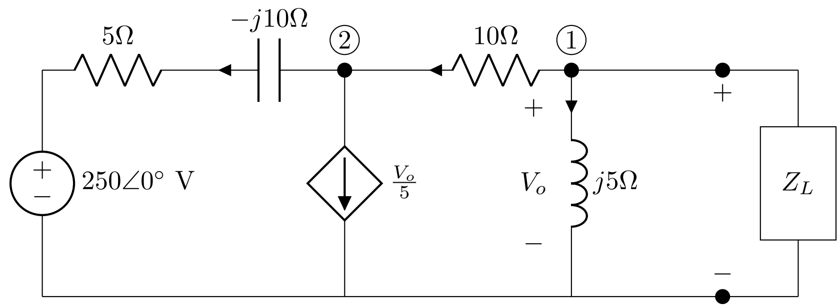
Solution (Detailed Explanation)

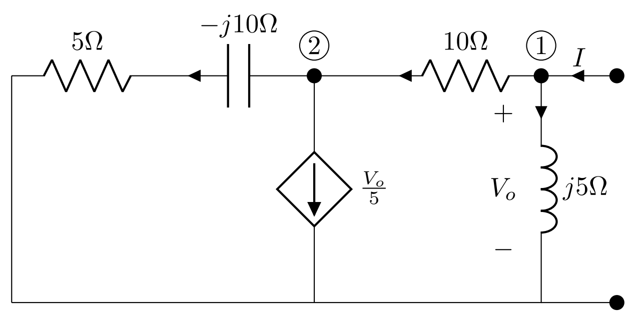
The load impedance Z_L in the circuit shown below is adjusted to ensure that the maximum average power is delivered to Z_L.

(a) Determine the maximum average power delivered to Z_L.

(b) Calculate the percentage of the total power developed in the circuit that is delivered to Z_L.



Objective: Determine the Thevenin impedance seen from the load terminals and identify the appropriate load for maximum power transfer. For this purpose, let us deactivate the independent source and calculate the impedance seen from the terminals where the load is connected.



Node Definitions

- V_2: the upper node of the dependent current source
- $V_0 = V_1$: the node where the load terminals are connected

Step 1: Writing the KCL Equations

KCL for node ②:

$$\frac{V_2}{5-10j} + \frac{V_0}{5} = \frac{V_0 - V_2}{10} \tag{1}$$

KCL at node ① ($V_O = V_1$):

$$I = \frac{V_0}{5j} + \frac{V_0 - V_2}{10} \tag{2}$$

Step 2: Expressing V_2 in terms of V_0

Equation (1) is rearranged:

$$\frac{5+10j}{125}V_2 + \frac{V_2}{10} = \frac{V_0}{10} - \frac{V_0}{5} \tag{3}$$

Factoring out the common term:

$$V_2\left(\frac{1}{25} + \frac{1}{10} + \frac{2}{25}j\right) = -\frac{V_0}{10} \tag{4}$$

Rewriting:

$$\frac{V_2}{50}(7+4j) = -\frac{V_0}{10} \tag{5}$$

Therefore,

$$V_2 = -\frac{5}{7+4j}V_0 \tag{6}$$

Step 3: Determining the Current Expression

Substituting for V_2:

$$I = \frac{V_0}{5j} + \frac{V_0}{10} - \frac{V_2}{10} \tag{7}$$

$$= \frac{V_0}{5j} + \frac{V_0}{10} + \frac{V_0}{14+8j} \tag{8}$$

Separating the real and imaginary parts:

$$I = (0.153 - 0.23j)\,V_0 \tag{9}$$

Step 4: Thevenin Impedance

$$Z_{th} = \frac{V_0}{I} = \frac{1}{0.153-0.23j} = 2+3j\ \Omega \tag{10}$$

Step 5: Load for Maximum Power

For maximum power transfer:

$$Z_L = Z_{th}^* \tag{11}$$

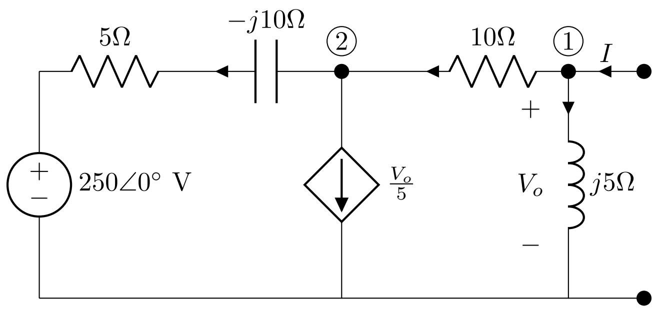
$$Z_L = 2-3j\ \Omega \tag{12}$$

Result

$$Z_{th} = 2 + 3j\ \Omega \tag{13}$$
$$Z_L = 2 - 3j\ \Omega \tag{14}$$

Objective: Let us compute the Thevenin open-circuit voltage. We leave the two terminals open and calculate the open-circuit voltage.

