Physically, a mechatronic system is composed of four prime components. They are sensors, actuators, controllers and mechanical components. Figure shows a schematic diagram of a mechatronic system integrated with all the above components.
Electrical Actuation

- **Switching devices**
  - Mechanical switches
    - Keyboards, limit switches, switches
  - Relays
  - Solid-state switches
    - Diodes, thyristors, Triacs transistors, MOSFET

- **Solenoids**
  - Starter solenoid, pneumatic or hydraulic valve

- **Drive systems**
  - D.C., A.C., or stepper motors
  - Usually under specific control element e.g. position, torque, or speed control
Relays are electrically operated switches in which changing a current in one electric circuit switches a current on or off in another circuit.

In the shown circuit when the solenoid is energized, a magnetic field is produced, which attracts the iron armature, moves the push rod and in result: closes the normally open contact and open the normally closed contact.

Relays are normally used in the control circuit together with transistor circuit to increase current from microcontroller system.

Diodes are used to protect against back voltage generated in the coil circuit during switching.

Figure 9.1 (a) A relay, (b) a driver circuit
Relays In control system Example-1

- When start switch is closed, solenoid A and B will be energized, thus in both A and B extending (A+, B+)

--switches a+ and b+ are then closed, the closure of a+ energized relay coil 1 which in turn operates relay contact 1, thus coil C is energized and result in extending of C (C+)

- Extension of C operates switch c+ and thus operates coils A- and B- and retraction of cylinder A and B start

Operation of switch a- energizes Relay coil 2 and thus operates contact 2 which allow cylinder C to retract...etc
Solid state switches: Diodes

Unidirectional uncontrolled switch used to rectify or permit current flow in one direction

Figure 9.3  (a) Diode characteristic, (b) half-wave rectification

Solid state switches: Thyristor

Unidirectional controlled switch used to control the flow of currents by controlling the gate circuit

- Linearly proportional POWER
- Gate controls when current flows
- Commonly used in heating control and motor speed control

Figure 9.4  (a) Thyristor characteristic, (b) thyristor circuit
Solid state switches: Triacs

Triac is similar to the thyristor and equivalent to a pair of thyristors connected in reverse parallel on the same chip.

Figure 9.5  Triac characteristic
Solid state switches: Triacs & thyristors

The figures show the types of effect that occur when a sinusoidal alternating voltage is applied across a thyristor and across a triac. Forward breakdown occurs when the voltage reaches the breakdown value and then the voltage across the device remains low.

![Figure 9.6 Voltage control: (a) thyristor, (b) triac](image)
Solid state switches: Triacs & thyristors

The circuit shows how these elements can be used to control the flow of dc power through a load in a form of chopper circuit.

Figure 9.7  Thyristor d.c. control
Figure 9.8  (a) Phase control, (b) snubber circuit
Solid state switches: BJT Transistors

Current Amplifier, Gain on order of 50-100, On-off, Like diodes, current flows only one direction

Darlington pair – higher gain, higher current

Figure 9.9 (a) Transistor symbols, (b), (c), (d), (e) transistor switch
Motor Control Example
Solid state switches: BJT Transistors

Darlington connections to increase drive current

Figure 9.10  (a) Switching a load, (b) and (c) Darlington pairs
Solid state switches: BJT Transistors

BJT is implemented by base currents and higher frequencies of switching are possible than with thyristors. The power handling capability is less than that of thyristors.

The circuit shows how a buffer might be used when transistor switching is used to control a dc motor by on/off switching.

Figure 9.11 Control of d.c. motor
Solid state switches: MOSFET

Here no current flows into the gate to exercise the control, the gate voltage is the controlling signal. Thus drive circuitry can be simplified, e.g. no need to concerned about the size of the current.

With MOSFET, higher frequency switching is possible, upto 1 MHz level voltage.

