Feedback Control system

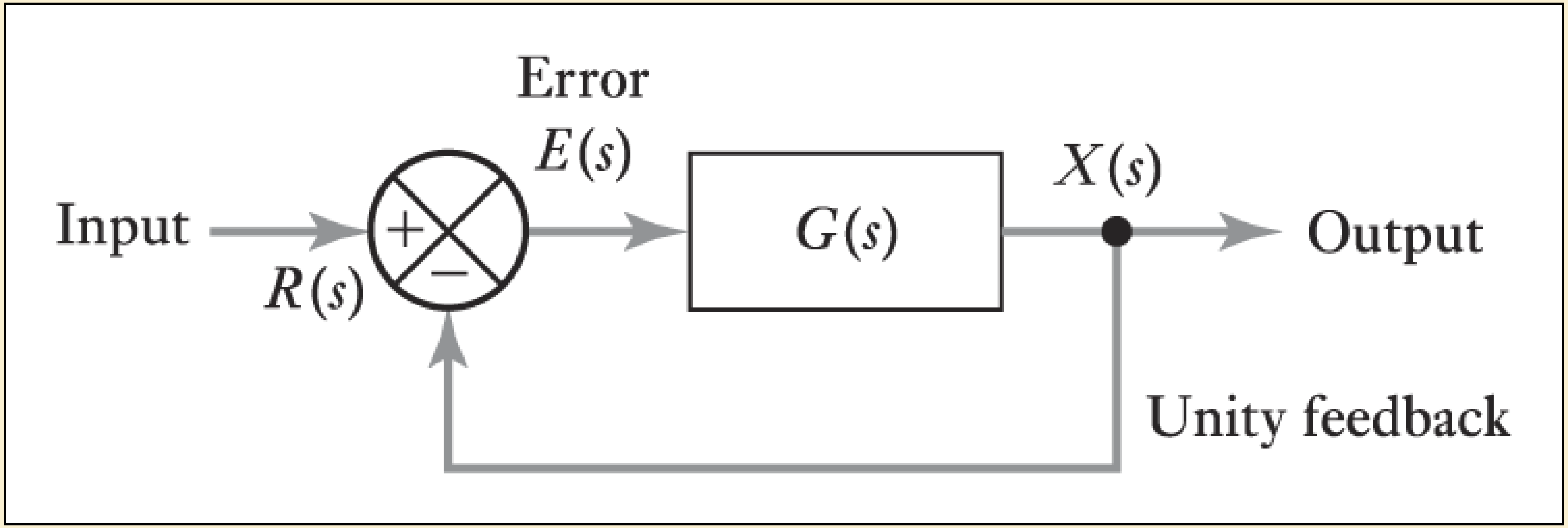


Figure 15.1 Unity feedback

Advantages of Feedback in Control

Compared to open-loop control, **feedback can be used to**:

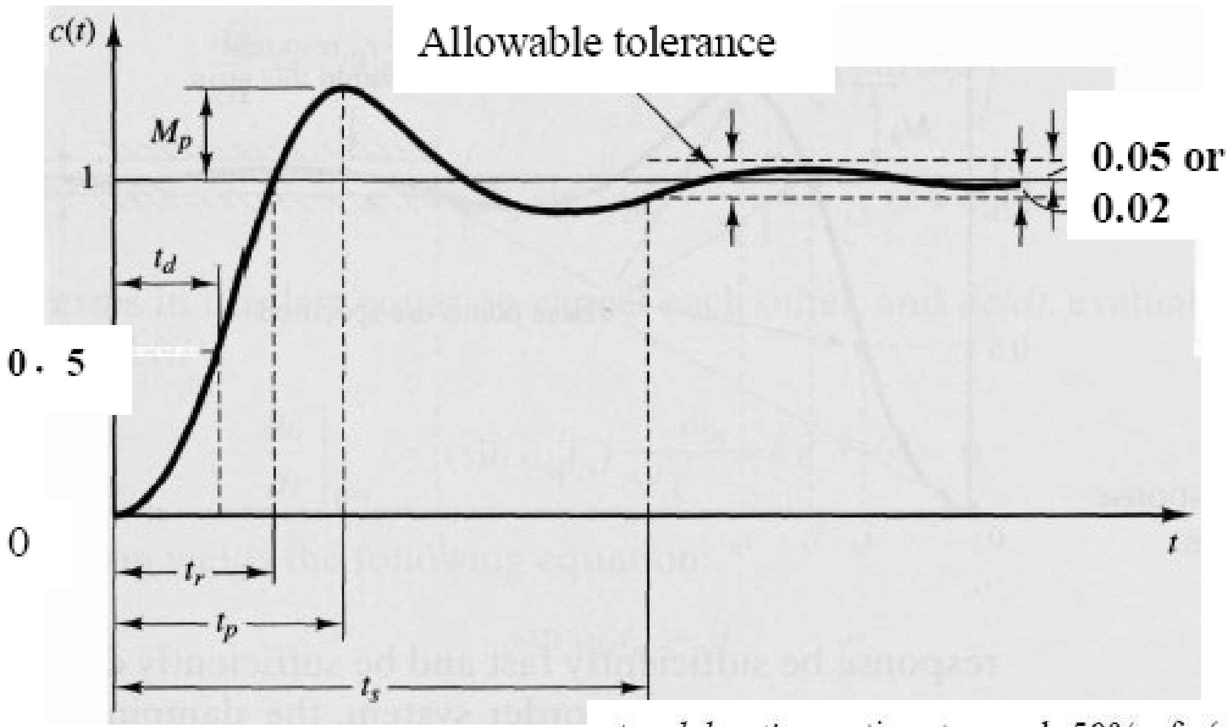
- ***Reduce the sensitivity*** of a system's transfer function ***to parameter changes***
- ***Reduce steady-state error*** in response to ***disturbances***,
- ***Reduce steady-state error*** in tracking a ***reference response*** *(& speed up the transient response)*
- ***Stabilize an unstable process***

Disadvantages of Feedback in Control

Compared to open-loop control,

- Feedback requires a ***sensor*** that ***can be very expensive and may introduce additional noise***
- Feedback systems are often ***more difficult to design and operate*** than open-loop systems
- Feedback changes the dynamic response (faster) but often makes the system ***less stable***.

Unit step response of a 2nd order under-damped system



t_d *delay time*: time to reach 50% of c(∞) for the first time.

t_r *rise time* : time to rise from 0 to 100% of c(∞).

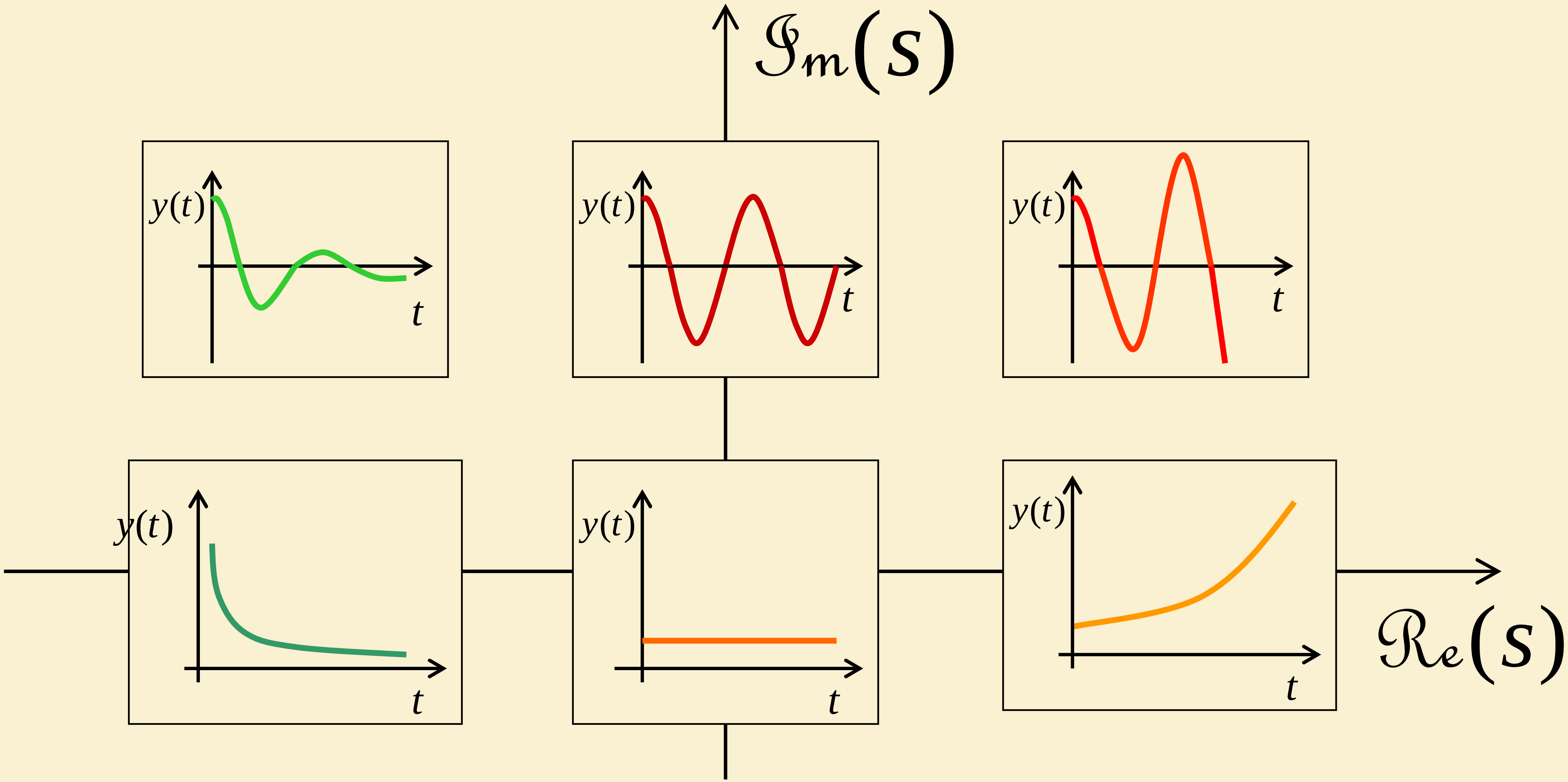
t_p *peak time* : time required to reach the first peak.

M_p *maximum overshoot* : $\frac{c(t_p)-c(\infty)}{c(\infty)} \cdot 100\%$

t_s *settling time* : time to reach and stay within a 2% (or 5%) tolerance of the final value c(∞).

$$t_r \cong \frac{1.8}{\omega_0},\; t_p \cong \frac{\pi}{\omega_0},\; t_s \cong \frac{4.6}{\zeta\omega_0},\; M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}},$$

Transient Response vs Simple Pole Locations



Controlled system

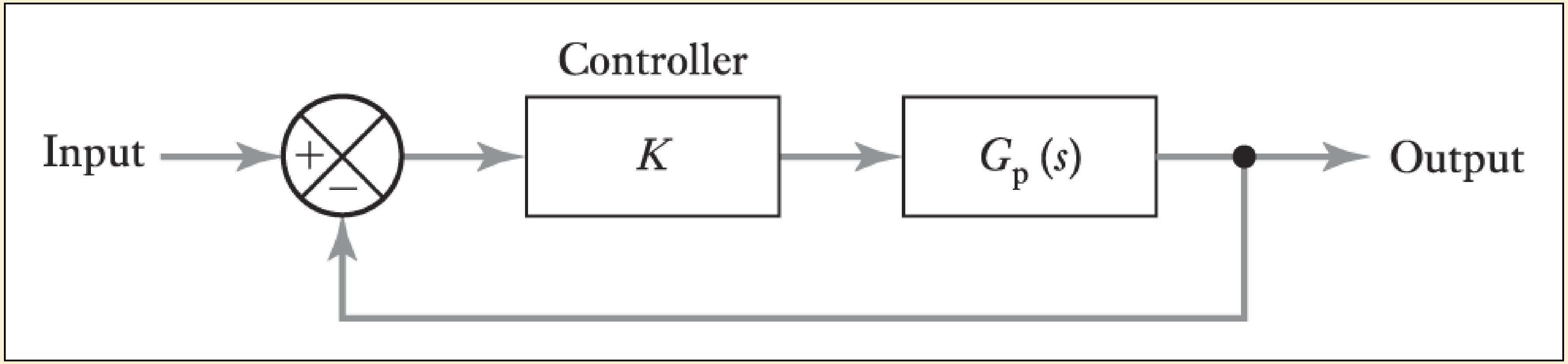


Figure 15.15 Control system

Control Modes

- There are a number of ways by which a control unit can be react to an error signal and supply an output for correcting elements:
- 1- **two step mode**: the controller is an ON-OFF correcting signal
- 2- **The proportional Mode**: the controller produce a control action proportional to the error.
- 3: **the derivative mode D**: the controller produces a control action proportional to the rate at which the error is changing. When there is a sudden change in the error signal, the controller gives alarge correcting signal, when there is a gradual change only a small correcting signal is produces. Normally used in conjunction with proportional control

Control Modes

- 4- The integral mode: the control action proportional to the integral of the error thus a constant error signal e=constant produces an increasing correcting signal
- 5- Combination of Modes PD, PI, PID
- A controller can achieve these modes by means of pneumatic circuit, analogue electronic circuits involving opamp or by the programming of a microprocessor or computer system

Two step mode

An example of two step mode of control id the bimetallic thermostat that might be used with a simple temperature control system.

