SEN301 OPERATIONS RESEARCH I
PREVIUOS EXAM QUESTIONS

1. A company is involved in the production of two items (X and Y). The resources need to produce X and Y are twofold, namely machine time for automatic processing and craftsman time for hand finishing. The table below gives the number of minutes required for each item:

	Machine time	Craftsman time
Item X	13	20
Item Y	19	29

The company has 40 hours of machine time available in the next working week but only 35 hours of craftsman time. Machine time is costed at £10 per hour worked and craftsman time is costed at £2 per hour worked. Both machine and craftsman idle times incur no costs. The revenue received for each item produced (all production is sold) is £20 for X and £30 for Y. The company has a specific contract to produce 10 items of X per week for a particular customer.
Formulate the problem of deciding how much to produce per week as a linear program.

2. Answer the questions related to the model below:

$$\begin{aligned} \text{max.} \quad & 3x_1 + 2x_2 \\ \text{st} \quad & 2x_1 + 2x_2 \le 5 \\ & 2x_1 + x_2 \le 4 \\ & x_1 + 2x_2 \le 4 \\ & x_1, x_2 \ge 0 \end{aligned}$$

a. Use the graphical solution technique to find the optimal solution to the model.
b. Use the simplex algorithm to find the optimal solution to the model.
c. For which objective function coefficient value ranges of x_1 and x_2 does the solution remain optimal?
d. Find the dual of the model.

3. Consider the following problem.

$$\begin{aligned} \max\ & 3x_1 + 2x_2 \\ \text{s.t.}\ & 3x_1 + x_2 \le 12 \\ & x_1 + x_2 \le 6 \\ & 5x_1 + 3x_2 \le 27 \\ & x_1, x_2 \ge 0. \end{aligned}$$

a) Solve the problem by the *original simplex method* (in tabular form). Identify the *complementary* basic solution for the dual problem obtained at each iteration.
b) Solve the *dual* of this problem manually by the *dual simplex method*. Compare the resulting sequence of basic solutions with the complementary basic solutions obtained in part (*a*).

4. Use the *revised simplex algorithm* manually to solve the following problem.

$$\begin{aligned} \min\ & 5x_1 + 2x_2 + 4x_3 \\ \text{s.t.}\ & 3x_1 + x_2 + 2x_3 \le 4 \\ & 6x_1 + 3x_2 + 5x_3 \le 10 \\ & x_1, x_2, x_3 \le 0 \end{aligned}$$

5. Maçka Police Station employs 30 police officers. Each officer works for 5 days per week. The crime rate fluctuates with the day of week, so the number of the police officers required each day depends on which day of the week it is: Monday, 18; Tuesday, 24; Wednesday, 25; Thursday, 16; Friday, 21; Saturday, 28; Sunday, 18. The Police Station wants to schedule police officers to minimize the number whose days off are not consecutive. *Formulate an LP* that will accomplish this goal.

6. Turkish national swimming team coach is putting together a relay team for the 400 meter relay. Each swimmer must swim 100 meters of breaststroke, backstroke, butterfly, or free style. The coach believes that each swimmer will attain the times (seconds) given in the Table below. To minimize the team's time for the race, *assign* each swimmer for a stroke.

	Free	Breast	Fly	Back
Derya	54	54	51	53
Murat	51	57	52	52
Deniz	50	53	54	56
Ceyhun	56	54	55	53

7. A shoe company forecasts the following demands during the next three months: 200, 260, 240. It costs \$7 to produce a pair of shoes with regular time labor (RT) and \$11 with overtime labor (OT). During each month regular production is limited to 200 pairs of shoes, and overtime production is limited to 100 pairs. It costs \$1 per month to hold a pair of shoes in inventory. Formulate a *balanced transportation* problem to minimize the total cost of meeting the next three months of demand on time (Do *not* try to *solve* it!).

8. Suppose that you only have the "range (sensitivity) analysis" part of a Lindo output for a minimization problem.

a. What is the *reduced cost* of a non-basic variable?
b. What is the *surplus* quantity for a constraint that is not binding?

9. For the following LP show the starting basis (bfs) of the Big M method (form the initial tableau only!). At the initial tableau, show the leaving and entering variables. (PS: Please do not try to solve the problem, optimal solution can be found at iteration 5.)

$$\begin{aligned} \min \quad & 4x_1 + x_2 \\ \text{s.t.} \quad & 3x_1 + x_2 \geq 10 \\ & x_1 + x_2 \geq 5 \\ & x_1 \geq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

10. A freight plane has three large compartments to carry cargo. Weight and volume limitations of these compartments are:

Compartment	Weight (Tons)	Volume (m^3)
Front	10	6800
Center	16	8700
Rear	8	5300

There are four cargos waiting to be loaded in this plane. Properties of these cargos are shown on the table below:

Cargo	Total Weight (Tons)	Total Volume (m^3)	Profit (TL/ton)
K1	18	8640	310
K2	15	9750	380
K3	23	13340	350
K4	12	4680	285

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the plane. Any proportion of these cargoes can be accepted.

