



Arches

THEORY OF STRUCTURES

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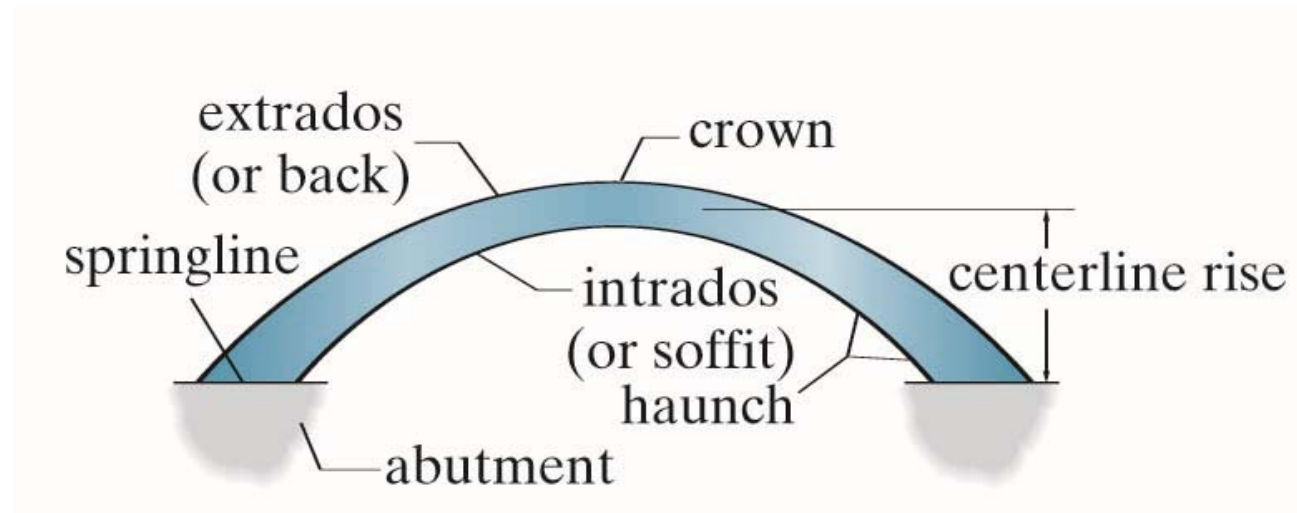
Arches

Arches can be used to reduce the bending moments in long-span structures. Essentially, an arch acts as an inverted cable, so it receives its load mainly in compression although, because of its rigidity, it must also resist some bending and shear depending upon how it is loaded and shaped.



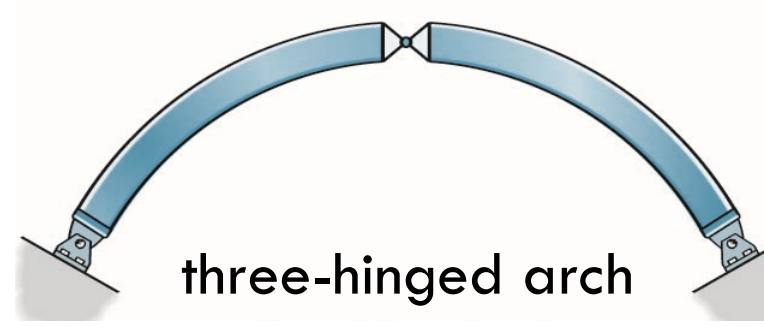
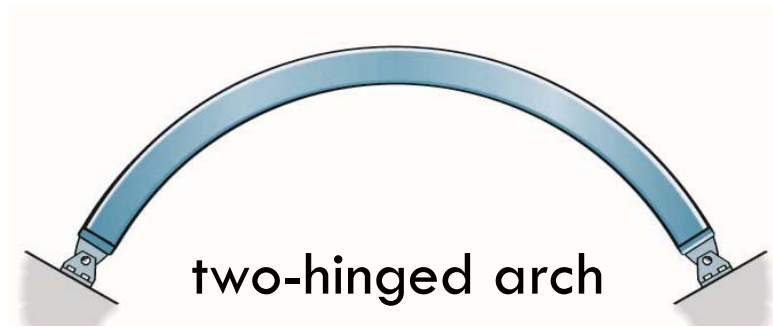
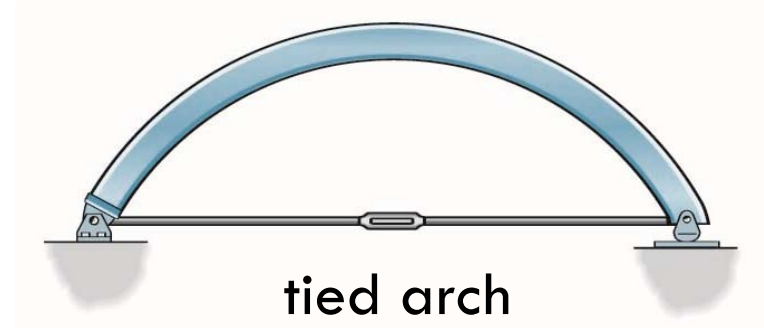
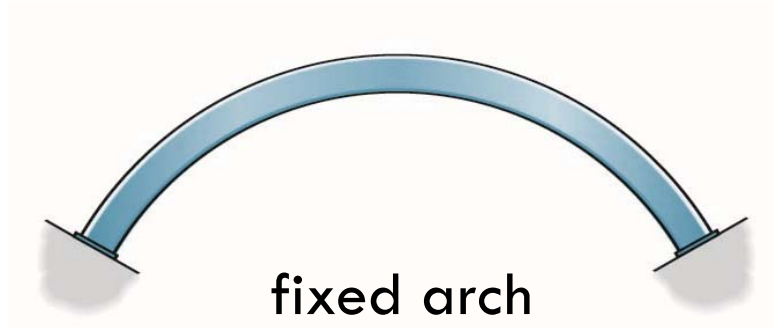
Arches

