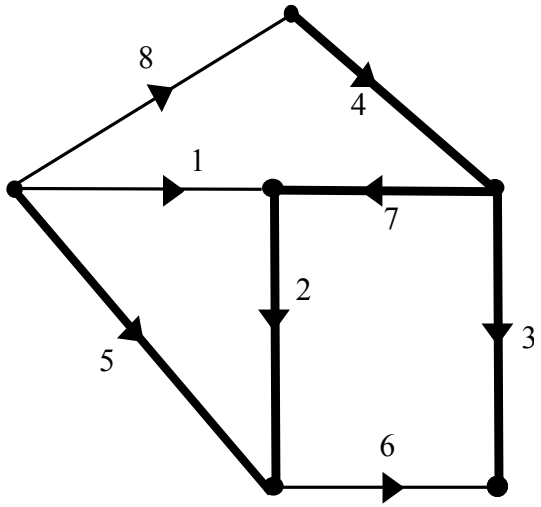
Generalized Chord Current Method

Linear Circuits Containing Two-Terminal Resistors and Independent/dependent Sources

- Follow the same steps as before after taking the dependent sources as a independent source.
- Place the dependent source in tree if it is a voltage source (or co-tree if it is a current source).
- At the last step, compare the number of the unknowns and the number of equations ! Currents and/or voltages of the dependent sources will be the additional unknowns.
- Using $v - i$ relations of the dependent sources new unknown variable are written in the terms of the link currents, voltage sources and current sources.

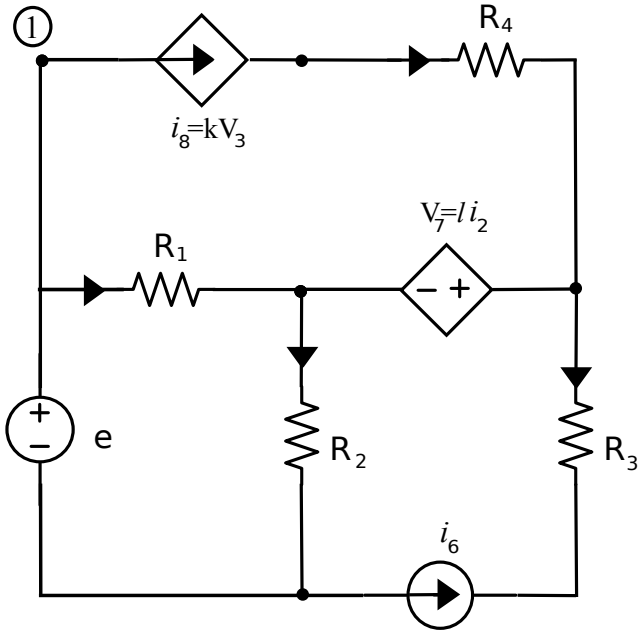
Generalized Chord Current Method

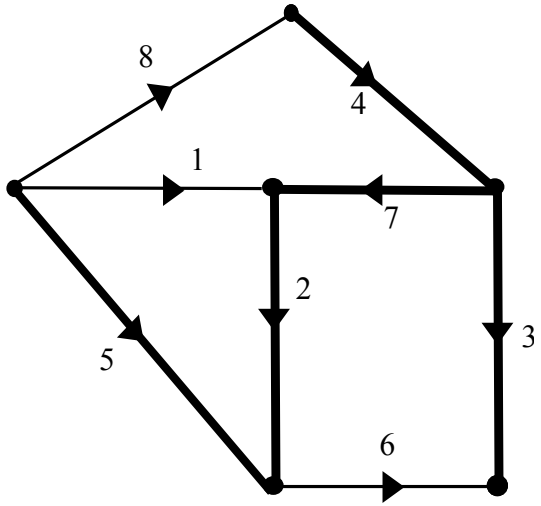




1. Proper tree $G_A = \{2, 3, 4, 5, 7\}$.

Generalized Chord Current Method





2. Fundamental loop equation for the link 1:

$$V_1 + V_2 + V_5 = 0$$

3. Write the $v - i$ relations of the resistors:

$$V_k = R_k i_k \quad k = \{1, 2, 3, 4\}$$

Generalized Chord Current Method

4. Substitute the equation in step 3 into the fundamental loop equations:

$$R_1 i_1 + R_2 i_2 - e = 0$$

5. Fundamental cut-set equation:

$$i_2 - i_8 - i_1 - i_6 = 0$$

6. Substitute $i_2 = i_8 + i_1 + i_6$ into $R_1 i_1 + R_2 i_2 - e = 0$ we have

$$R_1 i_1 + R_2 (i_8 + i_1 + i_6) - e = 0$$

which e voltage source, i_6 current source, i_1 link current (which is the unknown variable) what is i_8 ?

Generalized Chord Current Method

Write $v - i$ relation for dependent source:

$$i_8 = kV_3$$

Substitute the $v - i$ relation of the edge 3 into the above equ.

$$i_8 = kR_3 i_3$$

i_3 is written in the term of the link currents using fundamental cut-set equation for the branch 3.

$$i_8 = -kR_3 i_6$$

In this case we will have

$$(R_1 + R_2)i_1 + (R_2 - kR_3)i_6 - e = 0$$

Generalized Chord Current Method

- If the circuit includes multi-terminal component, it is thought to be an independent source. The type of the source is decided by its $v - i$ relation. Then a proper tree is picked.

- At the last step, compare the number of the unknowns and the number of equations ! Currents and/or voltages of the multi-terminal component will be the additional unknowns.
- Using $v - i$ relations of the dependent sources new unknown variable are written in the terms of the link currents, voltage sources and current sources.