Formulate a linear programming model to maximize the profit by choosing how many tons of which cargo to load on the plane under these circumstances.

11. Answer the following questions related with the model given below:

$$\begin{aligned} \min \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_1 \leq 5 \\ & x_2 \leq 4 \\ & \forall\, x_i \text{ urs} \end{aligned}$$

a. Use the Graphical Method to solve the model.
b. Use the Simplex Algorithm to solve the model.
c. Find the dual of the model.

12. Answer the following questions related with the model given below:

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_1 \le 5 \\ & x_2 \le 4 \\ & \forall\, x_i \text{ urs} \end{aligned}$$

a. Use the Simplex Algorithm to solve the model.
b. Find the dual of the model.

13. Consider the following LP model:

$$\begin{aligned} \text{mak. } z &= x_1 + 5x_2 + 3x_3 \\ x_1 + 2x_2 + x_3 &= 3 \\ 2x_1 - x_2 \quad &= 4 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

a. Find the dual of the model.
b. Given that basic variables are x_1 and x_3 in the primal optimal solution, find the dual optimal solution (dual variables value and dual objective function value) without solving the dual model.

INFORMATION FOR QUESTIONS 14-25

LP Model for "Modified Advertisement Example":

x_1: The number of comedy spots
x_2: The number of football spots

min $z = 30x_1 + 70x_2$

st $12x_1 + 3x_2 \ge 32$ (high income women)
$4x_1 + 9x_2 \ge 16$ (high income men)
$x_1, x_2 \ge 0$

Useful information (Please do not solve the model, use this information only):

The optimal solution implies that the 1st constraint is nonbinding with an excess value of 16 and the 2nd constraint is binding with a shadow price of -7.5.

14. Please fill in the blanks below at the story of the given LP model:

Dorian makes luxury cars and jeeps for high-income men and women. It wishes to advertise with 1 minute spots in comedy shows and football games. Each comedy spot costs $................. and is seen by .... high-income women and .... high-income men. Each football spot costs $................. and is seen by .... high-income women and .... high-income men. How can Dorian reach .... high-income women and .... high-income men at the least cost?

15. Please fill in the blanks below at the report (executive summary):

The minimum cost of reaching the target audience is $.................. , with .... comedy spots and .... football spots.

16. Find the dual of the LP model given. What is the economic interpretation of the dual model (*define decision variables and constraints as well as objective function in the dual model*)?

17. What is the optimal solution of the dual model? (*Please do not solve the dual model, only submit the optimal values for the objective function and the decision variables of the dual model*)

18. What is the allowable range for the objective function coefficient of x_2 in which current solution (basis) remains optimal? (*Hint: Use the relation of Duality-Sensitivity*)

19. What are the reduced costs for the decisions variables in the primal model? Why?

20. What is the allowable range for RHS of the first constraint in the primal model in which current solution (basis) remains optimal?

21. Please fill in the blanks at the Lindo output given below:

```
        OBJECTIVE FUNCTION VALUE

        1)       .............
  VARIABLE        VALUE          REDUCED COST
        X1        .............     .............
        X2        .............     .............
       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)        .............     .............
        3)        .............     .............
RANGES IN WHICH THE BASIS IS UNCHANGED:
                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
       X1         .............     1.111111        30.000000
       X2         .............     .............    .............
                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
        2         .............    .............     .............
        3         .............    INFINITY         5.333333
```

22. If the cost of a comedy spot is $40K, what would be the new optimal solution to the problem?

23. If the cost of a comedy spot is $25K, what would be the new optimal solution to the problem?

24. If Dorian wishes to reach 13M high income men, what would be the new optimal solution to the problem?

25. Assume that Dorian wishes to make additional advertisements with 1 minute spots in a reality show which is seen by 6M high income women and 9M high income men. What should be the cost of the reality show spot that would make it reasonable to be selected (to be recommended in the solution)?

26. Consider the following LP problem:

$$\min z = -3x_1 + 9x_2$$
$$x_1 + 4x_2 \leq 8$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Let the slack variables are x_3 and x_4. Find the optimal solution <u>with revised simplex</u> method given that current set of basic variables is $\{x_3, x_2\}$.

27. Consider the following problem:

$$\max \quad Z = -5x_1 + 5x_2 + 12x_3$$

such that

$$-x_1 + x_2 + 3x_3 \leq 20$$
$$12x_1 + 4x_2 + 10x_3 \leq 90$$
$$x_j \geq 0 \quad \forall j$$

Let x_4 and x_5 represents the slack variables for constraints 1 and 2 respectively. The optimal table is given below:

$$
\begin{array}{lllllllll}
(0) & Z & & & +3x_3 & +5x_4 & & = & 100 \\
(1) & & -x_1 & +x_2 & +3x_3 & +x_4 & & = & 20 \\
(2) & & 16x_1 & & -2x_3 & -4x_4 & +x_5 & = & 10
\end{array}
$$

Then consider the following changes in the model <u>independently</u>. State <u>for each case</u>, whether the set of current basic variables or their values change or not, and then if they change, then find the new optimal set of basic variables and the new optimal solution.

a. Add the following constraint: $2x_1 + 3x_2 + 5x_3 \leq 50$

b. Change the objective function coefficient 1: $c_1 = 1$.