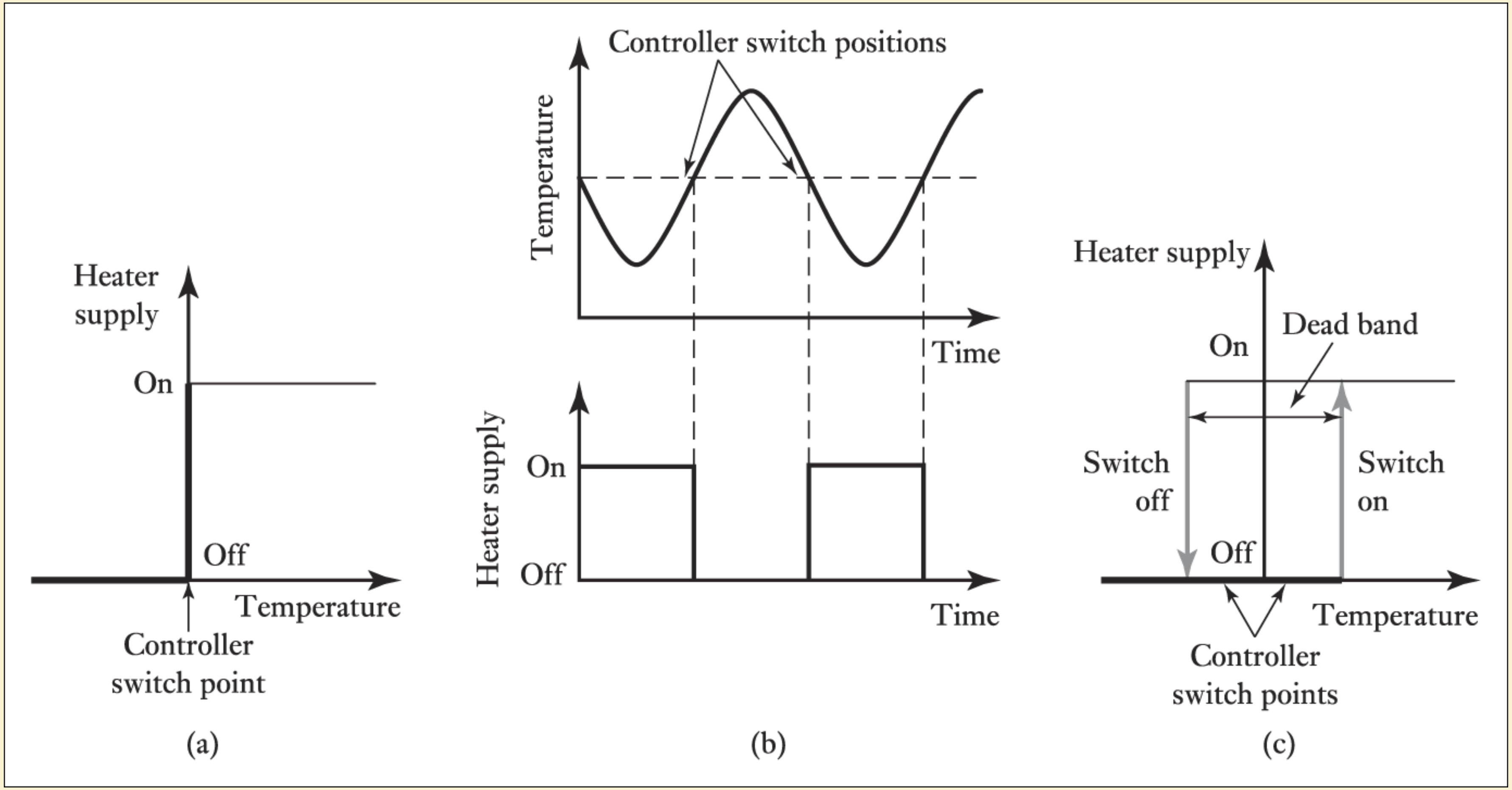
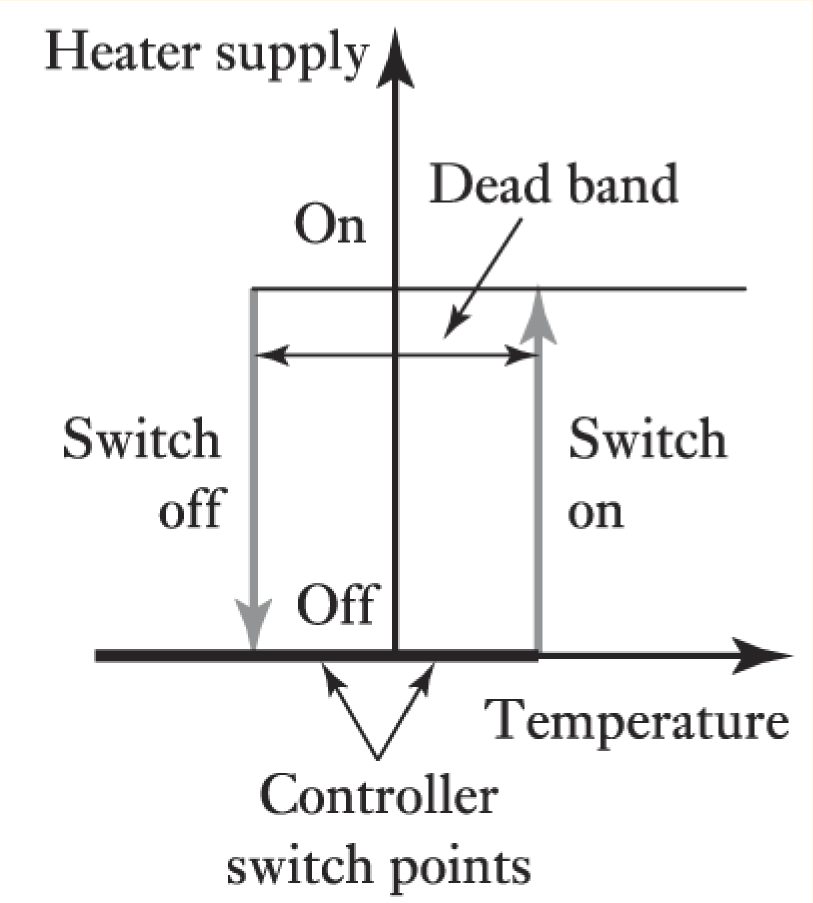


Figure 15.2 Two-step control

Two step mode

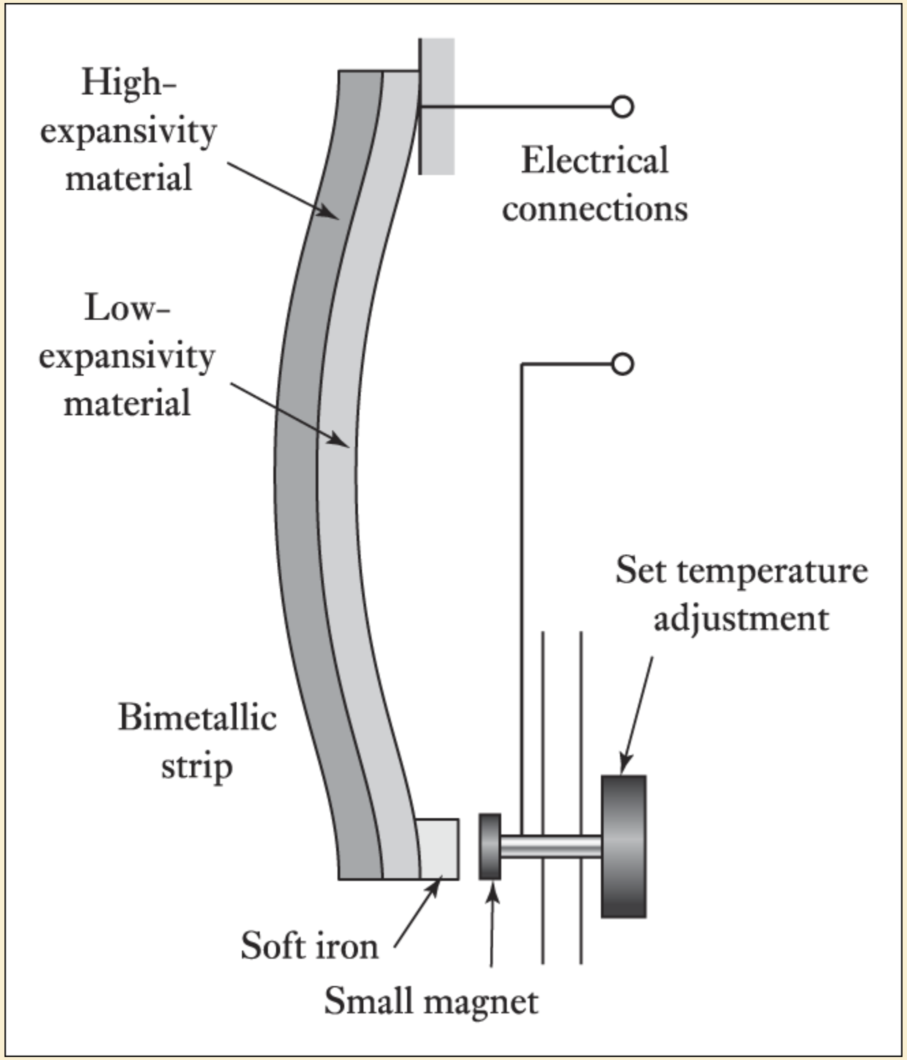
- **Consequences:**
- To avoid continuous switching ON and OFF on respond to slight change, two values are normally used, a dead band is used for the values between the ON and OFF values
- Large dead band implies large temp. fluctuations
- Small dead band increase switching frequency



Two step mode

- The bimetallic element has permanent magnet for a switch contact, this has the effect of producing a dead band.

Fast ON-OFF control can be used for motor control using controlled switching elements (IGBT, BJT, MOSFET, Thyrester)



Proportional Mode

- With the two step method of control, the controller output is either ON or OFF signal, regardless of the magnitude of error.
- With proportional control mode, the size of the controller output is proportional to the size of the error. The bigger the error the bigger the output from the controller
- Controller output $u(t) = K_p e$
- Or in s domain $U(s)=K_pE(s)$