Figure 9.12  MOSFETs: (a) n-channel, (b) p-channel, (c) used to control a d.c. motor
Magnetism is basis of their principles of operation. They use permanent magnets and/or electromagnets, and exploit the electromagnetic phenomenon in order to produce the actuation. Electromechanical actuators are DC, AC and stepper motors.
A DC motor converts the electrical energy to mechanical energy. The torque is produced due to input current. In reverse situation, the torque, which is equivalent to mechanical energy, can produce current that is equivalent to electrical energy. This reverse process is utilized for the design of DC generator. Figure-7.4 illustrates schematic diagram of typical DC motor and DC generator.
Basics of DC Motor

Figure 9.13  D.C. motor: (a) basics, (b) with two sets of poles
Figure 9.15  D.C. motors: (a) series-wound motor, (b) shunt-wound motor, (c) compound motor, (d) separately excited motor, (e) torque–speed characteristics
Control of brush-type dc Motor

The speed of the dc motor depends on the current through the armature coil

\[ \tau_m = K_T \cdot i_a \quad E_{emf} = K_b \cdot \omega \]

Electronically controlled high-frequency switch to chop the D.C.

Figure 9.16  PWM: (a) principles of PWM circuit, (b) varying the armature voltage by chopping the d.c. voltage
Control of brush-type dc Motor

- Two direction control of DC motor
- Direction and speed control of dc motor (with additional logic circuit)

Figure 9.17  (a) Basic transistor circuit, (b) H-circuit, (c) H-circuit with logic gates
Control of brush-type dc Motor

Figure 9.18  Speed control with feedback
Figure 9.20  (a) Single-phase induction motor, (b) three-phase induction motor, (c) three-phase synchronous motor
Figure 9.21  Variable speed a.c. motor
Servomotors

- Sometimes called control motors, are electric motors that are specially designed and built for use in feedback control system as output actuators.
- **Ratings**: fractional of watts to several 100 watts
- Higher speed response, smaller in diameter and longer in length
- Normally operate at low or zero speed
- Used in robots, radar, computers, tracking and guidance systems and in process control.
- Both DC and AC servomotors are used at present
DC Servomotors

- They are separately excited DC motors or PMDC
- The armature is designed to have large resistance so that the torque speed characteristics are linear and have a large negative slope.
- A step change in armature voltage results in a quick change in position or speed of the rotor.
AC Servomotors

- Most ac servomotors in control systems are of the two-phase squirrel cage IM
- Operating frequency normally 60 or 400Hz higher frequency is preferred in airborne system.
- A two phase ac servomotor is shown below:
The stator has two distributed windings displaced 90 electrical degrees apart. One winding, called the reference or fixed phase is connected to a constant-voltage source, \( V_m \perp -90^\circ \). The other winding, called the control phase, is supplied with a variable voltage of the same frequency as the reference phase but is phase-displaced by 90 electrical degrees. The control phase voltage is usually supplied from a servo amplifier. The direction of rotation

**Depends on the sign of the phase shift angle**
Application: Radar Position Control

Two potentiometers are used as position transducers. The reference potentiometer generates a voltage $E_{\text{ref}}$ depending on the desired position command $\theta_{\text{ref}}$. The second potentiometer coupled to the shaft of the servomotor produces a voltage $E$ proportional to the output shaft position $\theta$. The difference in the two voltages, $E_{\text{error}} (= E_{\text{ref}} - E)$, is therefore proportional to the position error $\theta_{\text{ref}} - \theta$. This error is fed to a servo amplifier, which generates the necessary voltage $V_a$ for the control phase winding of the servomotor to reduce the position error to zero.
Analysis of ac servomotors

Consider the servo system shown in Fig. 8.4. The input variable is the control phase voltage $V_a$ and the output variable is either position $\theta$ or speed $\omega_m$. Most loads are a combination of inertia $J_L$ and viscous friction $F_L$.