Solution



KCL for node ②:

$$\frac{V_2 - 250}{5 - 10j} + \frac{V_0}{5} = \frac{V_0 - V_2}{10} \tag{15}$$

KCL for node ①:

$$\frac{V_0}{5j} + \frac{V_0 - V_2}{10} = 0 \tag{16}$$

From here,

$$V_0(1 - 2j) = V_2 \tag{17}$$

Let us rearrange the equations

$$-\frac{250}{5 - 10j} + \left(\frac{1}{25} + \frac{1}{10} + \frac{2}{25}j\right) V_2 = -\frac{V_0}{10} \tag{18}$$

$$-\frac{250}{5 - 10j} + \left(\frac{7}{50} + \frac{4}{50}j\right)(1 - 2j)V_0 = -\frac{V_0}{10} \tag{19}$$

$$-\frac{250}{5 - 10j} + \left[\left(\frac{7}{50} + \frac{8}{50}\right) + \left(-\frac{4}{50} + \frac{4}{50}\right)j\right] V_0 = -\frac{V_0}{10} \tag{20}$$

$$-\frac{250}{5 - 10j} + (15 - 10j)\frac{V_0}{50} = -\frac{V_0}{10} \tag{21}$$

$$-\frac{250}{5-10j} = (-20+10j)\frac{V_0}{50} \tag{22}$$

$$V_0 = \frac{250}{5-10j} \cdot \frac{50}{20-10j} \tag{23}$$

$$= \frac{50}{1-2j} \cdot \frac{5}{2-j} \tag{24}$$

$$= \frac{250}{-5j} = 50j \tag{25}$$

Complex power of the load

$$P = \frac{1}{2}VI^* \tag{26}$$

$$I = \frac{50j}{4} = 12.5j \tag{27}$$

$$S_L = \frac{1}{2}I^2 Z_L = \frac{1}{2}(12.5j)(2-3j)(-12.5j) \tag{28}$$

$$= 78.125(2-3j) \tag{29}$$

$$P_L = 156.25 \text{ W} \tag{30}$$

Thevenin Circuit

$$Z_{th} = 2+3j \tag{31}$$

$$Z_L = 2-3j \tag{32}$$

Maximum Power Calculation

$$P = \frac{1}{2}VI^* \tag{33}$$

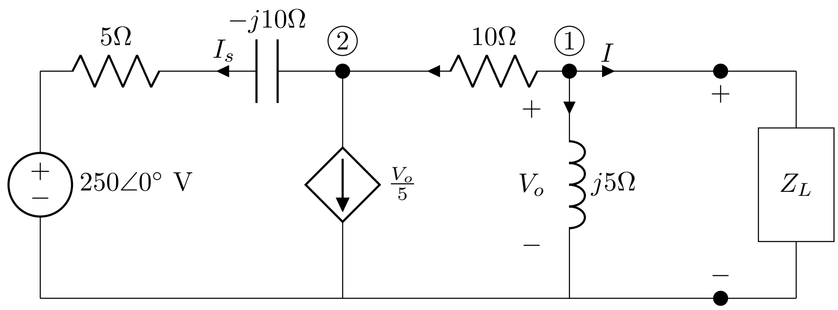
$$I = \frac{50j}{4} = 12.5j \tag{34}$$

$$S_L = \frac{1}{2}I^2 Z_L = \frac{1}{2}(12.5j)(2-3j)(-12.5j) \tag{35}$$

$$= 78.125(2-3j) \tag{36}$$

$$P_L = 156.25 \text{ W} \tag{37}$$

KCL Equations When the Load is Connected



KCL for node ①:

$$\frac{V_0}{5j} + \frac{V_0 - V_2}{10} + \frac{V_0}{2 - 3j} = 0 \tag{38}$$

$$V_0(0.25 + 0.03j) = \frac{V_2}{10} \tag{39}$$

KCL for node ②:

$$\frac{V_2 - 250}{5 - 10j} + \frac{V_0}{5} = \frac{V_0 - V_2}{10} \tag{40}$$

After rearranging the equations

$$\frac{V_2(5 + 10j)}{125} + \frac{V_0}{10} + \frac{V_2}{10} = \frac{250}{5 - 10j} \tag{41}$$

$$\frac{V_2}{25} + \frac{2jV_2}{25} + \frac{V_0}{10} + \frac{V_2}{10} = \frac{250}{5 - 10j} \tag{42}$$

$$\frac{7}{50}V_2 + \frac{4j}{50}V_2 + \frac{V_0}{10} = \frac{250}{5 - 10j} \tag{43}$$

$$\frac{7 + 4j}{50}(2.5 + 0.3j)V_0 + \frac{V_0}{10} = \frac{250}{5 - 10j} \tag{44}$$

$$V_0\left(\frac{16 + 12j}{50}\right) + \frac{V_0}{10} = \frac{250}{5 - 10j} \tag{45}$$

$$\left(\frac{21 + 12j}{50}\right)V_0 = \frac{250}{5 - 10j} \tag{46}$$

$$V_0 = 38.5 + 25.6j \tag{47}$$

$$V_2 = 88.5 + 75.5j \tag{48}$$

Current Calculation

$$I_s = \frac{V_2 - 250}{5 - 10j} \tag{49}$$

$$I_s = \frac{(88.5 + 75.5j) - 250}{5 - 10j} \tag{50}$$

$$I_s = -12.5 - 9.8j \tag{51}$$

Power Calculation

$$S = \frac{1}{2} V I_s^* \tag{52}$$

$$S = \frac{1}{2} \cdot 250 \cdot (-12.5 + 9.8j) \tag{53}$$

$$P = 1562 \text{ W} \tag{54}$$

Efficiency

$$\frac{P_L}{P_{\text{source}}} = \frac{156.2}{1562} \approx 10\% \tag{55}$$