28. Recall that the optimal solution for Powerco problem was z=$1,020 and the optimal tableau was:

	u_i/v_j	City 1	City 2	City 3	City 4	Supply
		6	6	10	2	
Plant 1	0	[8]	[6] 10	[10] 25	[9]	35
Plant 2	3	[9] 45	[12]	[13] 5	[7]	50
Plant 3	3	[14]	[9] 10	[16]	[5] 30	40
Demand		45	20	30	30	

a) For <u>what range</u> of values of the cost of shipping 1 million kwh of electricity from plant 3 to city 3 will the <u>current basis remain optimal</u>?

b) Suppose we increase both s_3 and d_3 by 3. Find the new value of the <u>cost</u> and new values of the <u>decision variables</u>.

29. Nicole Kidman, Jennifer Lopez, Catherine Zeta Jones, and Cameron Diaz are marooned on a desert island with Brad Pitt, Antonio Banderas, Robin Williams, and Tom Cruise. The "compatibility measures" in the table given below indicate <u>how much happiness</u> each couple would experience if they spent all their time together. Determine the partner for each person.

	NK	JL	CZJ	CD
BP	7	5	8	2
AB	7	8	9	4
RW	3	5	7	9
TC	5	7	6	9

30. The dean of a college must plan the school's course offerings for the fall semester. Student demands make it necessary to offer at least 30 undergraduate and 20 graduate courses in the term. Faculty contracts also dictate that at least 60 courses be offered in total. Each undergraduate course taught costs the college an average of $2500 in faculty wages, and each graduate course costs $3000. Formulate an LP model to identify number of undergraduate and graduate courses that should be taught in the fall so at total faculty salaries are kept to a minimum (What are the decision variables, objective, and the constraints? Indicate sign restrictions if any. Please do not solve the problem).

31. A company must deliver d_i units of its product at the end of the ith month (for i= 1,..5, d_i values are given in the following table). Material produced during a month at a cost of $100 can be delivered at the end of the same month or can be stored as inventory and delivered at the end of a subsequent month; however, there is a storage cost of 5 dollars per month for each unit of product held in inventory. At the beginning of month 1, 3 products are available. If the company produces x_i units in month i and x_{i+1} units in month $i+1$, it incurs a cost of $40|x_{i+1} - x_i|$ dollars, reflecting the cost of switching to a new production level. Formulate a linear programming problem whose objective is to minimize the total cost of the production and inventory schedule over a period of five months. Assume that inventory left at the end of the year has no value and does not incur any storage cost. (What are the decision variables, objective, and the constraints? Indicate sign restrictions if any. Please do not solve the problem).

Aylar	1	2	3	4	5
d_i	15	17	19	11	7

32. Max z = 100[67 – ($x_A + x_B + x_C$)]

S.t; $x_A + x_B + x_C \le 67$

$3x_A \ge 40$

$10x_C \le 70$

$5x_D \ge 60$

$(3x_A + 7x_B + 10x_C + 5x_D)/5 \ge 50$

$x_A + x_B + x_C = 1,5\ x_D$

$x_A, x_B, x_C \ge 0$; x_D urs

Considering the above given LP model;

a) Convert the model to standard form.
b) Build the initial tableau to solve it using Big M Method.
c) Determine whether the initial bfs is optimal. If it is not, determine which nonbasic variable should become a basic variable and which basic variable should become a nonbasic variable to improve the objective function (do not make any operation to find the next table)

33. max z = $3x_1 - 6x_2 + x_3$

s.t. $x_1 + x_2 + x_3 \ge 8$

$2x_1 - x_2 = 5$

$-x_1 + 3x_2 + 2x_3 \le 7$

$x_1, x_2, x_3 \ge 0$

While solving the above given LP problem using two-phase simplex method, we reach the following table as the optimal first Phase LP. Continue to the solution procedure with the second phase (if it is required) and find an optimal solution for the original problem.

w	x_1	x_2	x_3	e_1	a_1	a_2	s_3	Rhs
1	0	0	0	0	-1	-1	0	0
0	0	1	2/3	-2/3	2/3	-1/3	0	11/3
0	1	0	1/3	-1/3	1/3	1/3	0	13/3
0	0	0	1/3	5/3	-5/3	4/3	1	1/3

34. Silicon Valley Corporation (Silvco) manufactures transistors. An important aspect of the manufacture of transistors is the melting of the element germanium (a major component of a transistor) in a furnace. Unfortunately, the melting process yields germanium of highly variable quality.