The torque–speed characteristics of the unbalanced two-phase motor shown in Fig. 8.2b are assumed to be linear and equally spaced for equal increments of the control phase voltage. The motor torque can be written as

$$T = K_m V_a - F_m \omega_m$$  \hspace{1cm} (8.1)$$

where $K_m$ is the motor torque constant in N · m/volt

$F_m$ is the motor viscous friction in N · m/radian/sec

Note that $F_m$ is just the slope of the torque–speed curves at constant control phase voltage $V_a$. Also, $K_m$ is the change in torque per unit change in control phase voltage at constant speed.
Analysis of ac servomotors

The equation of motion of the servomotor driving the load is

\[ T = K_m V_a - F_m \omega_m = (J_m + J_L) \frac{d\omega_m}{dt} + F_L \omega_m \] (8.2)

where \( J_L \) is the load inertia
\( J_m \) is the motor inertia

If \( \theta \) is the angular position of the load

\[ \frac{d\theta}{dt} = \omega_m \] is the speed of the system

**FIGURE 8.4** Servo system using a two-phase motor.
AC servomotor Example:

Equation 8.2 can also be written as

\[ K_m V_a - F_m \frac{d\theta}{dt} = (J_m + J_L) \frac{d^2 \theta}{dt^2} + F_L \frac{d\theta}{dt} \]  \hspace{1cm} (8.3)

Equations 8.2 and 8.3 can also be written as

\[ K_m V_a = (J_m + J_L) \frac{d\omega_m}{dt} + (F_m + F_L)\omega_m \]  \hspace{1cm} (8.4)

\[ K_m V_a = (J_m + J_L) \frac{d^2 \theta}{dt^2} + (F_m + F_L) \frac{d\theta}{dt} \]  \hspace{1cm} (8.5)

Note that the negative slope \((F_m)\) of the torque–speed characteristic of the motor corresponds to viscous friction and therefore provides damping for the system.

Taking the Laplace transforms of Eqs. 8.4 and 8.5,

\[ \frac{\omega_m(s)}{V_a(s)} = \frac{K_m/F}{1 + s\tau_m} \]  \hspace{1cm} (8.6)

\[ \frac{\theta(s)}{V_a(s)} = \frac{K_m/F}{s(1 + s\tau_m)} \]  \hspace{1cm} (8.7)

where \( F = F_L + F_m \)

\( J = J_L + J_m \)

\( \tau_m = J/F \) is the mechanical time constant of the drive system.
AC servomotor Example:

Equations 8.6 and 8.7 are shown in block diagram forms in Fig. 8.5.

**Time Response for a Step Change in Control Phase Voltage: Open-Loop Operation**

Consider a step change in the control phase voltage $V_a$, as shown in Fig. 8.6:

$$V_a(s) = \frac{V}{s}$$

From Eq. 8.6

$$\omega_m(s) = \frac{K_m/F}{1 + s\tau_m} \frac{V}{s}$$

$$= \frac{K_mV}{F} \left( \frac{1}{s} - \frac{1}{s + 1/\tau_m} \right)$$

\[ \text{FIGURE 8.5 Transfer functions.} \]
FIGURE 8.6  Step response in a two-phase servo system. (a) Step change in $V_a$. (b) Response in speed. (c) Response in position.
The corresponding time function is

$$\omega_m(t) = \frac{K_m V}{F} (1 - e^{-t/\tau_m})$$

(8.8)

The steady-state speed is

$$\omega_m(\infty) = \frac{K_m V}{F}$$

(8.9)

From Eq. 8.7

$$\theta(s) = \frac{K_m/F}{s(1 + s \tau_m)} \frac{V}{s}$$

$$= \frac{K_m V}{F s^2} - \frac{K_m V \tau_m}{F s} + \frac{K_m V \tau_m}{F (s + 1/\tau_m)}$$

The corresponding time function is

$$\theta(t) = \frac{K_m V}{F} t - \frac{K_m V \tau_m}{F} + \frac{K_m V \tau_m}{F} e^{-t/\tau_m}$$

(8.10)
AC servomotor Example: 1

EXAMPLE 8.1

A two-phase servomotor has rated voltage applied to its reference phase winding. The torque–speed characteristic of the motor with $V_a = 115 \text{ V}, 60 \text{ Hz}$ applied to its control phase winding is shown in Fig. E8.1. The moment of inertia of the motor and load is $10^{-5} \text{ kg} \cdot \text{ m}^2$, and the viscous friction of the load is negligible (Fig. 8.4).