Electronic Proportional Mode

A summing opamp with an inverter can be used as proportional controller

The output from the summing amplifier is

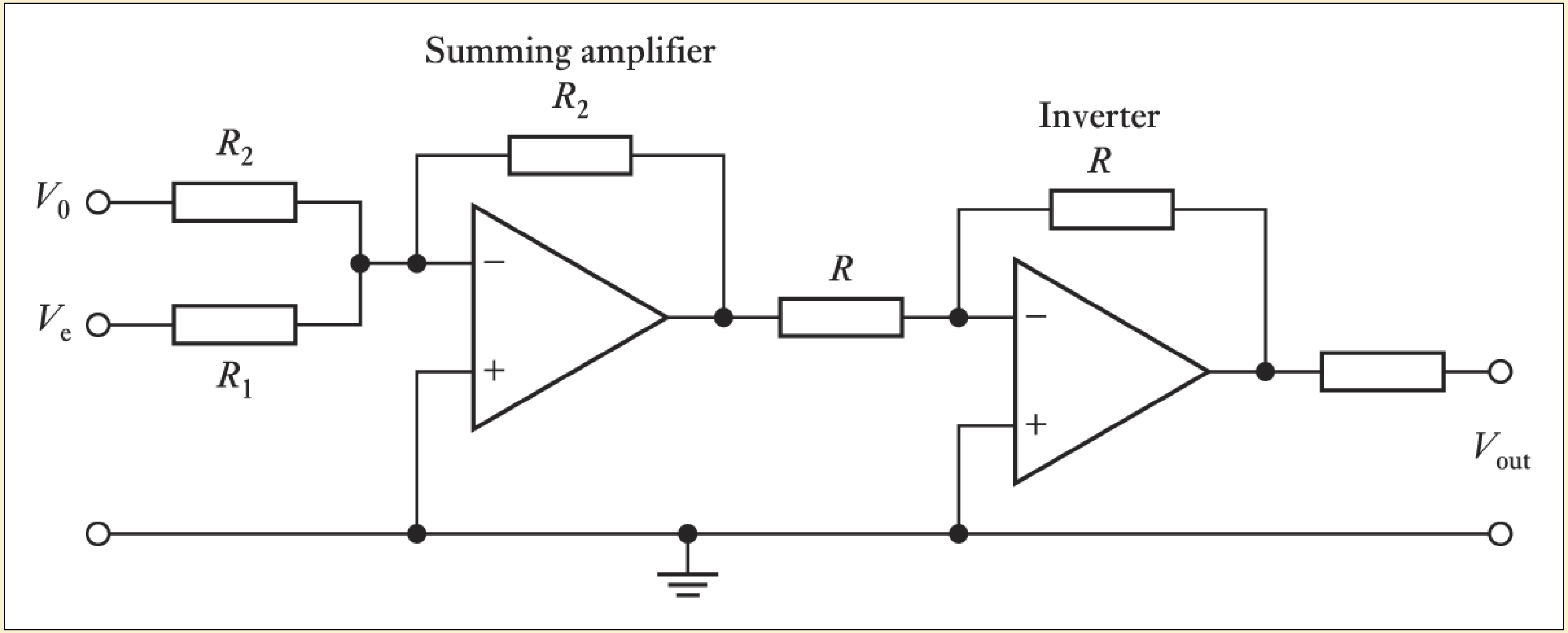


Figure 15.3 Proportional controller

Electronic Proportional Mode

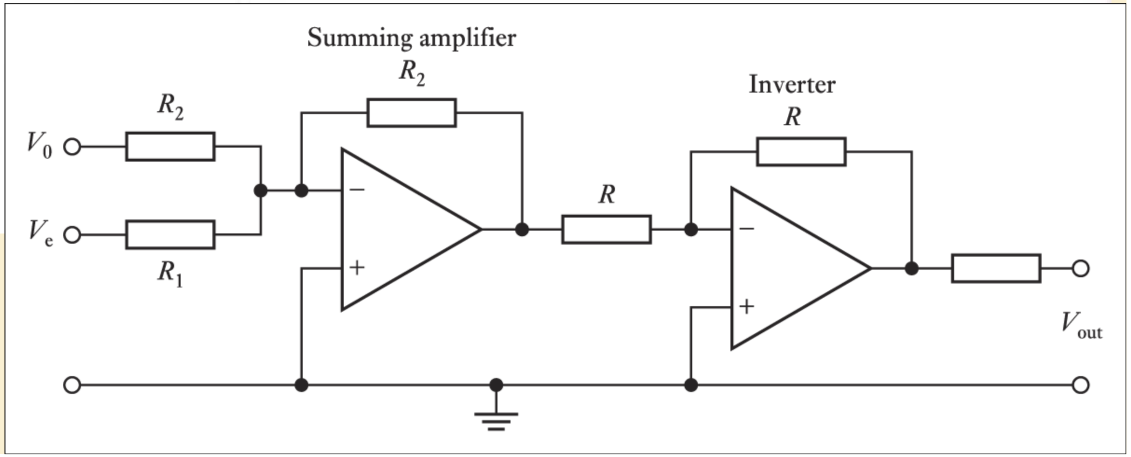
The input to the summing amplifier through R_2 is the zero error voltage value V_0, i.e. the set value, and the input through R_1 is the error signal V_e. But when the feedback resistor $R_f = R_2$, then the equation becomes

$$V_{out} = -\frac{R_2}{R_1}V_e - V_0$$

If the output from the summing amplifier is then passed through an inverter, i.e. an operational amplifier with a feedback resistance equal to the input resistance, then

$$V_{out} = \frac{R_2}{R_1}V_e + V_0$$

$$V_{out} = K_P V_e + V_0$$



Electronic Proportional Mode

where K_P is the proportionality constant. The result is a proportional controller.

As an illustration, Figure 15.4 shows an example of a proportional control system for the control of the temperature of a liquid in a container as liquid is pumped through it.

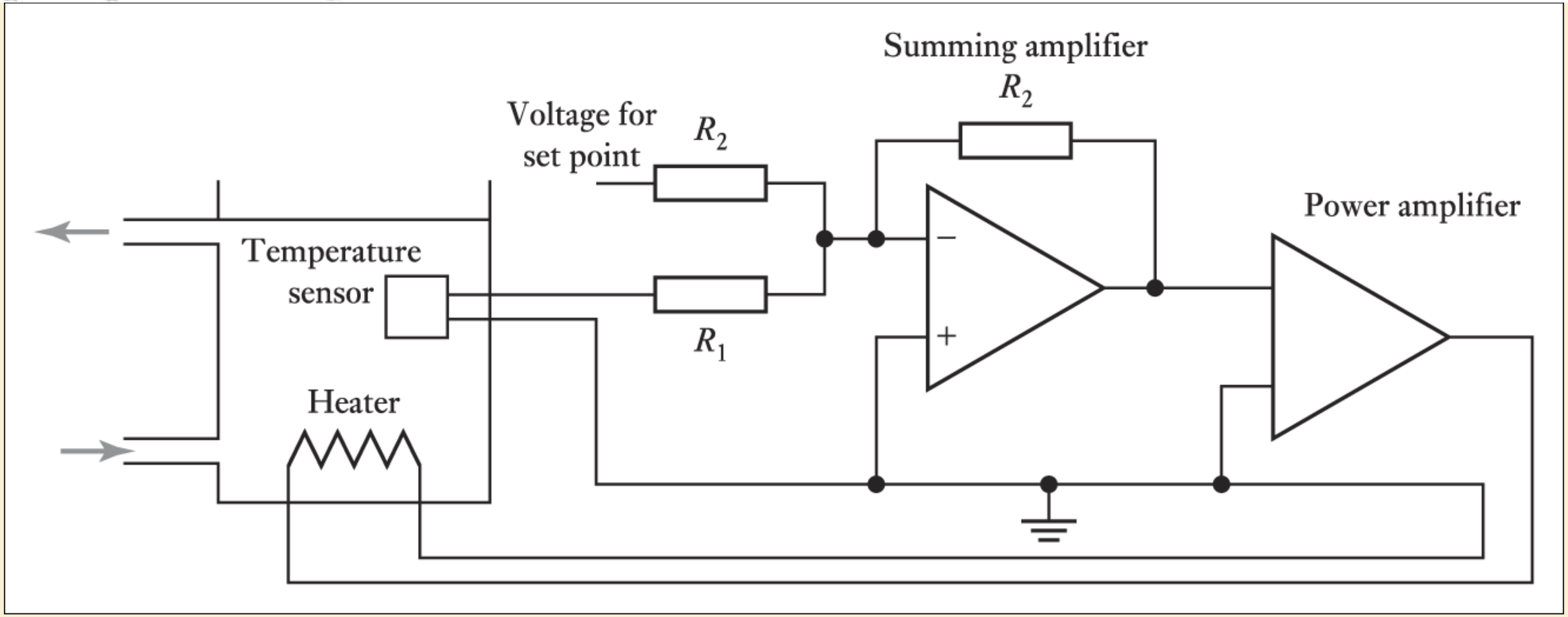
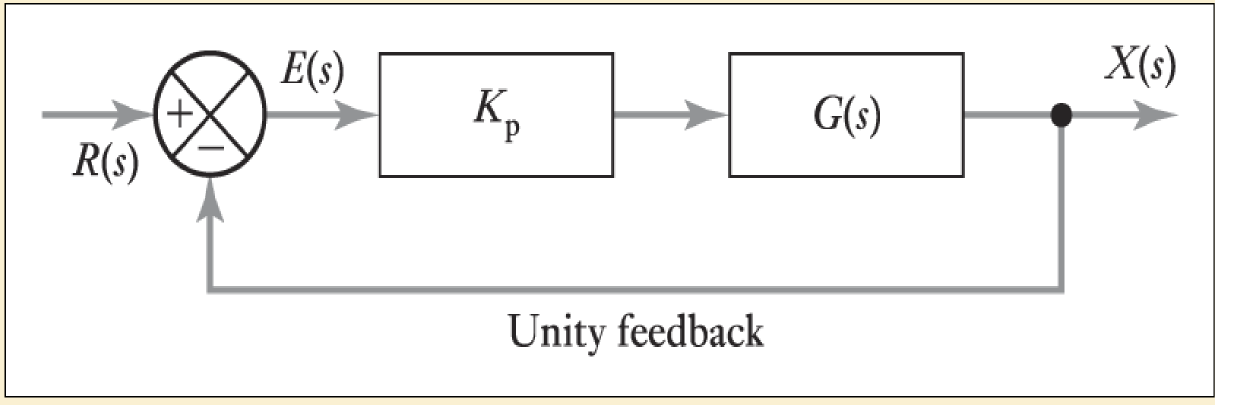


Figure 15.4 Proportional controller for temperature control

System response to proportional control

With proportional control we have a gain element with transfer function K_P in series with the forward-path element $G(s)$ (Figure 15.5). The error is thus

$$E(s) = \frac{K_P G(s)}{1 + K_P G(s)} R(s)$$



and so, for a step input, the steady-state error is

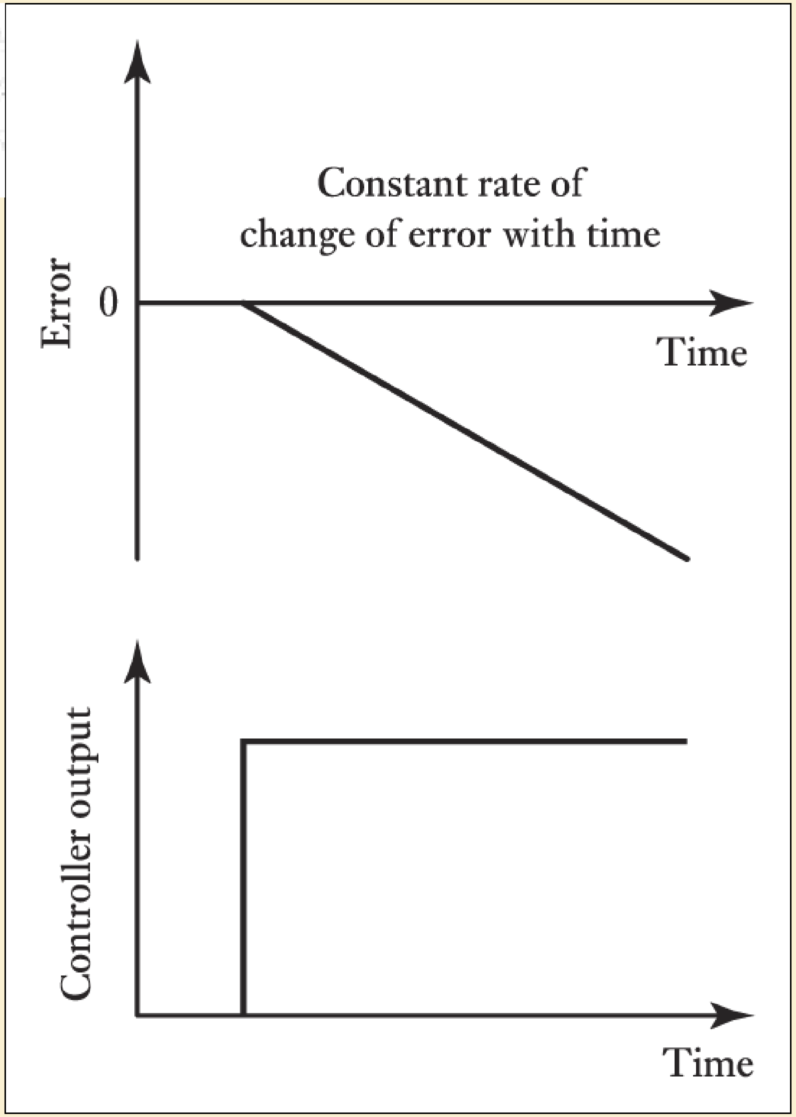
$$e_{SS} = \lim_{s \to 0} sE(s) = \lim_{s \to 0}\left[s \frac{1}{1 + 1/K_P G(s)} \frac{1}{s} \right]$$

This will have a finite value and so there is always a steady-state error. Low values of K_P give large steady-state errors but stable responses. High values of K_P give smaller steady-state errors but a greater tendency to instability.

Derivative control mode

$$\text{controller output} = K_D \frac{de}{dt}$$

$$\text{controller output}(s) = K_D s E(s)$$



As soon as the error signal begins to change, there can be quite a large controller output, thus rapid initial response to error signal occur

In Figure the controller output is constant is constant because the rate of change is constant.

Derivative control mode

- - They do not response to steady- state error signal, that is why always combined with P-controller
- P responds to all error
- D responds to rate of change
- Derivative action can also be a problem if the measurement of the process variable gives a noisy signal, the rapid fluctuations of the noise resulting in outputs which will be seen by the controller as rapid changes in error and so give rise to significant outputs from the controller.