Two methods can be used to melt germanium; method 1 costs \$50 per transistor, and method 2 costs \$70 per transistor. The qualities of germanium obtained by methods 1 and 2 are shown in Table 1. Grade 1 is poor while grade 4 is excellent. The quality of the germanium dictates the quality of the manufactured transistor.

Silvco can refire melted germanium in an attempt to improve its quality. It costs \$25 to refire the melted germanium for one transistor. The results of the refiring process are shown in Table 2. Silvco has sufficient furnace capacity to melt or refire germanium for at most 20,000 transistors per month.

Silvco's monthly demands are for 1,000 grade 4 transistors, 2,000 grade 3 transistors, 3,000 grade 2 transistors, and 3,000 grade 1 transistors. Formulate an LP model in general form (with $\sum$ ve $\forall$) to minimize the cost of producing the needed transistors.

Table 1. Percent yielded by melting (%)

Grade of Melted Germanium	Melting Methods	
	Method 1	Method 2
Defective	30	20
1	30	20
2	20	25

3	15	20
4	5	15

Table 2. Percent yielded by refiring (%)

Grade Yielded by Refiring	Refired Grade of germanium			
	Defective	Grade 1	Grade 2	Grade 3
Defective	30	0	0	0
1	25	30	0	0
2	15	30	40	0
3	20	20	30	50
4	10	20	30	50

35. Consider the following LP;

$$\min z = 5x_1 + 3x_2 - 2x_3$$
$$subject\ to;$$
$$x_1 + x_2 + x_3 \geq 4$$
$$2x_1 + 3x_2 - x_3 \geq 9$$
$$x_2 + x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

Optimal Tableau:

Z	x_1	x_2	x_3	e_1	a_1	e_2	a_2	s_3	STD	TD
1	-2.5	0	0	0	-M	-1.25	1.25-M	-0.75	7.5	Z = 7.5
	-0.5	0	1	0	0	0.25	-0.25	0.75	1.5	x_3 = 1.5
	0.5	1	0	0	0	-0.25	0.25	0.25	3.5	x_2 = 3.5
	-1	0	0	1	-1	0	0	1	1	e_1 = 1

Lindo Output:

```
 LP OPTIMUM FOUND AT STEP      3
        OBJECTIVE FUNCTION VALUE
        1)      7.500000
  VARIABLE        VALUE          REDUCED COST
        X1         0.000000          A
        X2         3.500000         0.000000
        X3         B                0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         1.000000         0.000000
        3)         0.000000        -1.250000
        4)         C                D

 NO. ITERATIONS=       3
 RANGES IN WHICH THE BASIS IS UNCHANGED:
                           OBJ COEFFICIENT RANGES
 VARIABLE          CURRENT        ALLOWABLE        ALLOWABLE
                    COEF          INCREASE         DECREASE
       X1        5.000000          INFINITY         2.500000
       X2        3.000000          E                    F
       X3       -2.000000          1.000000         5.000000

                           RIGHTHAND SIDE RANGES
      ROW          CURRENT        ALLOWABLE        ALLOWABLE
                    RHS           INCREASE         DECREASE
        2        4.000000          1.000000         INFINITY
        3        9.000000          6.000000        14.000000
        4        5.000000          INFINITY         1.000000
```

Answer the following questions according to the optimal tableau and the lindo output. (Each question is independent from the others, give necessary information about your answers, and show your calculations in details):

a) What are the values of A, B, C and D on the Lindo output?

b) What are the values of E and F on the Lindo output?

c) If the objective function coefficient of x_3 (currently -2) would be -1, what would be the new z?

d) If the right hand side value of the second constraint (current 9) would be 16, what would be the new z?
e) If the right hand side value of all constraints changed as b = $[3, 10, 4.5]^T$, does the current optimal basis change? If not, what would be the new z?

36. Graphically solve the following LP. Indicate the values of z, x_1, x_2 and x_3 in the optimal solution. (Hint: Use the Theorem of Complementary Slackness)

$$
\begin{aligned}
&max\ z = 6x_1 + 4x_2 + 20x_3 \\
&subject\ to; \\
&\qquad 2x_1 + x_2 + 2x_3 \le 5 \\
&\qquad x_1 + x_2 + 10x_3 \le 1 \\
&\qquad x_1, x_2, x_3 \ge 0
\end{aligned}
$$

37. Answer the following questions, considering the following LP.

$$
\begin{aligned}
&max\ z = 2x_1 + x_2 + x_3 \\
&subject\ to; \\
&\qquad x_1 + x_3 \le 1 \\
&\qquad x_2 + x_3 \le 2 \\
&\qquad x_1 + x_2 \le 3 \\
&\qquad x_1, x_2, x_3 \ge 0
\end{aligned}
$$

a. Show that the basic solution with basic variables x_1, x_2 and x_3 is optimal. Find the optimal solution.
b. Show that if we multiply the right hand side of each constraint by a non-negative constant k, then the new optimal solution is obtained simply by multiplying the value of each variable in the original optimal solution by k.
c. Find the range of values for the right hand side of the second constraint for which the current basis remains optimal.
d. A new decision variable (x_4) is added and model is changed as following. What would be the new optimal solution?