(a) Obtain the transfer function between shaft position $\theta$ and control voltage $V_a$.

(b) Obtain an expression for the shaft position due to the application of a step voltage $V_a = 115 \text{ V}$ to the control phase winding.
AC servomotor Example: 1

Solution

(a) \[ K_m = \frac{T}{V_a} \bigg|_{\omega_m = \text{constant}} = \frac{0.2}{115} = 0.00174 \text{ N} \cdot \text{m/V} \]

\[ F_m = \frac{T}{\omega_m} \bigg|_{V_a = \text{constant}} = \frac{0.2}{3000 \times 2\pi/60} = 0.0006366 \text{ N} \cdot \text{m/rad/sec} \]

\[ F = F_m + F_L = F_m + 0 = F_m \]

\[ J = 10^{-5} \text{ kg} \cdot \text{m}^2 \]

\[ \tau_m = \frac{J}{F} = \frac{10^{-5}}{0.0006366} = 15.71 \times 10^{-3} \text{ sec} \]

\[ \frac{K_m}{F} = \frac{0.00174}{0.0006366} = 2.733 \]
AC servomotor Example: 1

From Eq. 8.7

\[
\frac{\theta(s)}{V_a(s)} = \frac{2.733}{s(1 + 0.01571s)}
\]

\[
V_a(s) = \frac{115}{s}
\]

\[
\frac{K_m V}{F} = 2.733 \times 115 = 314.3
\]

From Eq. 8.10

\[
\theta(t) = 314.3t - 4.94 + 4.94e^{-t/0.01571}
\]

\[
\approx 314.3t
\]
AC servomotor Example:

EXAMPLE 8.2

For the position control system shown in Fig. 8.7, let the potentiometer transducers give a voltage of 1 volt per radian of position. The transfer function of the servo amplifier is \( G(s) = \frac{10(1 + 0.01571s)}{(7 + s)} \). Assume that the initial angular position of the radar is zero. The transfer function between the motor control phase voltage \( V_a \) and radar position \( \theta \) is \( M(s) = \frac{2.733}{s(1 + 0.01571s)} \).

(a) Derive the transfer function of the system.

(b) For a step change in the command angle of 180° (= \( \pi \) radians), find the time response of the angular position of the antenna.
AC servomotor Example:

Solution

(a) The block diagram is shown in Fig. E8.2a. This can be simplified to the block diagram shown in Fig. E8.2b. From Fig. E8.2b

\[ \frac{\theta(s)}{\theta_{\text{ref}}(s)} = \frac{27.33/s(s + 7)}{1 + 27.33/s(s + 7)} = \frac{27.33}{s^2 + 7s + 27.33} \]

(b) \[ \frac{\theta_{\text{ref}}(s)}{\theta(s)} = \frac{\frac{27.33}{s(s + 7)}}{\theta(s)} \]

(c) \[ \frac{\frac{\theta_{\text{ref}}(s)}{\theta(s)}}{\frac{27.33}{s^2 + 7s + 27.33}} \]

(d) 

AC servomotor Example:

This equation represents a second-order system. The corresponding block diagram is shown in Fig. E8.2c.

\begin{equation}
\theta_{\text{ref}}(s) = \frac{\pi}{s}
\end{equation}

\begin{equation}
\theta(s) = \frac{27.33 \frac{\pi}{s^2 + 7s + 27.33}}{s^2 + 7s + 27.33}
\end{equation}

\begin{equation}
= \frac{27.33}{s(s^2 + 7s + 27.33)}
\end{equation}

\begin{equation}
= \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}
\end{equation}

where \( \omega_n = \sqrt{27.33} = 5.228 \text{ rad/sec} \)

\begin{equation}
\xi = \frac{7}{2\omega_n} = \frac{7}{2 \times 5.288} = 0.67
\end{equation}