Derivative control mode

The Figure shows the form of an electronic derivative controller circuit, it composed of two operational amplifier: integrator + Inverter

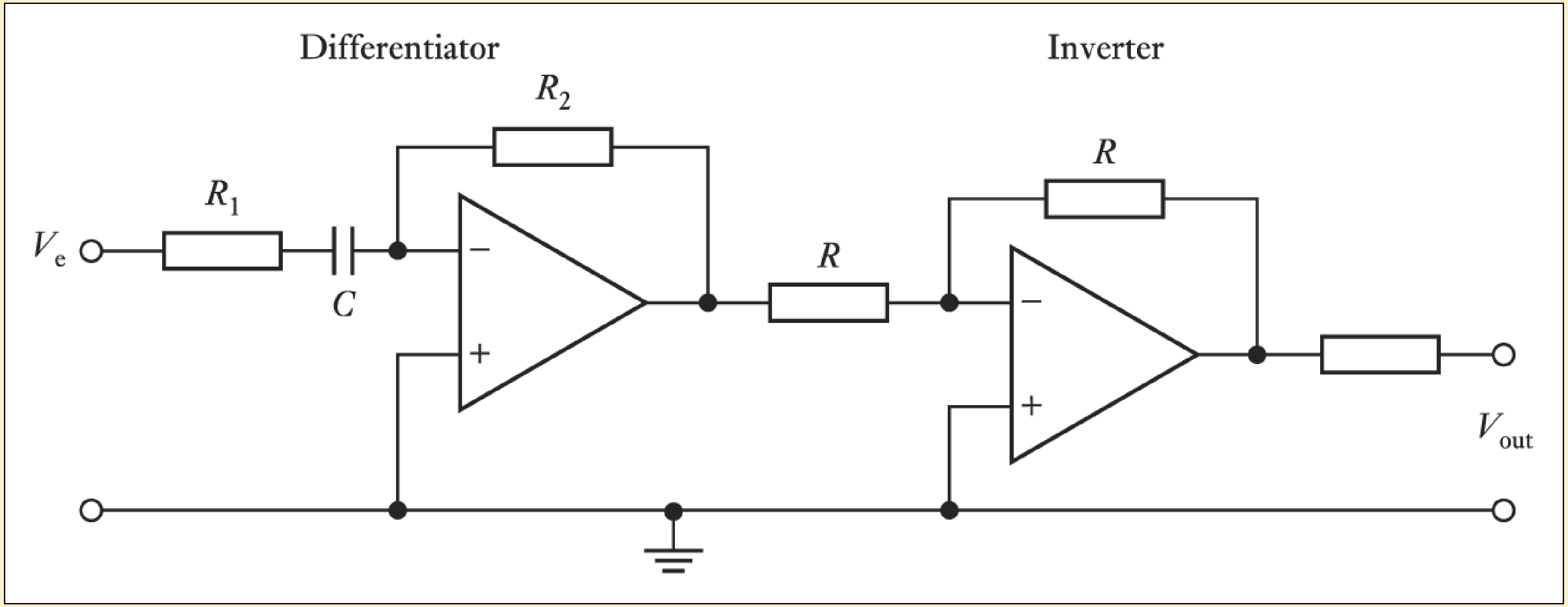


Figure 15.7 Derivative controller

PD controller

With proportional plus derivative control the controller output is given by

$$\text{controller output} = K_\mathrm{P}e + K_\mathrm{D}\frac{\mathrm{d}e}{\mathrm{d}t}$$

K_P is the proportionality constant and K_D the derivative constant, $\mathrm{d}e/\mathrm{d}t$ is the rate of change of error. The system has a transfer function given by

$$\text{controller output}\,(s) = K_\mathrm{P}E(s) + K_\mathrm{D}sE(s)$$

Hence the transfer function is $K_\mathrm{P} + K_\mathrm{D}s$. This is often written as

$$\text{transfer function} = K_\mathrm{D}\left(s + \frac{1}{T_\mathrm{D}}\right)$$

where $T_\mathrm{D} = K_\mathrm{D}/K_\mathrm{P}$ and is called the **derivative time constant.**

PD controller

Figure 15.8 shows how the controller output can vary when there is a constantly changing error. There is an initial quick change in controller output

due to the derivative followed by the gradual change due to proportional action.

This form of control thus deal with fast process changes

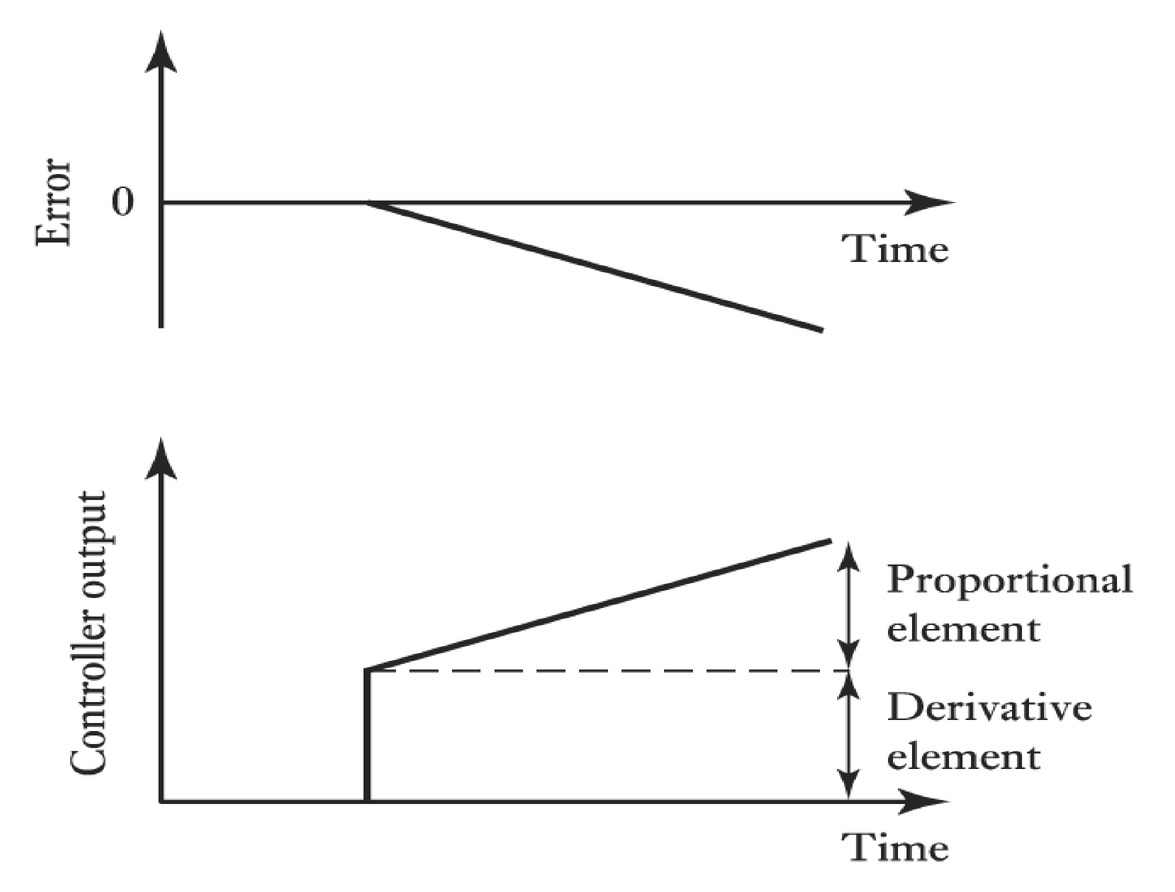


Figure 15.8 PD control

Integral control

The **integral mode** of control is one where the rate of change of the control output I is proportional to the input error signal e:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = K_{\mathrm{I}}e$$

K_{I} is the constant of proportionality and has units of $1/s$. Integrating the above equation gives

$$\int_{I_0}^{I_{\mathrm{out}}} \mathrm{d}I = \int_0^t K_{\mathrm{I}}e\,\mathrm{d}t$$

$$I_{\mathrm{out}} - I_0 = \int_0^t K_{\mathrm{I}}e\,\mathrm{d}t$$

$$(I_{\mathrm{out}} - I_0)(s) = \frac{1}{s}K_{\mathrm{I}}E(s)$$

and so

$$\text{transfer function} = \frac{1}{s}K_{\mathrm{I}}$$

I_0 is the controller output at zero time, I_{out} is the output at time t.

Integral control

Figure shows the action of an integral controller when there is a constant error input to the controller

When the controller output is constant, the error is zero; when controller output is varies at a constant rate, the error has a constant value.

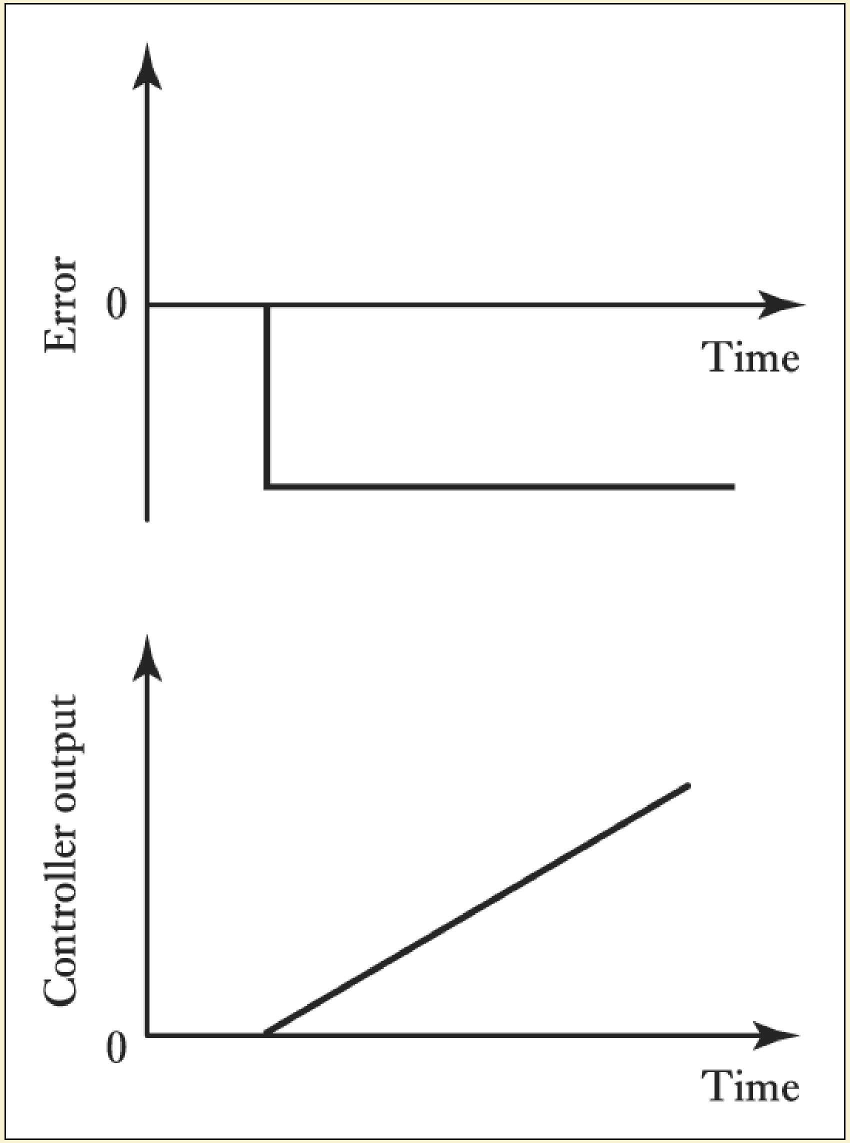


Figure 15.9 Integral control

Integral control

The integrator is connected to the error signal at time t, while the second integrator is connected to the error at t=0- i.e. (t-Ts)

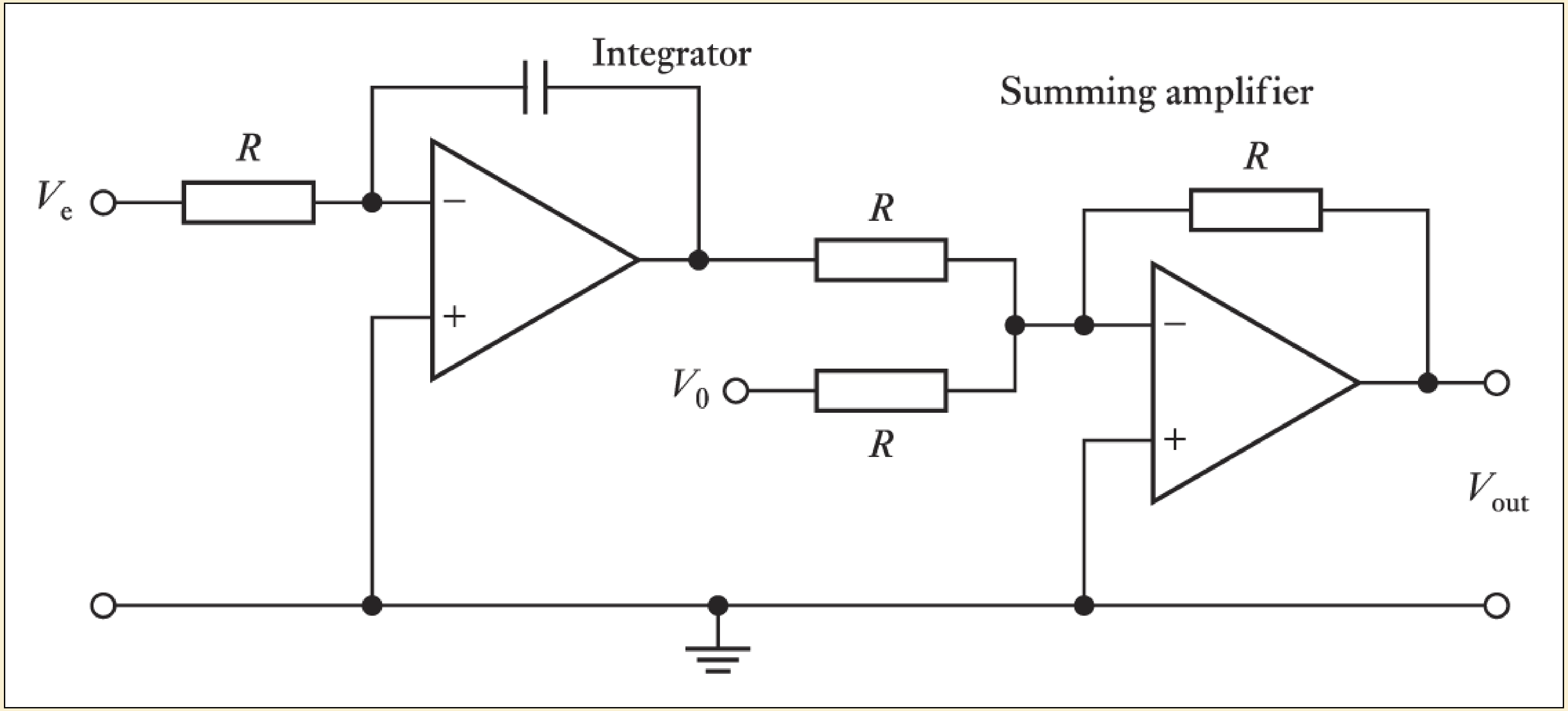


Figure 15.10 Electronic Integral controller

PI control

The integral mode of control is not usually used alone but is frequently used in conjunction with the proportional mode. When integral action is added to a proportional control system the controller output is given by