In particular, if the arch has a parabolic shape and it is subjected to a uniform horizontally distributed vertical load, then only compressive forces will be resisted by the arch. Under these conditions the arch shape is called a funicular arch because no bending or shear forces occur within the arch.



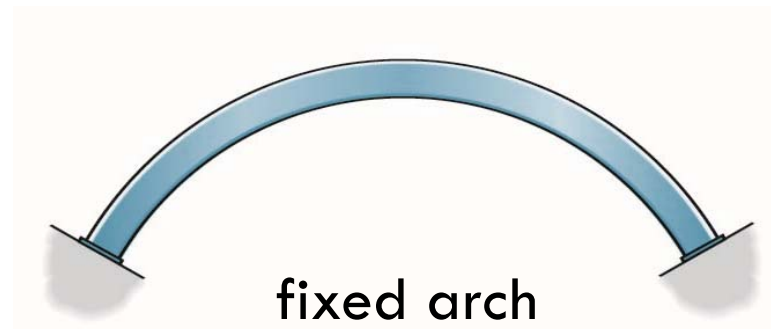
Arches

Depending on its uses, several types of arches can be selected to support a loading



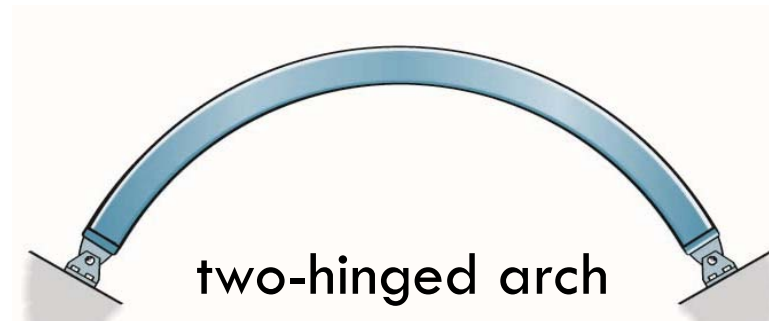
Arches

A *fixed arch* is often made from reinforced concrete. Although it may require less material to construct than other types of arches, it must have solid foundation abutments since it is indeterminate to the third degree and, consequently, additional stresses can be introduced into the arch due to relative settlement of its supports.



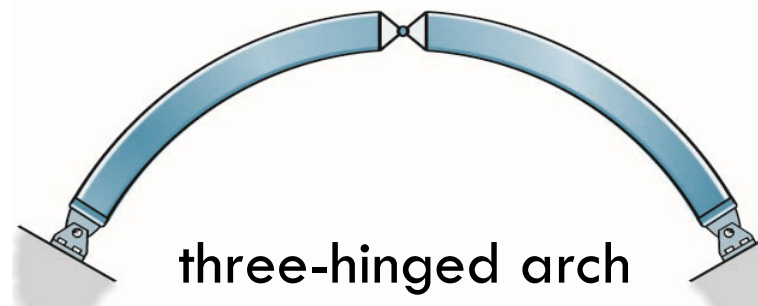
Arches

A *two-hinged arch* is commonly made from metal or timber. It is indeterminate to the first degree, and although it is not as rigid as a fixed arch, it is somewhat insensitive to settlement. We could make this structure statically determinate by replacing one of the hinges with a roller. Doing so, however, would remove the capacity of the structure to resist bending along its span, and as a result it would serve as a curved beam, and not as an arch.