The time response is

\begin{equation}
\theta(t) = \pi \left[ 1 - \frac{e^{-\xi_0 t}}{\sqrt{1 - \xi^2}} \sin(\omega_n \sqrt{1 - \xi^2} t + \cos^{-1} \xi) \right]
\end{equation}

\begin{equation}
= \pi [1 - 1.347e^{-3.5t} \sin(3.88t + 48^\circ)] \text{ radian}
\end{equation}

The position response is shown in Fig. E8.2d.
Stepper Motor

- A stepper motor rotates by specific number of degrees in response to an input electrical pulse. Typical step size are $2^0$, $2.5^0$, $5^0$, $7.5^0$, and $15^0$ for each electrical pulse.

- It is electromechanical incremental actuator that can convert digital pulses into analog output shaft motion.

- In normal situation No position sensor or feedback is required to make the output response.

- Typical applications: printers, tape and disk drives, machine tools, process control, X-Y recorders or motion, and robotics.
Stepper Motor

• Stepper motors have been built to follow signals as rapid as 1200 pulses per second with power ratings up to several horsepower

• Types of Stepper motor:
  • 1- Variable reluctance:
    a- single stack
    b- multistack
  • 2- Permanent magnet type
Variable Reluctance Stepper Motor

- **Single stack Stepper Motor**
- A basic of 4 phase – 2 pole, single stack stepper motor is shown

When the stator phases are excited with dc current in proper sequence, the resultant airgap field steps around and the rotor follows the axis of the airgap field by virtue of reluctance torque. This reluctance torque is generated because of the tendency of the ferromagnetic rotor to align itself along the direction of the resultant magnetic field.
Single stack Stepper Motor

- Figures show the mode of operation for 45° step in the clockwise direction. The windings are energized in the sequence A, A+B, B, B+C, and so forth.
- The direction of rotation can be reversed by reversing the sequence of switching the windings, that is A, A+D, D, D+C, etc.
Single stack Stepper Motor

- In order to obtain smaller step sizes, multipole rotor construction is used.
- Figure shows four phase, six pole stepper motor

When phase A is excited pole P1 is aligned with the axis of phase A, next phase A+B is excited, the resultant field axis moves in the clockwise direction by 45° and pole P2, nearest to this new resultant field axis is pulled to align with it. The motor therefore steps in the anticlockwise direction by 15°.

So, if the windings are energized in the sequence A, A+B, B, B+C, etc, the rotor rotates in step of 15° in anticlockwise direction.

Setp size = 360 / (No. Of phases X no of rotor teeth)
Multistack Stepper Motor

- They are widely used to give smaller step sizes
- The motor is divided along its axial length into magnetically isolated sections ("stacks"), and each of these sections can be excited by separate winding "phase"
- Three-phase arrangements are most common, but motors with up to seven stacks and phases are available.

Longitudinal cross section of a three-stack variable reluctance stepper motor
Multistack Stepper Motor

The stator of each stack has a number of poles. Both stator and rotor has the same number of teeth. Therefore when a particular phase is excited, the position of the rotor relative to the stator in that stack is accurately defined.

\[ \text{Stepsiz} \Delta \theta = \frac{360}{N \times \text{Rotor Teeth}} \]

When stack A is energized, the rotor and stator teeth in stack A are aligned but those in stacks B and C are not. Next if excitation is changed to B, the stator and rotor teeth in stack B are aligned, this is made possible by a rotor movement in the clockwise direction.