$$\text{controller output} = K_{\mathrm{P}}e + K_{\mathrm{I}}\int e\,\mathrm{d}t$$

where K_{P} is the proportional control constant, K_{I} the integral control constant and e the error e. The transfer function is thus

$$\text{transfer function} = K_{\mathrm{P}} + \frac{K_{\mathrm{I}}}{s} = \frac{K_{\mathrm{P}}}{s}\left(s + \frac{1}{T_{\mathrm{I}}}\right)$$

where $T_{\mathrm{I}} = K_{\mathrm{P}}/K_{\mathrm{I}}$ and is the **integral time constant.**

Figure 15.11(a) shows how the system reacts when there is an abrupt change to a constant error. The error gives rise to a proportional controller

PI control

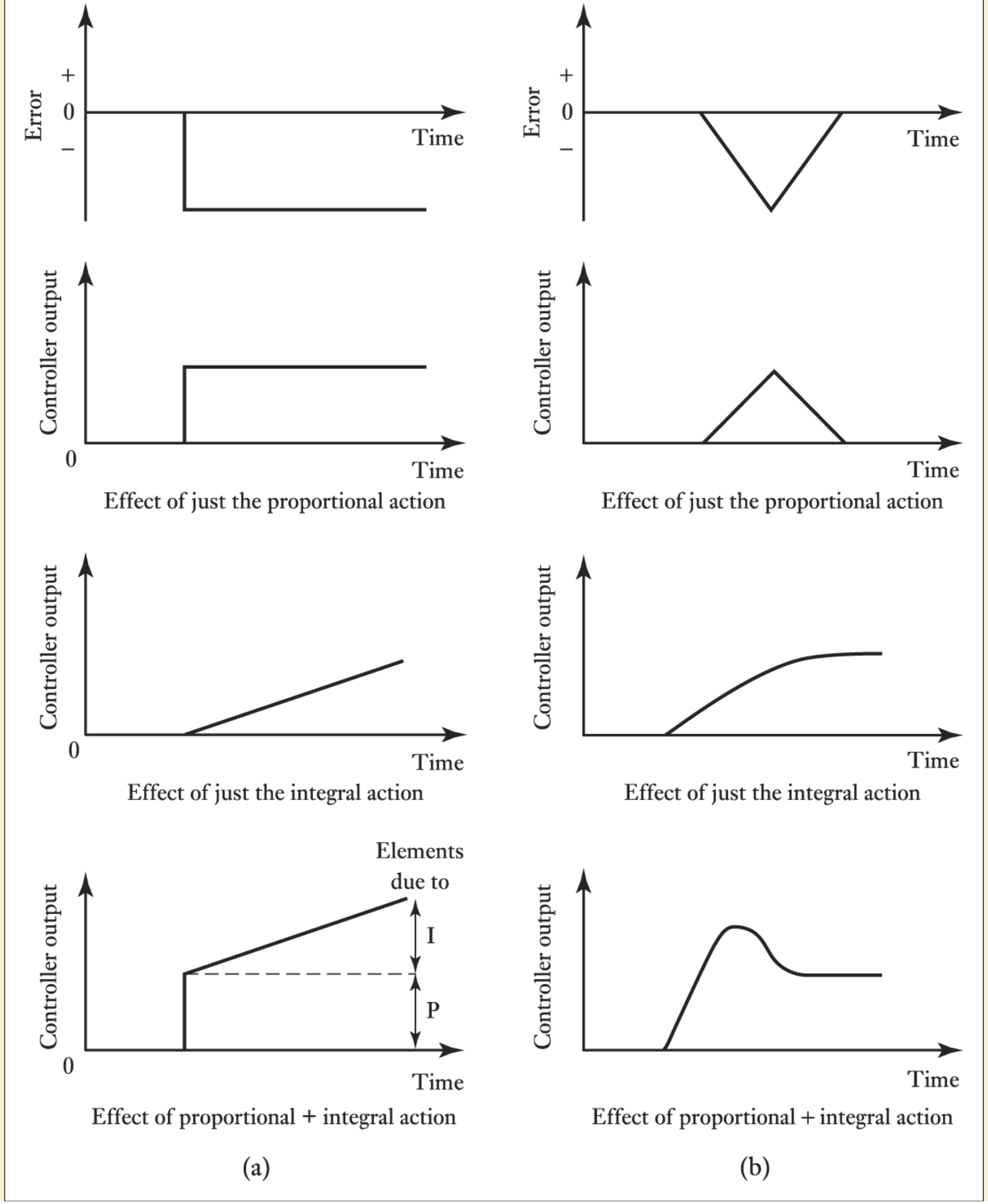


Figure 15.11 PI control

PID Controller

Combining all three modes of control (proportional, integral and derivative) gives a controller known as a **three-mode controller** or **PID controller**. The equation describing its action can be written as

$$\text{controller output} = K_{\mathrm{P}}e + K_{\mathrm{I}}\int e\,\mathrm{d}t + K_{\mathrm{D}}\frac{\mathrm{d}e}{\mathrm{d}t}$$

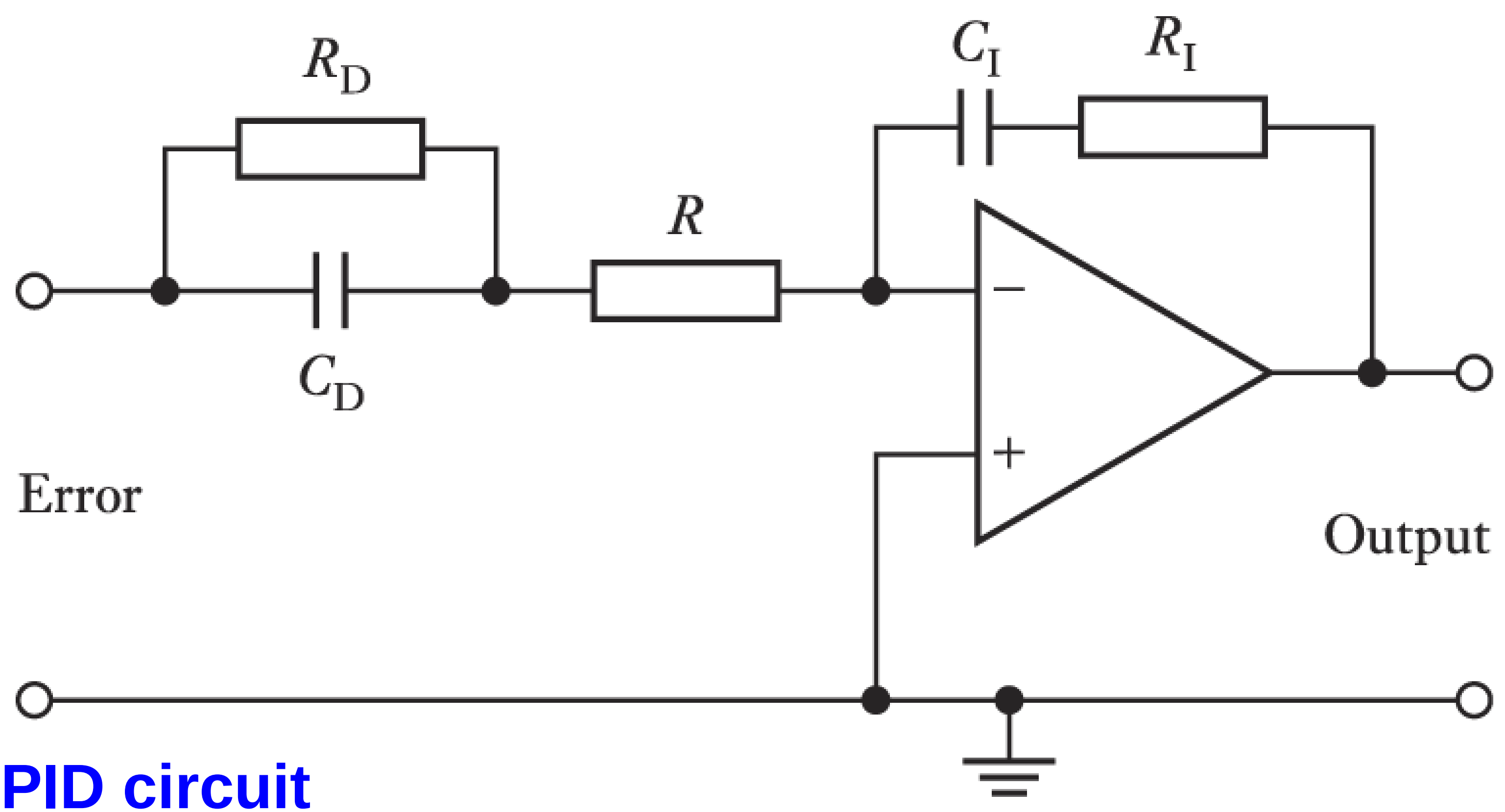
where K_{P} is the proportionality constant, K_{I} the integral constant and K_{D} the derivative constant. Taking the Laplace transform gives

$$\text{controller output }(s) = K_{\mathrm{P}}E(s) + \frac{1}{s}K_{\mathrm{I}}E(s) + sK_{\mathrm{D}}(s)$$

and so

$$\text{transfer function} = K_{\mathrm{P}}e + \frac{1}{s}K_{\mathrm{I}} + sK_{\mathrm{D}} = K_{\mathrm{P}}\left(1 + \frac{1}{T_{\mathrm{I}}s} + T_{\mathrm{D}}s\right)$$

A three-mode controller can be produced by combining the various circuits described earlier in this chapter for the separate proportional, derivative and integral modes. A more practical controller can, however, be produced with a single operational amplifier. Figure 15.12 shows one such circuit. The proportional constant K_P is $R_I/(R + R_D)$, the derivative constant K_D is $R_D C_D$ and the integral constant K_I is $1/R_I C_I$.



PID circuit

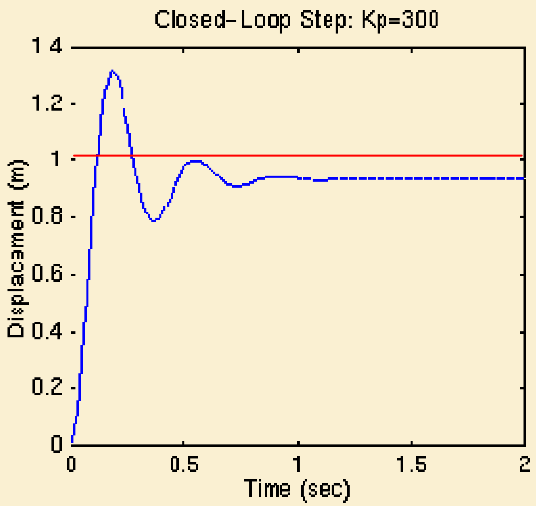
PID & Closed-loop Response

	Rise time	Maximum overshoot	Settling time	Steady-state error
P	**Decrease**	Increase	Small change	**Decrease**
I	**Decrease**	Increase	Increase	**Eliminate**
D	Small change	**Decrease**	**Decrease**	Small change

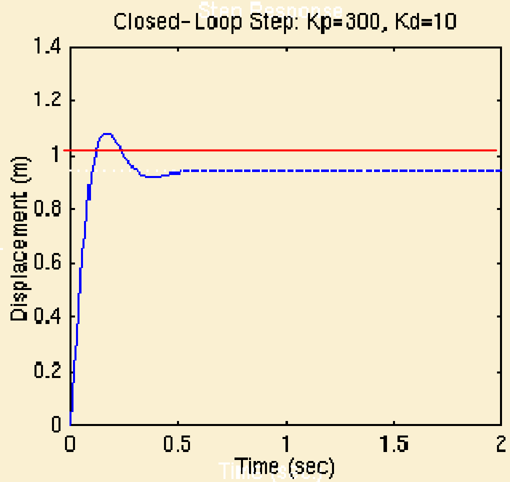
- Note that these correlations may not be exactly accurate, because P, I and D gains are dependent of each other.