$$
\begin{aligned}
&max\ z = 2x_1 + x_2 + x_3 + \mathbf{2.5x_4} \\
&subject\ to; \\
&\qquad x_1 + x_3 + \mathbf{2x_4} \le 1 \\
&\qquad x_2 + x_3 - \mathbf{x_4} \le 2 \\
&\qquad x_1 + x_2 \le 3 \\
&\qquad x_1, x_2, x_3, \mathbf{x_4} \ge 0
\end{aligned}
$$

Hint: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

38. Web Mercantile sells many household products through an on-line catalog. The company needs substantial warehouse space for storing its goods. Plans now are being made for leasing warehouse storage space over the next 5 months. Just how much space will be required in each of these months is known. However, since these space requirements are quite different, it may be most economical to lease only the amount needed each month on a month-by-month basis. On the other hand, the additional cost for leasing space for additional months is much less than for the first month, so it may be less expensive to lease the maximum amount needed for the entire 5 months. Another option is the intermediate approach of changing the total amount of space leased (by adding a new lease and/or having an old lease expire) at least once but not every month.
The space requirement and the leasing costs for the various leasing periods are given at the table.
The objective is to minimize the total leasing cost for meeting the space requirements. Formulate a linear programming model for this problem.

Month	Required Space (Sq.Ft.)
1	30,000
2	20,000
3	40,000
4	10,000
5	50,000

Leasing Period (Months)	Cost per Sq. Ft. Leased
1	$ 65
2	$100
3	$135
4	$160
5	$190

39. A company is taking bids on four construction jobs. Three people have placed bids on the jobs. Their bids (in thousands of dollars) are given in the table (a * indicates that the person did not bid on the given job). Person 1 can do only one job, but persons 2 and 3 can each do as many as two jobs.
a) built an assignment table to determine the minimum cost assignment of persons to jobs,
b) use Hungarian method to find the assignment of persons to jobs.

	Job			
Person	1	2	3	4
1	50	46	42	40
2	51	48	44	*
3	*	47	45	45

40. Consider the following problem.

$$\text{Minimize } W = 5y_1 + 4y_2 ,$$
$$\text{s.t.} \quad 5y_1 + 4y_2 \geq 4$$
$$2y_1 + y_2 \geq 3$$
$$y_1 + 2y_2 \geq 1$$
$$y_1 + y_2 \geq 2$$
$$y_1 , \; y_2 \geq 0$$

Because this primal problem has more functional constraints than variables, suppose that the simplex method has been applied directly to its dual problem. If we let s_1 and s_2 denote the slack variables for this dual problem, the resulting final simplex tableau is

Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS	BV
1	3	0	2	0	1	1	9	Z = 9
0	1	1	-1	0	1	-1	1	$x_2 = 1$
0	2	0	3	1	-1	2	3	$x_4 = 3$

For each of the following independent changes in the original primal model, you now are to conduct sensitivity analysis by directly investigating the effect on the dual problem and then inferring the complementary effect on the primal problem.

a) Find the allowable range in the objective function coefficient of y_1 to remain the current basis optimal.
b) If the objective function coefficient of y_1 is changed to 3, what would be the new optimal W?
c) Find the allowable range in the right hand side of the first constraint to remain the current basis optimal.
d) If the right hand side of the first constraint is changed to 5, what would be the new optimal W?

41. Basic variables of a solution of the following transportation problem are $x_{11}, x_{12}, x_{23}, x_{31}, x_{33}, x_{43}$ and x_{44}.

	1	2	3	4	s_i
1	9	7	12	8	18
2	15	12	12	15	4
3	8	9	6	12	6
4	14	12	11	12	12
d_j	6	14	15	5	

a) Is the solution basic?
b) Show that the solution is optimal.
c) Give the original linear programming problem and its dual.
d) Derive the optimal solution to the dual problem.
e) Suppose that c_{43} is increased from 11 to 13. Is the solution still optimal? If not, find the new optimal solution.
f) If the supply of 2 and demand of 4 increase by 2 units, what will be the optimal solution?

42. FSK manufactures three products P1, P2, and P3 on two machines M1 and M2. Each of the products must be processed on both machines in arbitrary order. Operators are required to operate the machines. 0.5 hours of operator hour is required to operate a machine for one hour. The unit profits of the products are 8TL, 11TL, and 7TL, respectively. The machine capacities are 40 and 30 hours per planning period. 30 operator hours are available per planning period. The following table indicates how many units of the products can be made each hour.

	P1	P2	P3
M1	4	6	9
M2	7	3	13

Additionally, it is required that at least fifteen units of the second product are made. Formulate a profit-maximizing LP model (What are the decision variables, objective, and the constraints? Indicate sign restrictions if any. Please do not try to solve the problem).