**FIGURE 8.20** Teeth position in a four-pole, three-stack, variable-reluctance stepper motor. (a) Phase A excited. Rotor and stator teeth are aligned. (b) Developed diagram for rotor and stator teeth for phase A excitation.
Permanent Magnet Stepper Motor

- It has a stator construction similar to that of the single–stack variable reluctance type, but the rotor is made of permanent magnet material.
- Figure shows two pole permanent magnet stepper motor.
- The rotor poles align with two stator teeth (or poles) according to the winding excitation.
- The current polarity is important in the PM stepper motor.

PM two-phase stepper motor with 90° steps. (a), (b), (c) and (d) show the positions of the magnet rotor as the coils are energised in different directions.
Hybrid Stepper Motor

Hybrid stepper motors are also commercially available in which the rotor has an axial permanent magnet at the middle and ferromagnetic teeth at the outer sections as shown in Fig. 8.22. Smaller step sizes can be obtained from these motors, but they are more expensive than the variable-reluctance-type stepper motors.
Stepper motor control: Drive circuit

When a two phase motor have 4 connecting wires for signals to generate the required signals, it termed bipolar motor

Figure 9.26  (a) Bipolar motor, (b) H-circuit

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Switching sequence for full-stepping bipolar stepper

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Half-steps for bipolar stepper
Stepper motor control: Drive circuit

When a two phase motor have 6 connecting wires for signals to generate the required signals, it is termed bipolar motor, they can be switched with just 4 transistors.

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Switching sequence for full-stepping unipolar stepper

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Switching sequence for Half-steps for unipolar stepper
Analysis of Stepper Motor Drives

FIGURE 8.24 One phase of a bipolar drive circuit.
EXAMPLE 8.3

A three-phase variable-reluctance stepper motor has the following parameters:

\[ R_w = 1 \Omega \]

\[ L_w = 30 \text{ mH}, \quad \text{average phase winding inductance} \]

\[ I = 3A, \quad \text{rated winding current} \]

Design a simple unipolar drive circuit such that the electrical time constant is 2 msec at phase turn-on and 1 msec at turnoff. The stepping rate is 300 steps per second.

Solution

The turn-on time constant

\[ \tau_{\text{on}} = \frac{L_w}{R_w + R_{\text{ext}}} \]

\[ R_w + R_{\text{ext}} = \frac{30}{2} = 15 \Omega \]

\[ R_{\text{ext}} = 15 - 1 = 14 \Omega \]
This resistance must be able to dissipate the power lost when rated current flows through the phase winding continuously, namely

\[ P_{\text{ext}} = 3^2 \times 14 = 126 \text{ W} \]

The required dc supply voltage, from Eq. 8.27, is

\[ V_s = 3 \times 15 = 45 \text{ V} \]

The turnoff time constant

\[ \tau_{\text{off}} = \frac{L_w}{R_w + R_{\text{ext}} + R_f} \]

\[ R_w + R_{\text{ext}} + R_f = \frac{30}{1} = 30 \ \Omega \]

\[ R_f = 30 - 15 = 15 \ \Omega \]

Energy stored in the phase winding at turnoff = \( \frac{1}{2} L_w I^2 \)

\[ = \frac{1}{2} \times 30 \times 10^{-3} \times 3^2 \text{ J} \]

\[ = 0.135 \text{ J} \]
This energy is dissipated in $R_f$, $R_{ext}$, and $R_w$. Since $R_f = R_{ext} + R_w (= 15 \, \Omega)$ the energy dissipated in $R_f$ is 0.0675 J.

Stepping rate = 300 steps/sec

Number of turnoffs in each phase = 100

Average power dissipated in $R_f = 100 \times 0.0675 \, W = 6.75 \, W$

When the transistor conducts, the reverse voltage across the diode $D_f$ is $V_s = 45 \, V$. The peak current of the freewheeling diode is 3A, which is the phase winding current at the instant the transistor turns off.

From Eq. 8.28,

$$V_{CE(max)} = 45 + 3 \times 15 = 90 \, V$$

Current rating of the transistor is 3 A.