PID Response

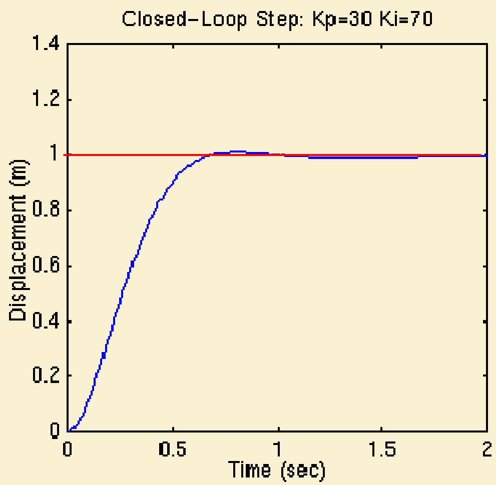
P



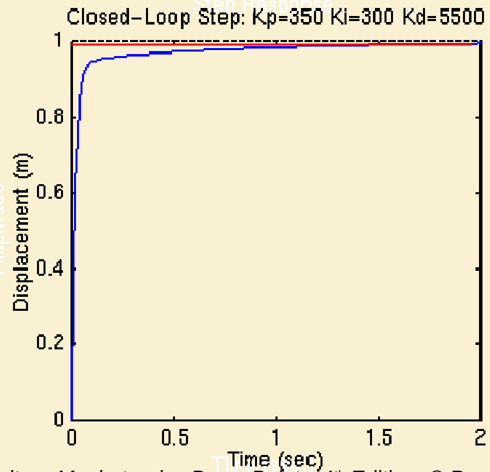
PD



PI



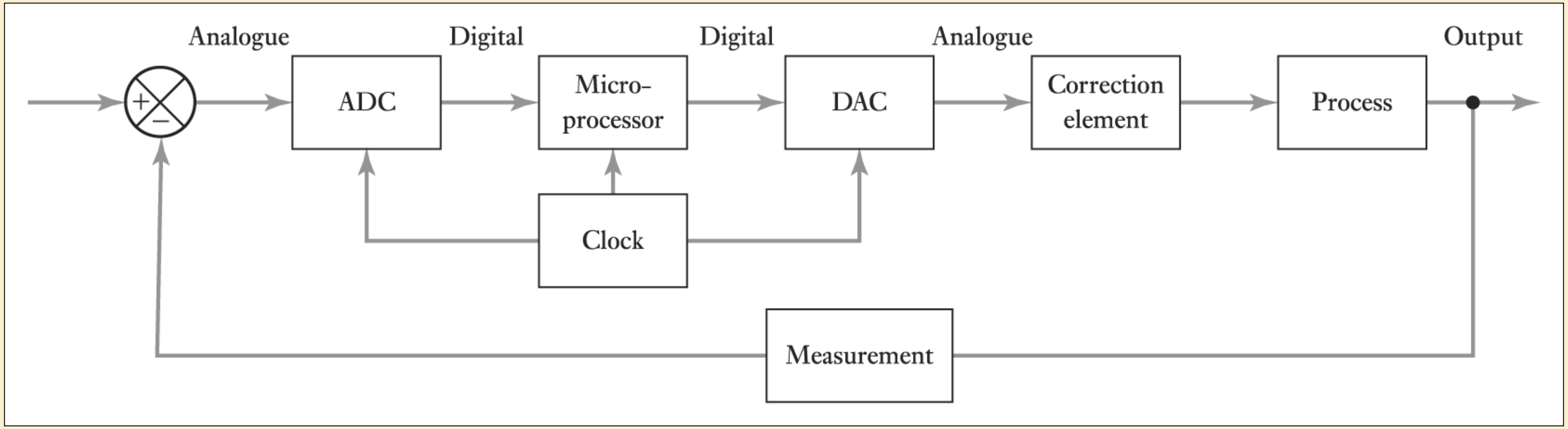
PID



PID Conclusions

- **Increasing the proportional feedback gain** ***reduces steady-state errors***, but high gains almost always ***destabilize the system.***
- **Integral control** provides ***robust reduction in steady-state errors***, but often ***makes the system less stable.***
- **Derivative control** usually ***increases damping and improves stability,*** but has almost ***no effect on the steady state error***
- These ***3 kinds of control combined*** from the **classical PID controller**

Digital closed-loop control system



A digital controller basically operates through the following cycle

1- samples the measured value

2- compares it with the set value and establishes the error

3- carries out calculation based on the error value and store values of previous inputs and outputs to obtain the output signal

4- sends the output signal to the DAC.

5- Waits until the next samples time before repeating the cycle.

Digital PID Controller

The transfer function for a PID analogue controller is

$$\text{transfer function} = K_P + \frac{1}{s}K_I + sK_D$$

$$x_n = K_P e_n + K_I\left(\frac{(e_n + e_{n-1})T_s}{2} + \text{Int}_{\text{prev}}\right) + K_D\frac{e_n - e_{n-1}}{T_s}$$

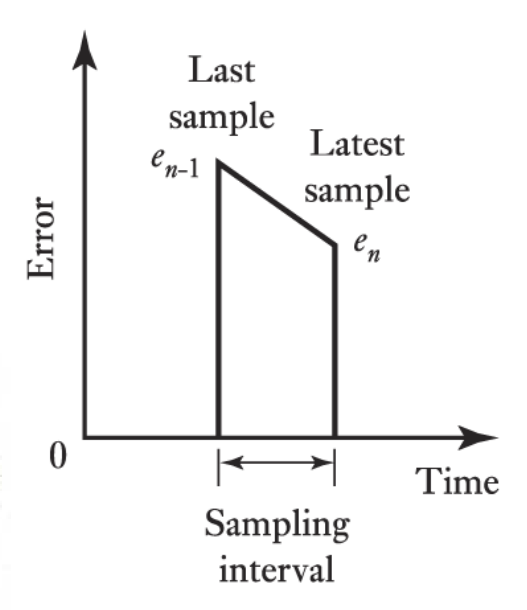
We can rearrange this equation to give

$$x_n = Ae_n + Be_{n-1} + C(\text{Int}_{\text{prev}})$$

where $A = K_P + 0.5K_IT_s + K_D/T_s$, $B = 0.5K_IT_s - K_D/T_s$ and $C = K_I$.

The program for PID control thus becomes:

1 Set the values of K_P, K_I and K_D.
2 Set the initial values of e_{n-1}, Int_{prev} and the sample time T_s.
3 Reset the sample interval timer.
4 Input the error e_n.
5 Calculate y_n using the above equation.
6 Update, ready for the next calculation, the value of the previous area to $\text{Int}_{\text{prev}} + 0.5(e_n + e_{n-1})T_s$.
7 Update, ready for the next calculation, the value of the error by setting e_{n-1} equal to e_n.
8 Wait for the sampling interval to elapse.
9 Go to step 3 and repeat the loop.



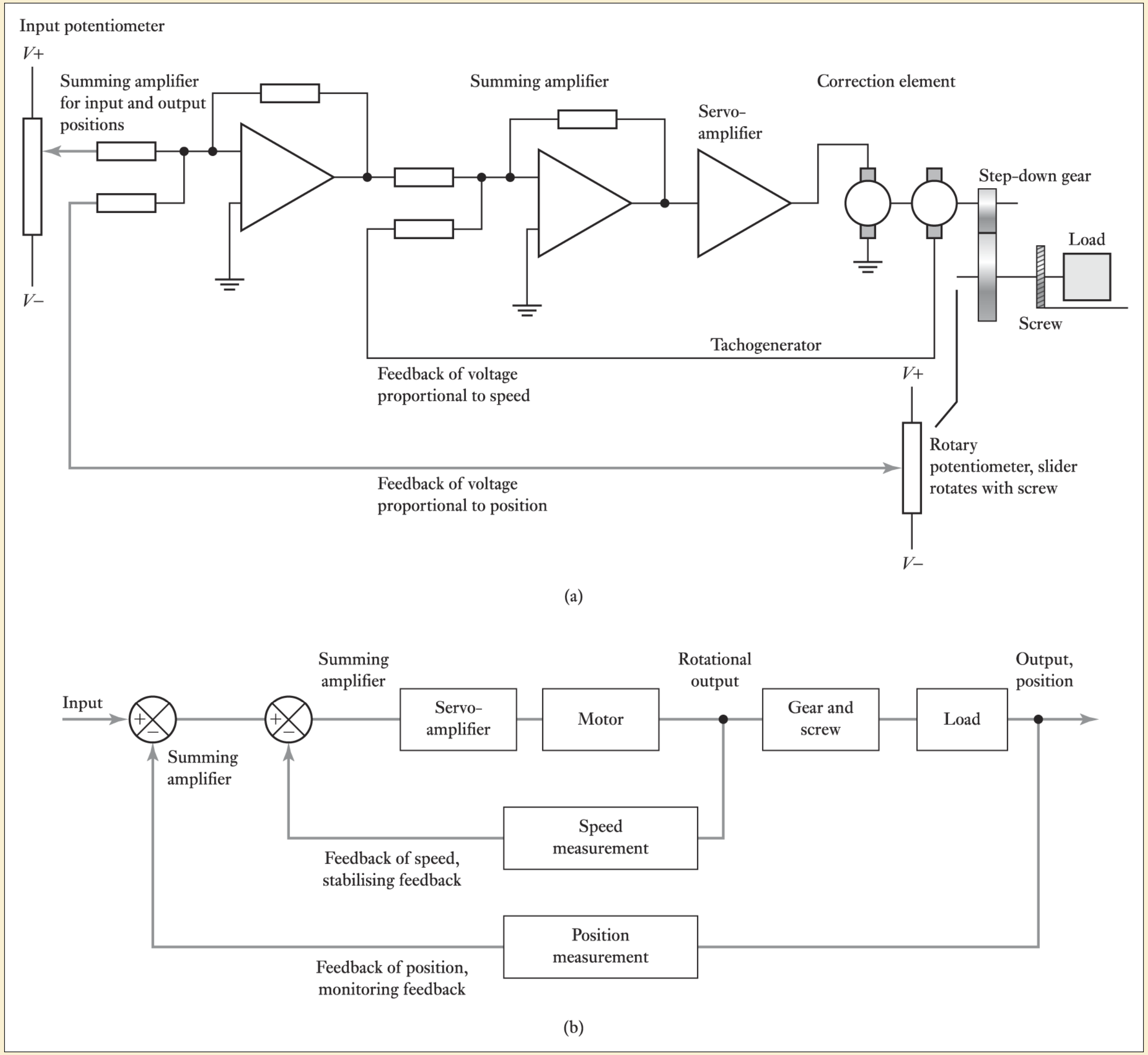
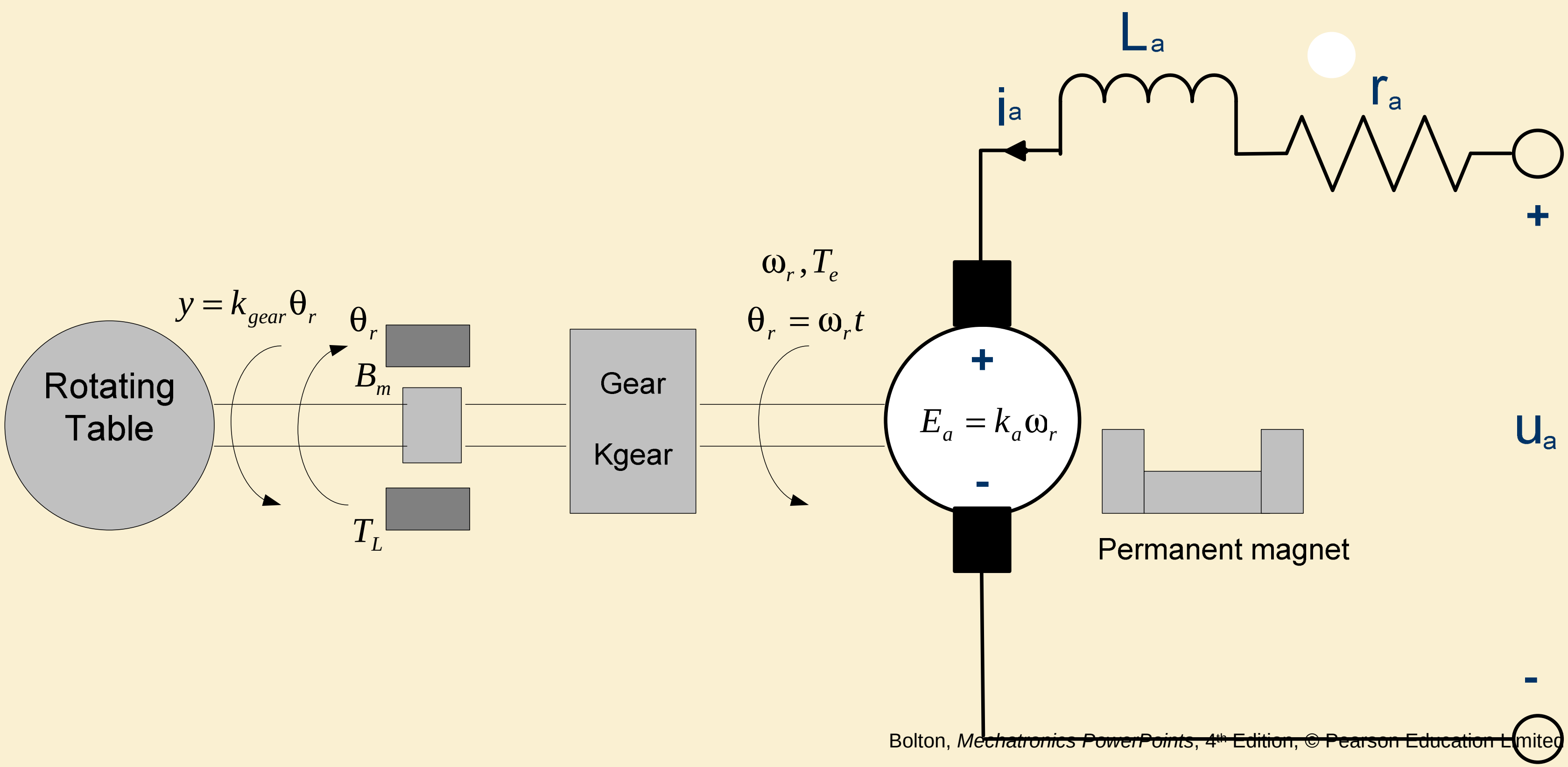