Arches

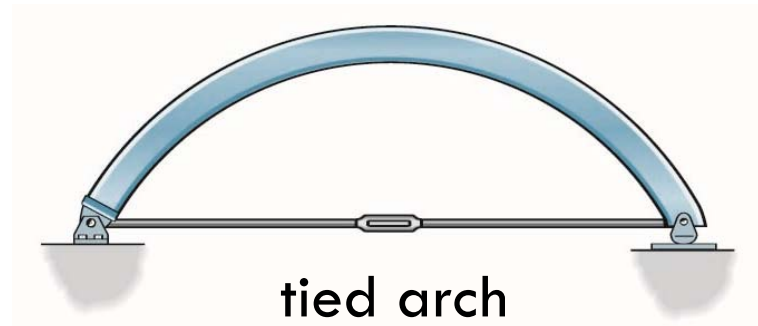
A *three-hinged arch* which is also made from metal or timber, is statically determinate. Unlike statically indeterminate arches, it is not affected by settlement or temperature changes.



Arches

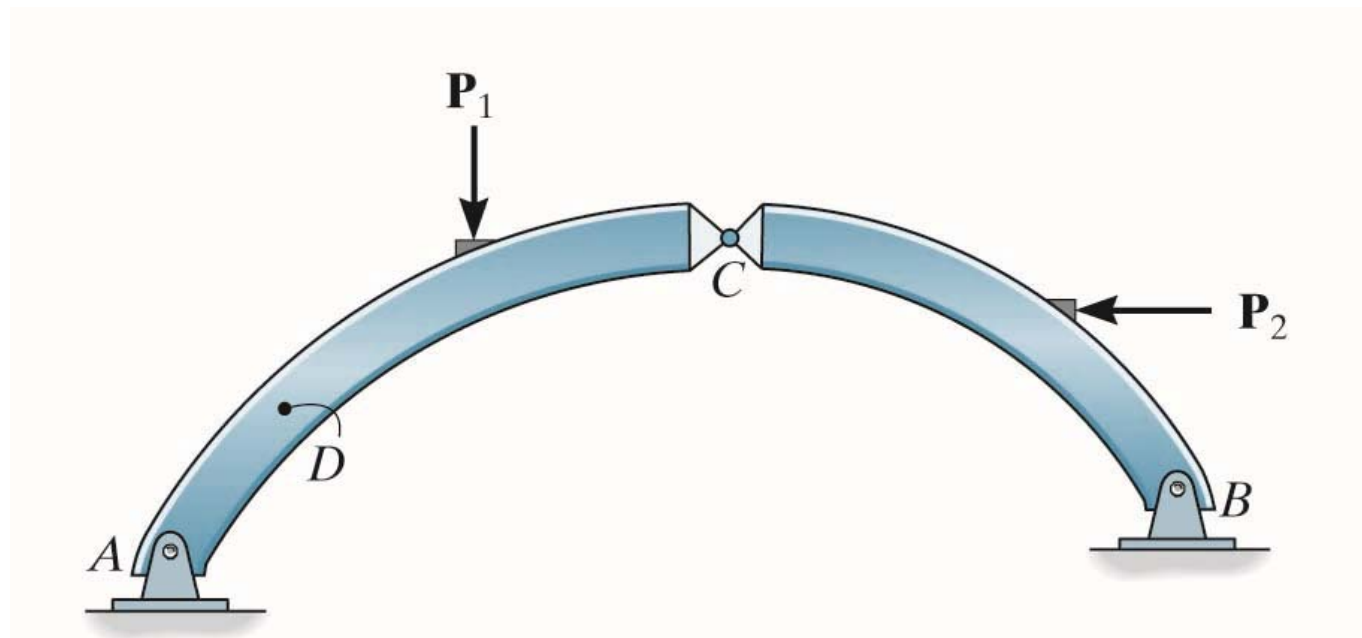
If two and three-hinged arches are to be constructed without the need for larger foundation abutments and if clearance is not a problem, then the supports can be connected with a tie rod.

A tied arch allows the structure to behave as a rigid unit, since the tie rod carries the horizontal component of thrust at the supports. It is also unaffected by relative settlement of the supports.