43. Solve the LP model given below using the Simplex Method and submit an executive summary for your solution.

$$\max z = 4x_1 + x_2$$
s.t.
$$2x_1 + 3x_2 \leq 4$$
$$x_1 + x_2 \leq 1$$
$$4x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

44. Solve the LP model given below utilizing two-phase simplex method and submit an executive summary for your solution.

$$\min z = 3x_1$$
s.t.
$$2x_1 + x_2 \geq 6$$
$$3x_1 + 2x_2 = 4$$
$$x_1, x_2 \geq 0$$

45. A bank employs both full-time and part-time tellers. The bank is open from 9 to 17. Full-time tellers work from 9 to 17, except for 1 hour off for lunch, and they are paid 64 TL per day. Lunch for full time tellers may be between 12 and 13 or between 13 and 14. Part-time tellers work at least 3 hours and at most 4 hours each working day without lunch break and they are paid 5 TL per hour. The number of part-time tellers should not exceed 5. Teller requirement of the bank is shown in the table below. Formulate an LP model to minimize the cost. (What are the decision variables, objective, and the constraints? Indicate sign restrictions if any. Please do not try to solve the problem).

Time Period	Teller Requirement
9 a.m. – 10 a.m.	4
10 a.m. – 11 a.m.	3
11 a.m. - noon	4
Noon – 1 p.m.	6
1 p.m – 2 p.m.	5
2 p.m – 3 p.m.	6
3 p.m – 4 p.m.	8
4 p.m – 5 p.m.	8

46. Consider the following LP

$$\begin{aligned} \max z &= 2x_1 + 4x_2 + 3x_3 + x_4 \\ \text{s.t.} \quad & 3x_1 + x_2 + x_3 + 4x_4 \le 12 \\ & x_1 - 3x_2 + 2x_3 + 3x_4 \le 7 \\ & 2x_1 + x_2 + 3x_3 - x_4 \le 10 \\ & x \ge 0 \end{aligned}$$

The optimal solution of this LP: $z = 42$; $x_1 = 0$; $x_2 = 10.4$; $x_3 = 0$; $x_4 = 0.4$

a. Find the dual of the LP.
b. Using optimal solution and complementary slackness conditions, find solution of the dual model.
c. Using the dual model solution, find shadow prices and reduced costs of the primal model.

47. ATK-Sugar can manufacture three types of candy bar. Each candy bar consists totally of sugar and chocolate. The first type of candy bar is composed of one units of sugar and two units of chocolate. The second type is composed of one units of sugar and three units of chocolate and the third type is composed of one unit of sugar and one unit of chocolate. Profits earned from sales of one unit of each candy are 3 TL, 6 TL and 4 TL, respectively. Fifty units of sugar and 80 units of chocolate are available. After defining xi to be the number of Type i candy bars manufactured, ATK-Sugar should solve the following LP:

$$\begin{aligned} \max z &= 3x_1 + 6x_2 + 4x_3 \\ \text{S.t.} \quad & x_1 + x_2 + x_3 \le 50 \qquad \text{(sugar constraint)} \\ & 2x_1 + 3x_2 + x_3 \le 100 \qquad \text{(chocolate constraint)} \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

The optimal solution of the given LP is on the basic feasible solution where x_2 and x_3 are basic variables and Z= 260, $x_1 = 0$, $x_2 = 30$, $x_3 = 20$. Answer the following questions using simplex method:

Hint : $\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1/2 & 1/2 \\ 3/2 & -1/2 \end{bmatrix}$

a) If the profit for a Type 1 candy bar were 4TL, what would be the new optimal solution to the problem?
b) For what values of Type 3 candy bar profit would the current basis remain optimal? If the profit for a Type 3 candy bar were 7 TL, then what would be the new optimal solution to the problem?
c) For what amount of available sugar would the current basis remain optimal?
d) If 70 units of sugar were available, what would be ATK-Sugar's profit? How many of each candy bar should the company make?

e) Suppose ATK-Sugar develops a new candy bar, which is composed of 2 units sugar and 1 unit chocolate and its marginal profit is 5 TL. Is it profitable to produce this new product? If it is profitable, find the new production plan. If it is not profitable, what should be the marginal profit of the new product to make it profitable?

48. A person who has established a new search engine, plans to earn his revenues by advertisement on his site. This person has obtained advertisements from 3 local retailer companies, namely A, B and C, and has decided to broadcast their advertisements. On the other hand, A, B and C have declared that their advertisements should appear on the site if one of these words are searched in the engine: market - shopping - retail - discount

The site owner predicts that the word "market" will get 35000 hits, the word "shopping" will get 25000 hits, the word "retail" will get 15000 hits and the word "discount" will get 5000 hits. Furthermore, according to his predictions, 20% of the users who see an advertisement, will click on the advertisements shown. A,B and C make their payments show in the table below for each advertisement that is clicked on:

	Market	Shopping	Retail	Discount
A	2	3	11	7
B	1	2	6	1
C	5	8	15	9

The websites of A, B and C have different capacities. A can get maximum 6000 visitors, B can get maximum 2000 visitors and C can get maximum 9000 visitors.