Figure 15.18 System with velocity feedback: (a) descriptive diagram of the system, (b) block diagram of the system

Analog control of permanent magnet DC Motor

- Permanent magnet DC Motor are widely used in servo systems.
- Figure: Schematic diagram of rotating table actuated by permanent magnet motor



Analog control of permanent magnet DC Motor

- The angular displacement of the positioning table = the output equation

$$y(t) = k_{gear}\theta_r$$

- To control the motor angular velocity w_r, as well as angular displacement θ_r, and rotating table $k_{gear}\ \theta_r$, one regulates the armature voltage applied to the motor winding u_a.
- To guarantee the stability, to attain the desired accuracy, to ensure tracking and disturbance attenuation of the servo system, one should design the control algorithm, and the coefficient of PID controller must be found.
- Find the transfer function. Obtained using differential equations that describe the system dynamics.

- Induced emf : $E_a = k_a \omega_r$

k_a : Back emf constant

ω_r : angular velocity

Analog control of permanent magnet DC Motor

- Using KVL: $$\frac{di}{dt} = -\frac{r_a}{L_a} i_a - \frac{k_a}{L_a} \omega_r + \frac{1}{L_a} u_a$$
- Applied Newtonian mechanics to find the differential equations for mechanical systems.
- Using Newton's second law: $$\sum \vec{T} = J\vec{\alpha} = J \frac{d\vec{\omega}}{dt}$$ J : equivalent moment of inertia
- Electromagnetic torque developed by permanent magnet DC motor: $$T_e = k_a i_a$$ k_a : Torque constant = Back emf constant
- Viscous torque : $$T_{viscous} = B_m \omega_r$$
- Load torque : T_L

Analog control of permanent magnet DC Motor

- Using Newton's second law :

$$\frac{d\omega_r}{dt} = \frac{1}{J}\left(T_e - T_{viscous} - T_L\right) = \frac{1}{J}\left(k_a i_a - B_m \omega_r - T_L\right)$$

- Dynamics of rotor angular displacement :

$$\frac{d\theta_r}{dt}\omega_r$$

- The derived three first order differential equations are rewritten in the s-domain

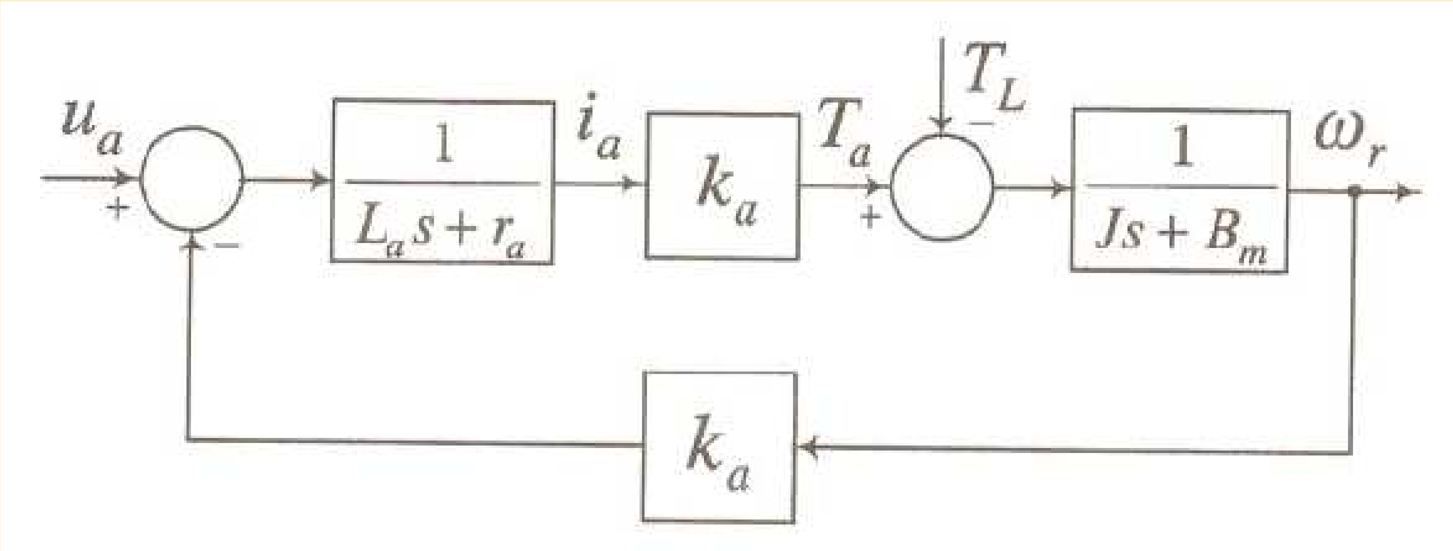
$$\left(s + \frac{r_a}{L_a}\right) i_a(s) = -\frac{k_a}{L_a}\omega_r(s) + \frac{1}{L_a}u_a(s)$$

$$\left(s + \frac{B_m}{J}\right)\omega_r(s) = \frac{1}{J}k_a i_a(s) - \frac{1}{J}T_L(s)$$

$$s\theta_r(s) = \omega_r(s)$$

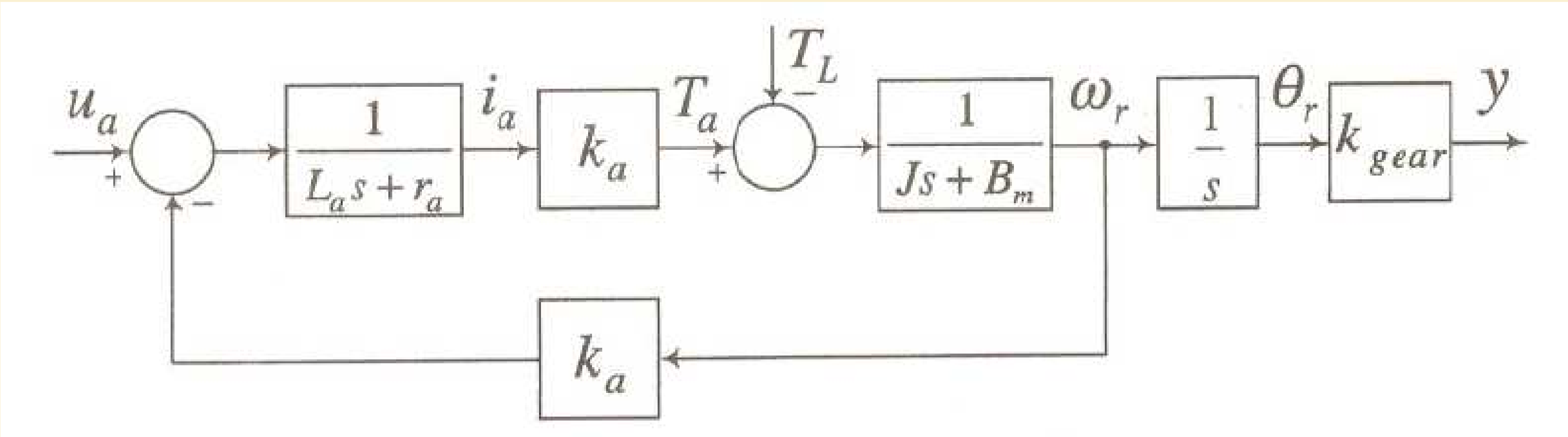
Analog control of permanent magnet DC Motor

- Block diagram of closed loop the permanent magnet DC motor :



Analog control of permanent magnet DC Motor

- The controller should be designed, and the output equation: $y(t) = k_{gear}\theta_r$
- Using this output equation, as well as $s\theta_r = \omega_r(s)$
- Block diagram of open loop servo actuated by permanent magnet DC motor :

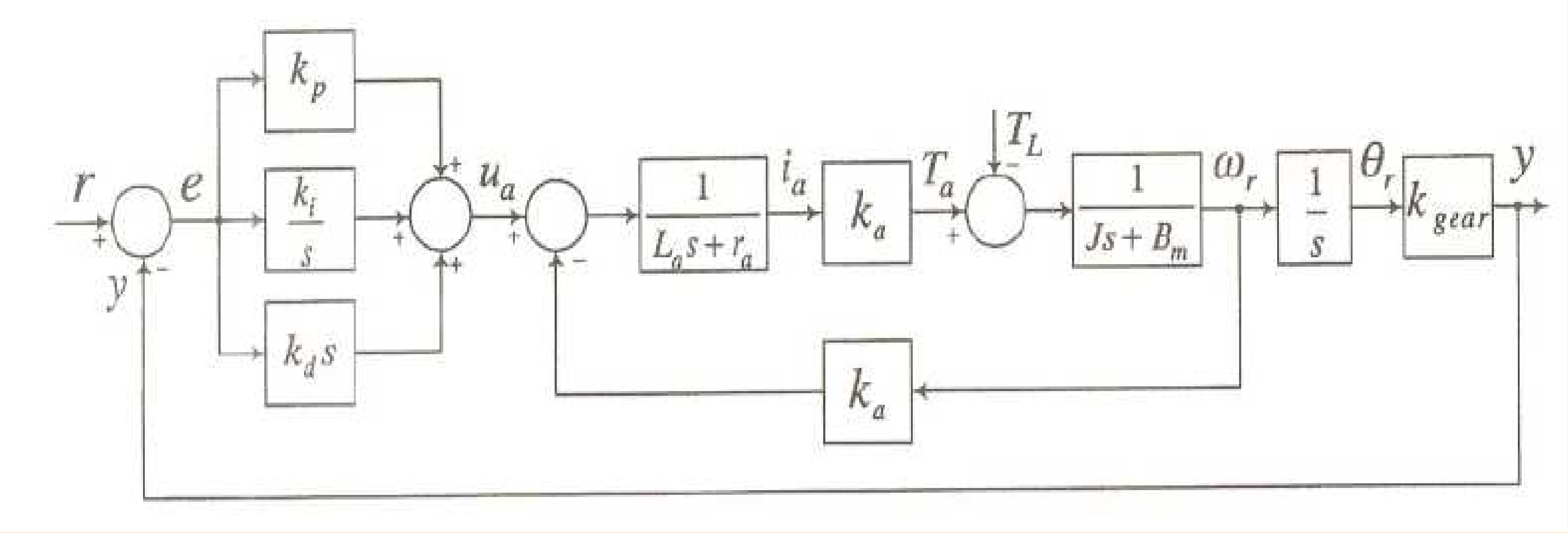


Analog control of permanent magnet DC Motor

- Using the linear PID controller:

$$u_a(t) = k_p e(t) + \frac{k_i}{s} e(t) + k_d s e(t)$$

- Block diagram of closed loop servo actuated by permanent magnet DC motor with the linear PID controller :