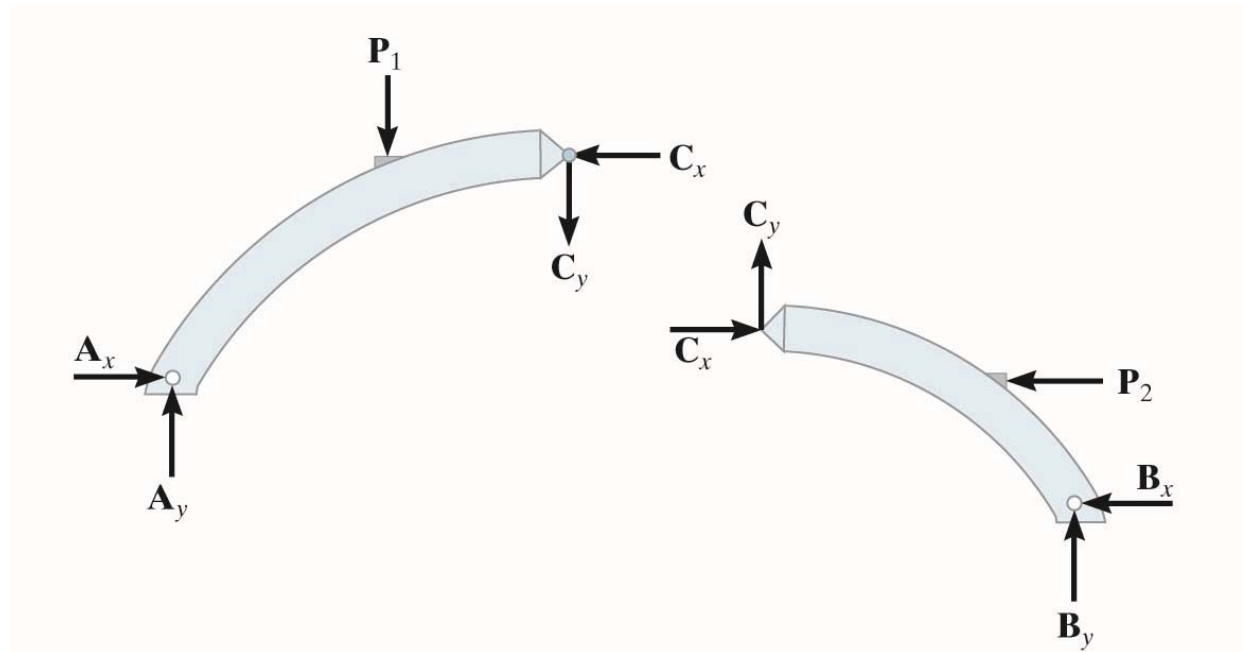


Three-Hinged Arches

- The third hinge is located at the crown & the supports are located at different elevations
- To determine the reactions at the supports, the arch is disassembled

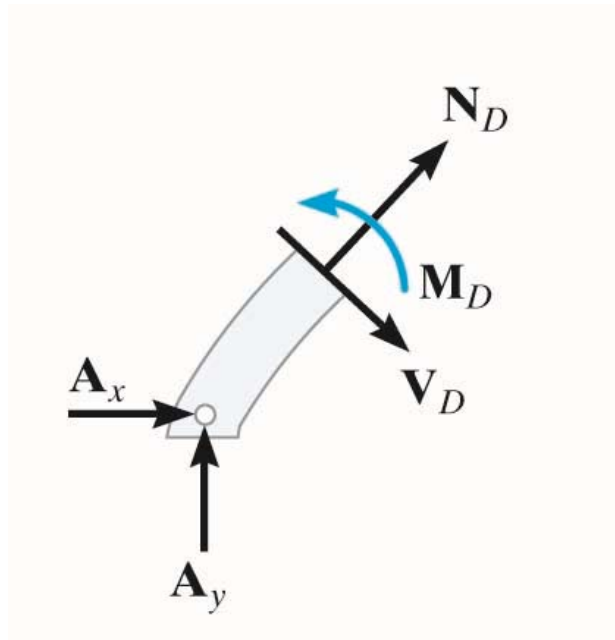


Three-Hinged Arches



In order to determine the reactions at the supports, the arch is disassembled and the free-body diagram of each member. Here there are six unknowns for which six equations of equilibrium are available. One method of solving this problem is to apply the moment equilibrium equations about points A and B . Simultaneous solution will yield the reactions C_x and C_y . The support reactions are then determined from the force equations of equilibrium.