For each word searched, only one advertisement for one firm can be shown on the search engine. The site owner wants to maximize his revenue by deciding on which word and how many of that word should result in showing advertisement of each firm.

a) Model this problem as a transportation problem by constructing a balanced transportation table.

b) Find an initial basic feasible solution using Vogel's Approximation Method (VAM).

49. The chair of the ITU Industrial Engineering Department will assign two professors (A and B) to the new three courses. One professor will be assigned at most two courses. The professors ranked the courses in a 1-10 scale. A rating of 10 means that the professor wants to teach that course and a ranking of 1 means that he or she does not want to teach that course. The cell including (--) in the table indicates that the chair of the department will not assign the related course to the related professor.

Professors	**Courses**		
	END500	END501	END502
A	8	10	7
B	4	8	--

a) Formulate an assignment problem to assign professors to courses that maximizes the total satisfaction of the professors (formulate the assignment table).

b) Solve the assignment problem you have formulated in part (a) using Hungarian Method. Determine an assignment of professors to courses. What will be the total satisfaction?

50. A company manufactures four products (1,2,3,4) on two machines (X and Y). The time (in minutes) to process one unit of each product on each machine is shown below:

Product	Machine X	Machine Y
1	10	27
2	12	19
3	13	33
4	8	23

The profit per unit for each product (1,2,3,4) is 10 TL, 12 TL, 17 TL and 8 TL respectively. Product 1 must be produced on *both* machines X and Y but products 2, 3 and 4 can be produced on *either* machine.

The factory is very small and this means that floor space is very limited. Only one week's production is stored in 50 square meters of floor space where the floor space taken up by each product is 0.1, 0.15, 0.5 and 0.05 (square meters) for products 1, 2, 3 and 4 respectively.
Customer requirements mean that the amount of product 3 produced should be related to the amount of product 2 produced. Over a week approximately twice as many units of product 2 should be produced as product 3.
Machine X is out of action (for maintenance/because of breakdown) 5% of the time and machine Y 7% of the time.
Assuming a working week 35 hours long formulate the problem of how to manufacture these products as an LP.

51. Solve the following LP utilizing dual simplex method.

$$\min w = 12y_1 + 7y_2 + 10y_3$$

$$\begin{aligned} \text{s.t.} \quad & 3y_1 + y_2 + 2y_3 \geq 2 \\ & y_1 - 3y_2 + y_3 \geq 4 \\ & y_1 + 2y_2 + 3y_3 \geq 3 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

52.

$$\begin{array}{lrrrrrrl} \text{Max} & -2x_1 & -3x_2 & +x_3 & & +3x_5 & +2x_6 & \\ \text{Subject to} & 2x_1 & +x_2 & +x_3 & +3x_4 & +2x_5 & +4x_6 & = 6 \\ & 2x_1 & -x_2 & -2x_3 & +5x_4 & +3x_5 & +2x_6 & = 2 \\ & -x_1 & +x_2 & +3x_3 & +5x_4 & +x_5 & -x_6 & = 1 \\ & x_1, & x_2, & x_3, & x_4, & x_5, & x_6 & \geq 0 \end{array}$$

For the above given LP, the revised simplex tableau related to the BV={x_1, x_2, x_3} is as follows:

	-3	4	4	-6
x_1	0.2	0.4	0.2	2.2
x_2	0.8	-1.4	-1.2	0.8
x_3	-0.2	0.6	0.8	0.8

a) Prove that this basic feasible solution is not optimal.
b) Use revised simplex method for one step to find a new basic feasible solution. Determine whether the new solution is optimal or not.

53. Consider the following transportation table for a minimization problem.

	1	2	3	4	Supply
1	3	4	3	3	60
2	6	5	9	4	70
3	3	2	1	2	90
Demand	100	60	40	20	

a) A basic feasible solution for the given transportation is given as BV: {x_{11}, x_{13}, x_{21}, x_{24}, x_{32}, x_{33}}. Find the values of the basic variables. Prove that this solution is not optimal.

b) Find the optimal solution using transportation simplex method starting from the basic feasible solution given in part a.

c) Find the range of values of the c_{24} (the cost related to x_{24}) for which the current basis remains optimal.

54. Gimignano Company produces a hot dog mixture in 1000 pound batches. The mixture contains two ingredients: chicken and beef. The cost per pound of chicken is \$3; the cost per pound of beef is \$5. The ratio of chicken to beef must be at least 2 to 1. Each batch has the following recipe requirements:

- At least 500 pounds of chicken
- At least 200 pounds of beef

The company wants to know the optimal mixture of ingredients that will minimize cost.