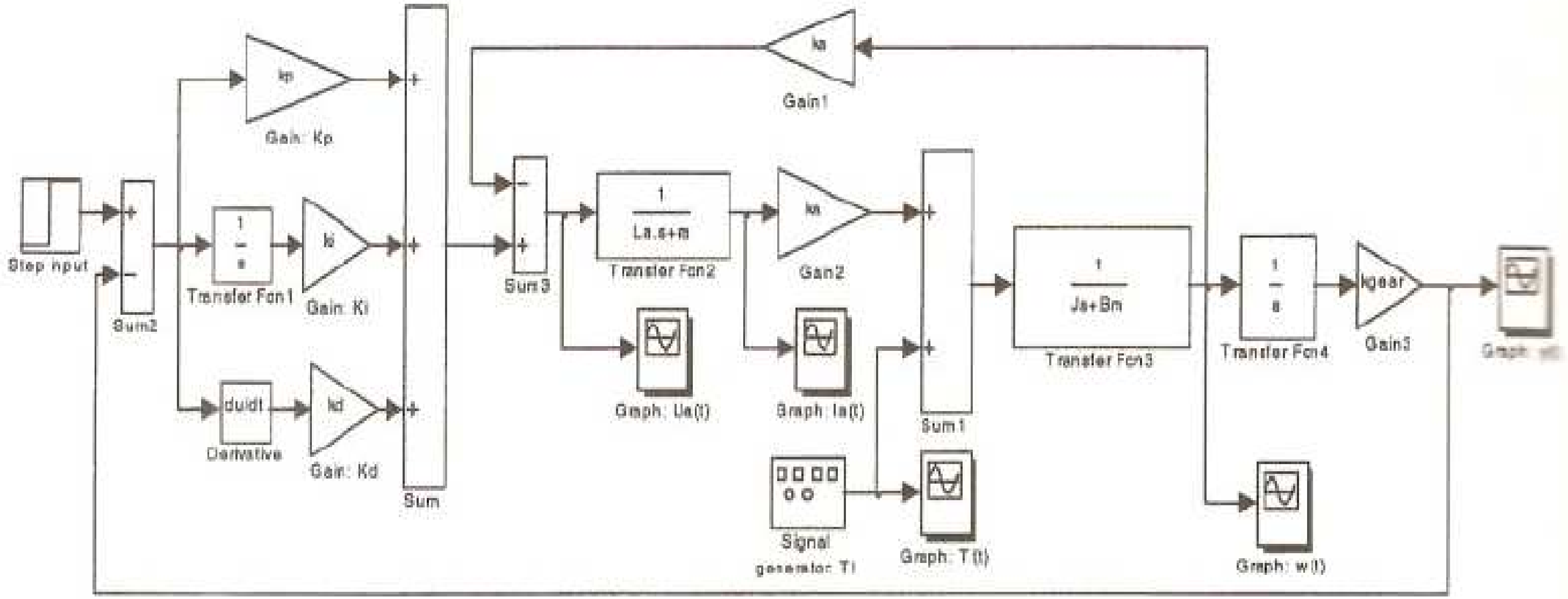


TUNING THE PID CONTROLLER

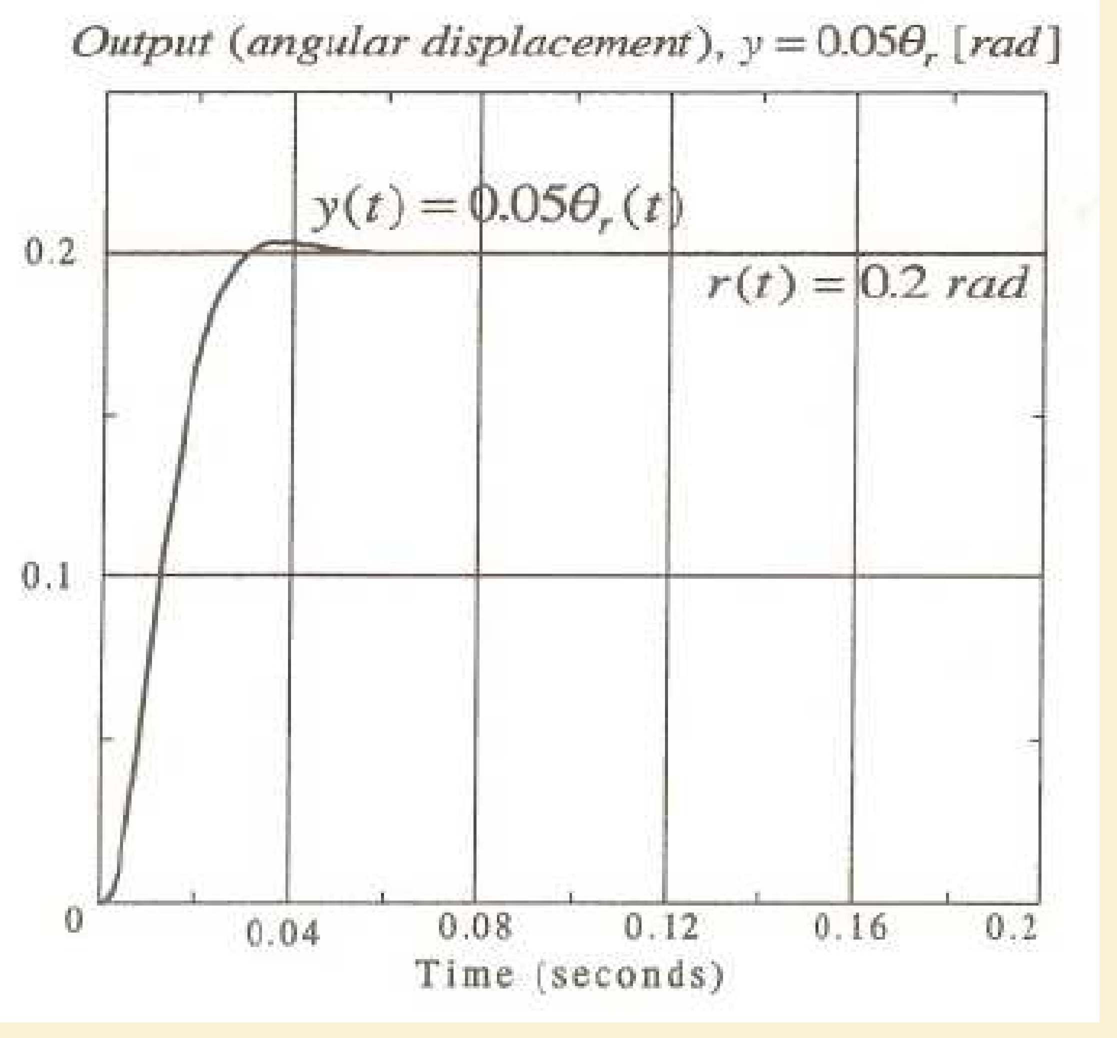
Ziegler-Nichols Tuning Rules

- 1. SET KP. Starting with KP=0, KI=0 and KD=0, increase KP until the output starts overshooting and ringing significantly.
- 2. SET KD. Increase KD until the overshoot is reduced to an acceptable level.
- 3. SET KI. Increase KI until the final error is equal to zero.

Analog control of permanent magnet DC Motor



Analog control of permanent magnet DC Motor



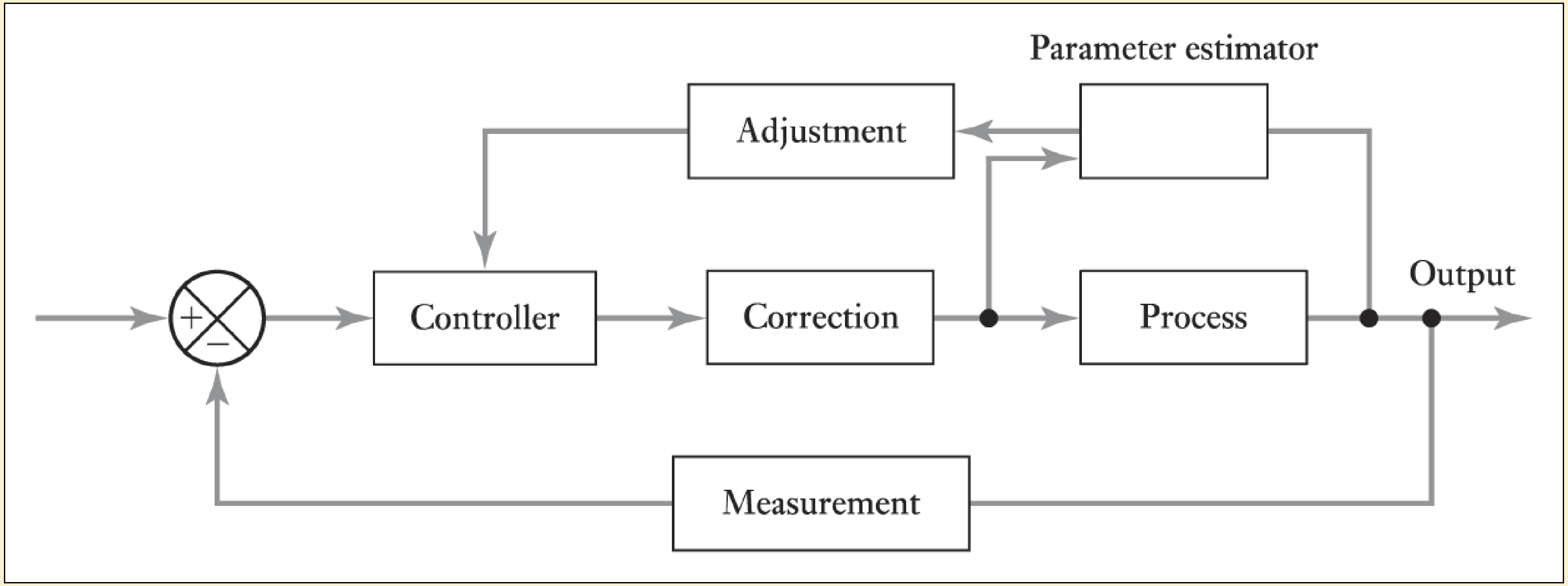


Figure 15.20 Self-tuning

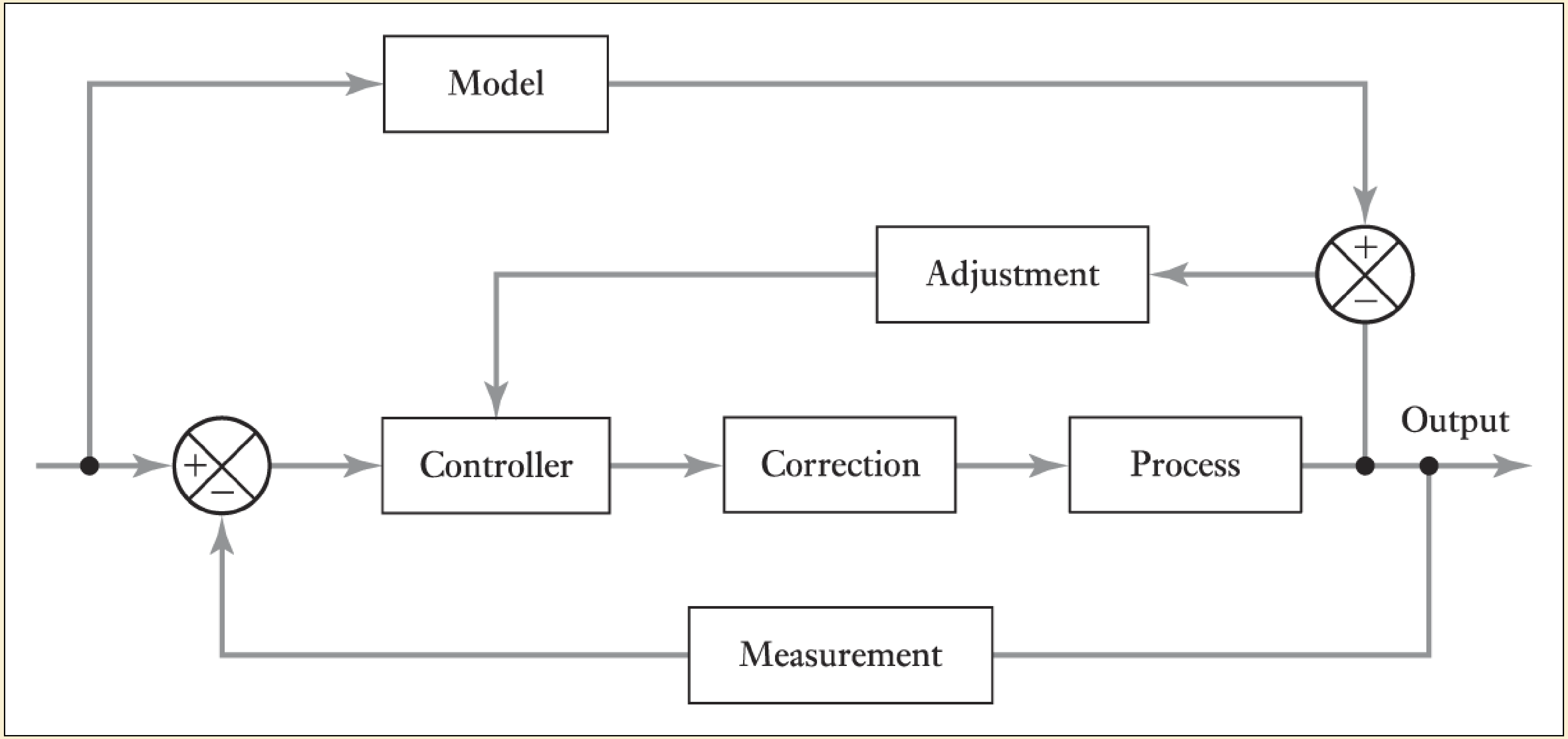


Figure 15.21 Model-referenced control