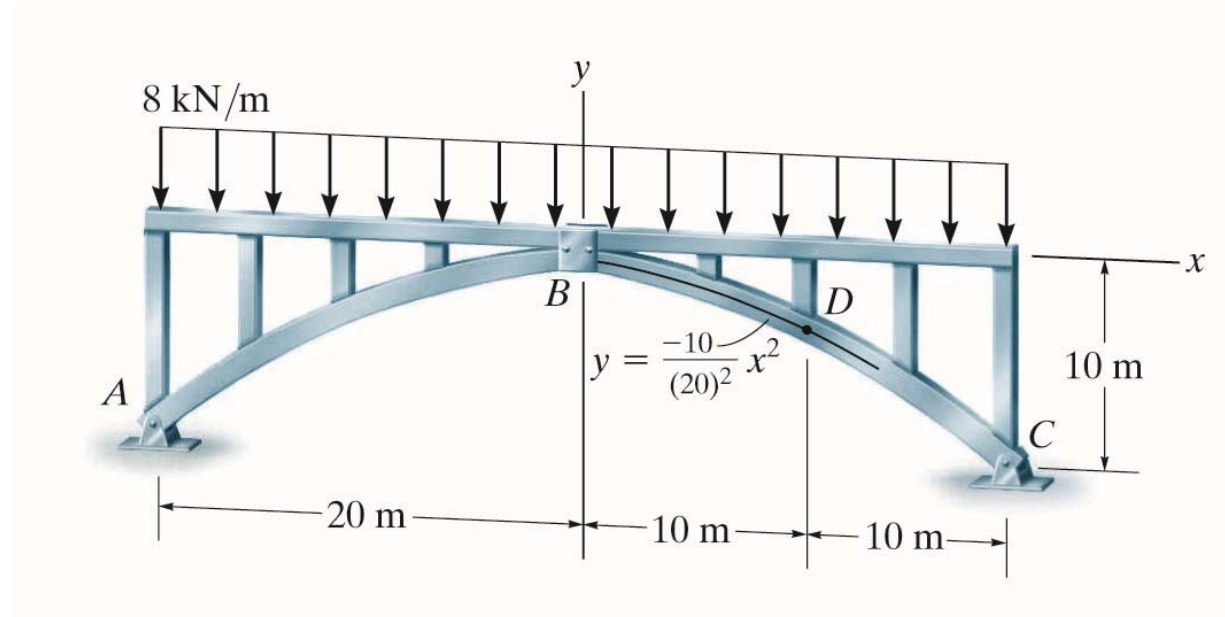
Three-Hinged Arches



Once all support reactions obtained, the internal normal force, shear, and moment loadings at any point along the arch can be found using the method of sections. Here, of course, the section should be taken perpendicular to the axis of the arch at the point considered.

Example

The three-hinged open-spandrel arch bridge has a parabolic shape and supports a uniform load. Show that the parabolic arch is subjected *only to axial compression* at an intermediate point such as point *D*. Assume the load is uniformly transmitted to the arch ribs.



Solution

Applying the equations of equilibrium, we have:

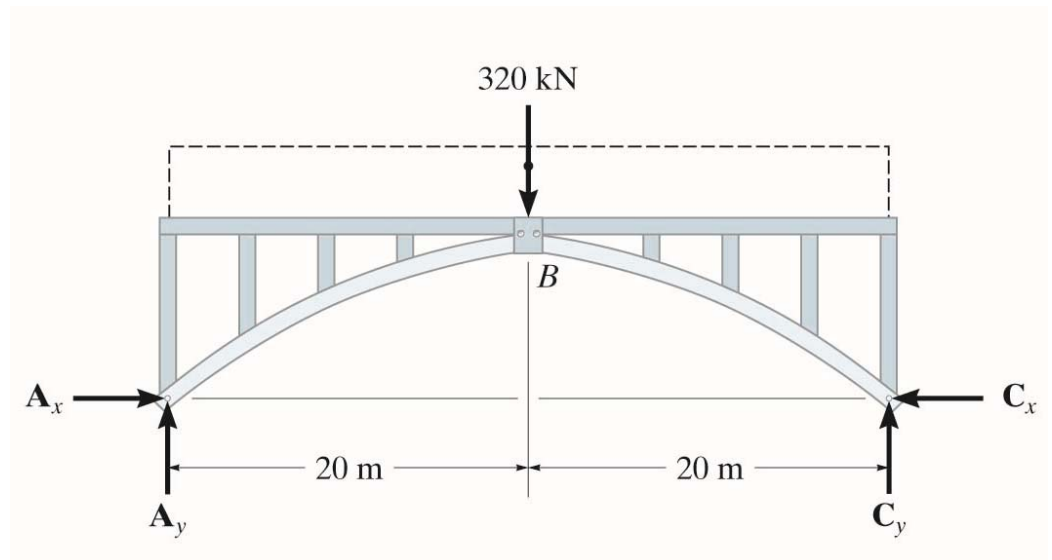
Entire Arch:

With anti-clockwise direction moments as +ve,

$$\sum M_A = 0$$

$$C_y(40\text{m}) - 320\text{kN}(20\text{m}) = 0$$

$$C_y = 160\text{kN}$$



Solution

Arch segment BC:

With anti-clockwise direction moments as + ve,

$$\sum M_B = 0$$

$$-160\text{kN}(10\text{m}) + 160\text{kN}(20\text{m}) - C_x(10\text{m}) = 0$$

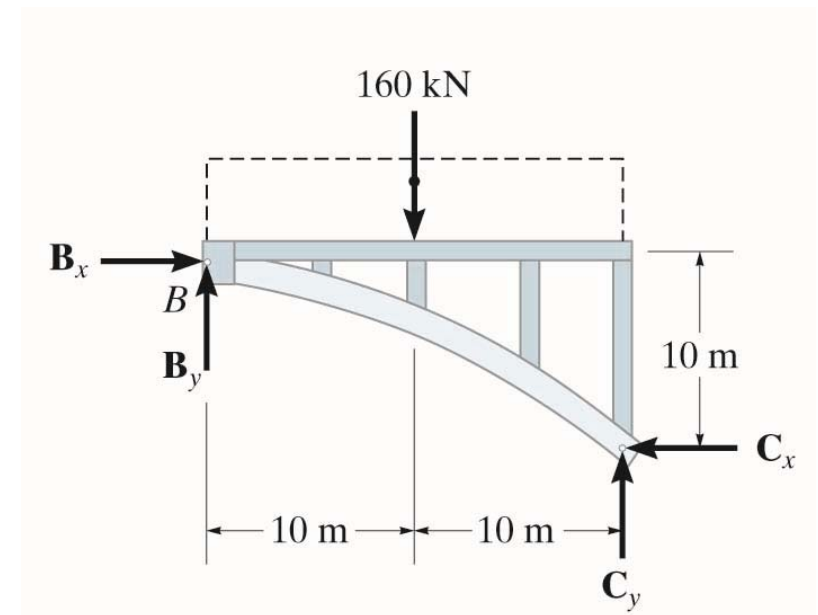
$$C_x = 160\text{kN}$$

$$\pm \sum F_x = 0 \Rightarrow B_x = 160\text{kN}$$

$$+\uparrow \sum F_y = 0$$

$$B_y - 160\text{kN} + 160\text{kN} = 0$$

$$B_y = 0$$



Solution

A section of the arch taken through point D

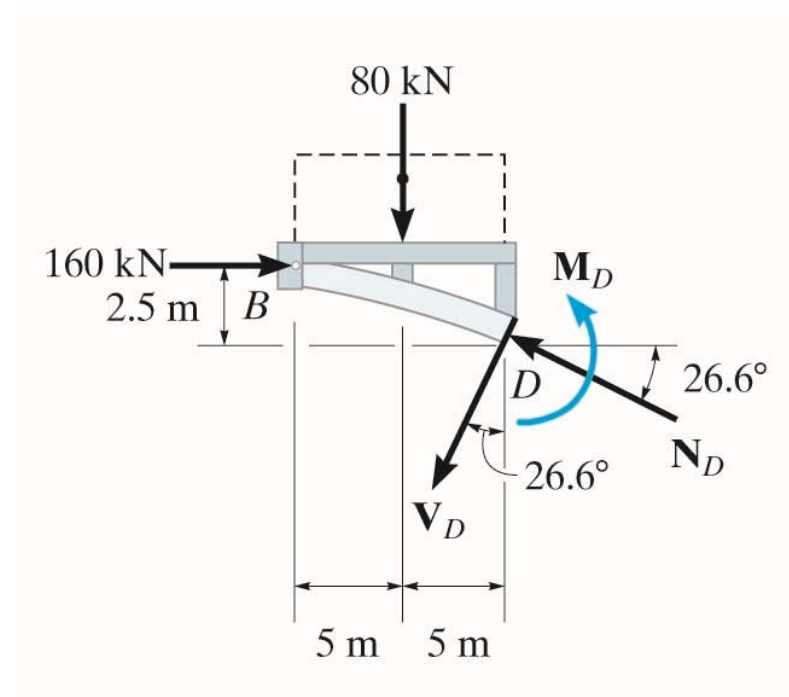
$$x = 10\text{m}$$

$$y = -10(10)^2 / (20)^2 = -2.5\text{m}$$

The slope of the segment at D is:

$$\tan \theta = \frac{dy}{dx} = \frac{-20}{(20)^2} x \bigg|_{x=10\text{m}} = -0.5$$

$$\theta = 26.6^\circ$$



Solution

Applying the eqn of equilibrium, Fig 5.10(d), we have:

$$\pm \sum F_x = 0$$

$$160\text{kN} - N_D \cos 26.6^\circ - V_D \sin 26.6^\circ = 0$$

$$+\uparrow \sum F_y = 0$$

$$-80\text{kN} + N_D \sin 26.6^\circ - V_D \cos 26.6^\circ = 0$$

$$\Rightarrow N_D = 178.9\text{kN}$$

$$\Rightarrow V_D = 0$$

With anti-clockwise moments as + ve:

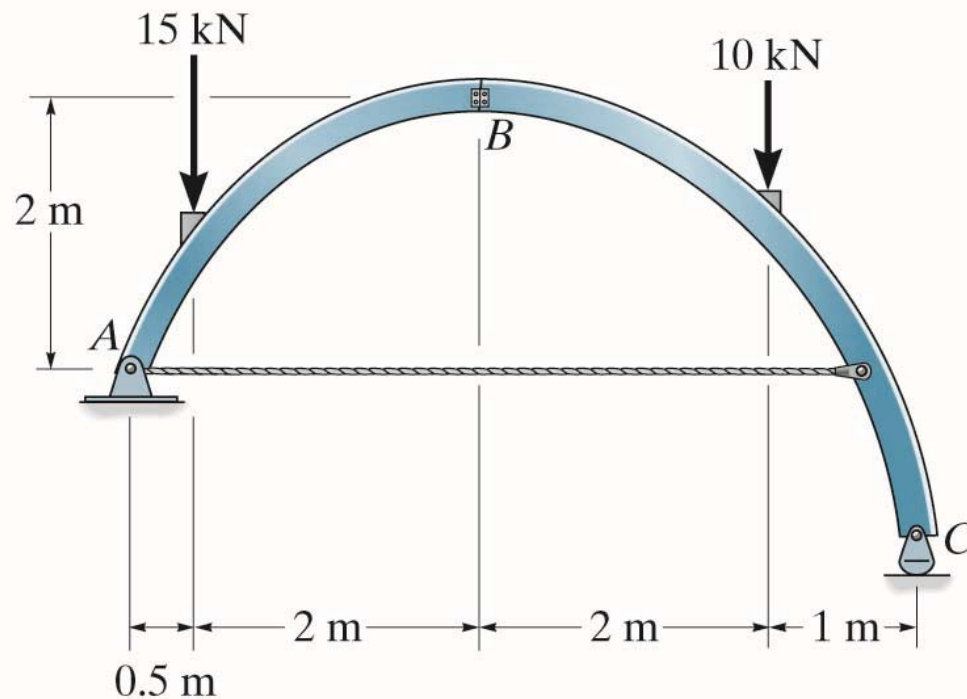
$$\sum M_D = 0$$

$$M_D + 80\text{kN}(5\text{m}) - 160\text{kN}(2.5\text{m}) = 0$$

$$\Rightarrow M_D = 0$$

Example

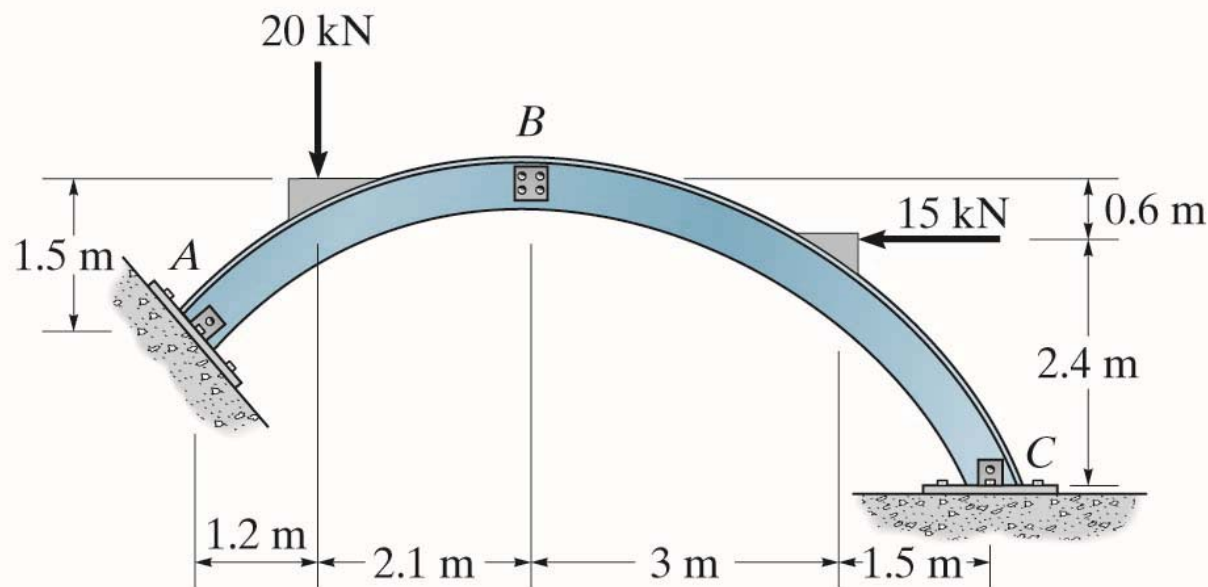
The tied three-hinged arch is subjected to the loading shown. Determine the components of reaction at A and C and the tension in the cable.



Solution on whiteboard

Example

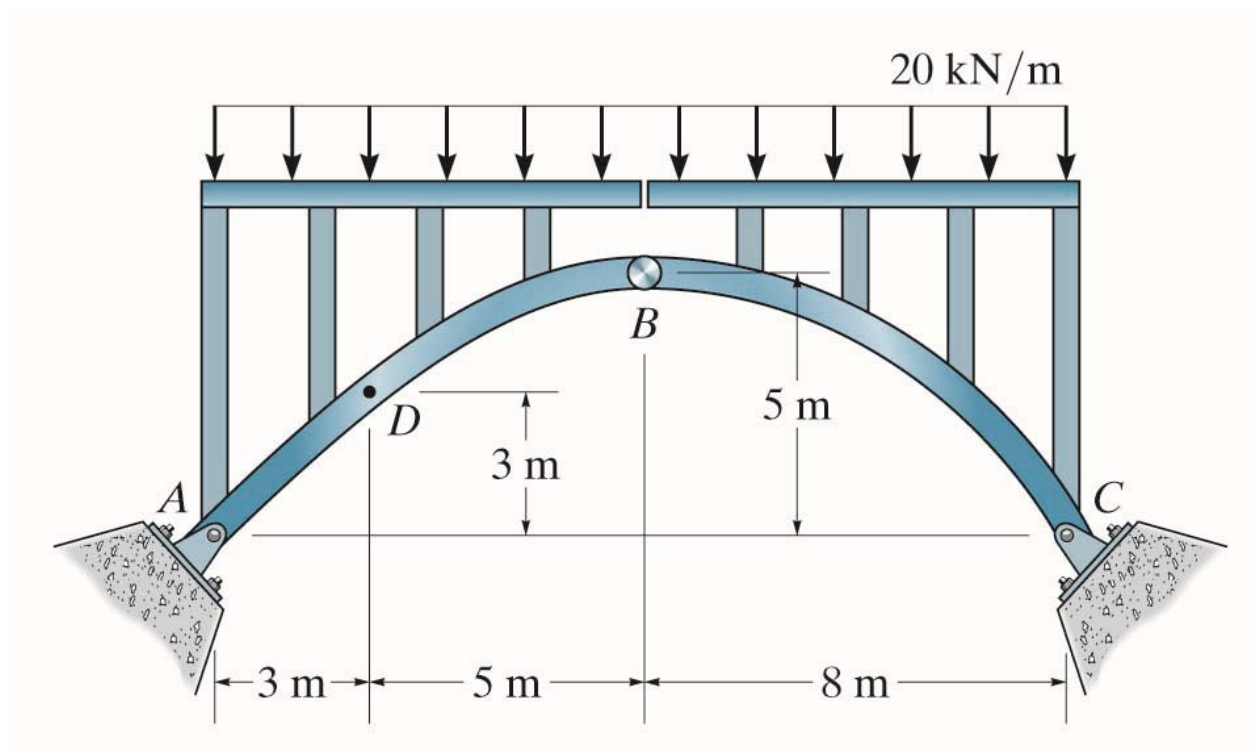
Determine the horizontal and vertical components of reaction at A, B, and C of the three-hinged arch. Assume A, B, and C are pin connected.



Solution on whiteboard

Example

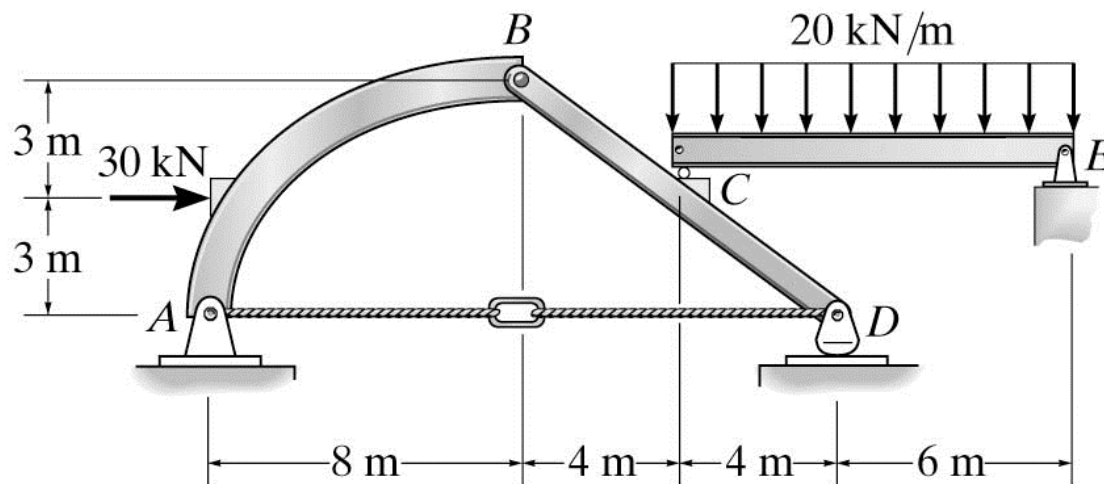
The three-hinged spandrel arch is subjected to the uniform load of 20 kN/m . Determine the internal moment in the arch at point D.



Solution on whiteboard

Example

The arch structure is subjected to the loading shown. Determine the horizontal and vertical components of reaction at A and D, and the tension in the rod AD.



Solution on whiteboard