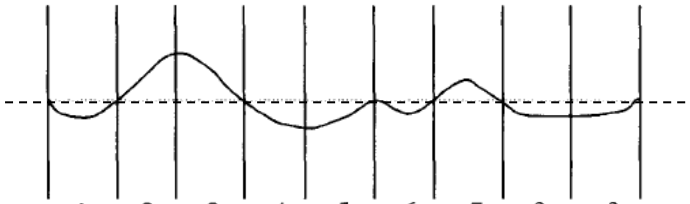
a) Formulate an LP model (*What are the decision variables, objective, and the constraints? Indicate sign restrictions if any*).

b) Assume that there is no recipe requirement. Solve the modified problem utilizing two phase method.

55. Write down the closed form LP formulation of the problem given in GAMS code below.

```
sets
        i       plants            / p1,p2,p3 /
        j       screning plant    / s1, s2 /;

scalar   TM     unit transportation cost   /3/;

parameters
         U(i)    plant capacity
         / p1  700
           p2  600
           p3  200/

         KE(j)   screning plant capacity
         / s1 700
           s2 800 /

         M(j)   screning cost
         / s1   20
           s2   30 /;

table DF(i,j)  distance between plants and screning plants
                     s1        s2
           p1        30        5
           p2        36        42
           p3        20        34 ;

positive variable
      X(i,j)  transportation between plant i to screning plant j;
variable
      Z      total cost;

equations
      obj                        objective function
      plantConstraint(i)        total transportaion from each plant
      ScreningConstraint(j)      total transportaion to each screning plant;

obj..  Z =e= sum((i,j), TM* DF(i,j)* X(i,j))+ sum(j, M(j) * sum(i,X(i,j)));
plantConstraint(i)..    sum(j, X(i,j)) =e= U(i);
ScreningConstraint(j)..    sum(i, X(i,j)) =l= KE(j);

model midterm1 /all/;
solve midterm1 using lp minimizing Z;
```

56. A highway contract requires a contractor to alter the terrain of a section of road work. The work involves cut and fill of earth so that the original profile will be much flatter. The original profile and the finished profile are shown in solid line and dotted line respectively in following figure:



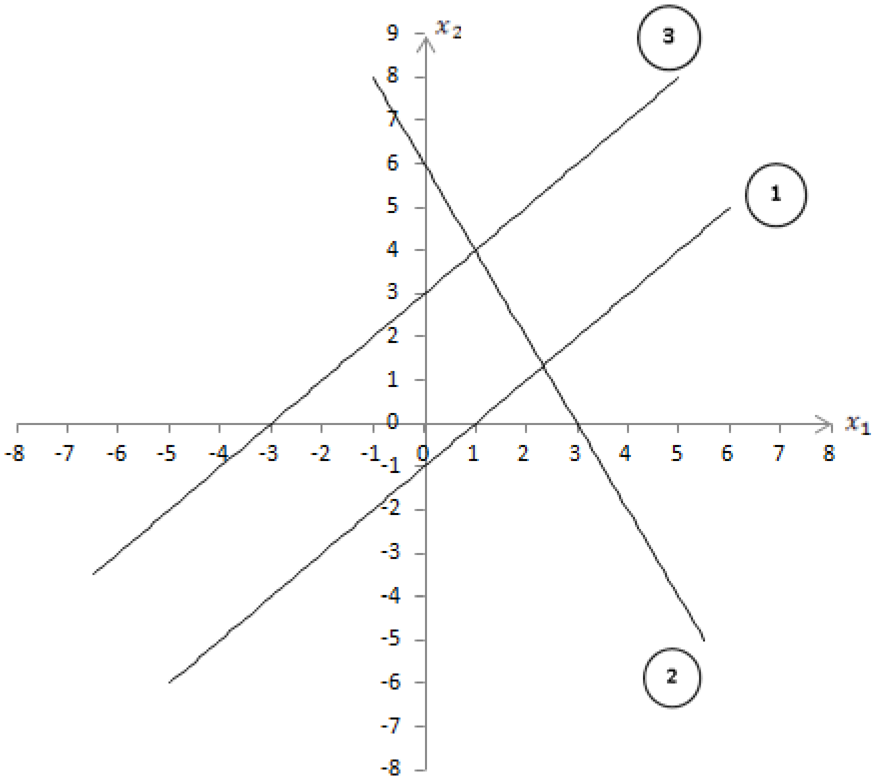
Section No.	1	2	3	4	5	6	7	8	9
Cut ($m^3 x 10^3$)		28	32				13		
Fill ($m^3 x 10^3$)	10			14	16	10		12	11

The roadway is divided into nine sections. For example; 10 x 10^3 m^3 of fill is required in section 1; 28 x 10^3 m^3 of cut in section 2; and so on. The cost for cutting including loading is \$8 per m^3 and that for filling including compaction is \$12 per m^3.The unit cost of transporting earth by trucks from one section to another is \$2 per m^3 per section (for instance if 3 m^3 of earth is moved from section 2 to section 6, transportation cost will be 2*3*4=\$24).
Formulate an LP to determine how much earth should be moved from where to where with minimum cost.

57. The lines defining the following constraints are graphed:

$$
\begin{aligned}
x_2 - x_1 &\geq -1 \quad (1)\\
x_2 + 2x_1 &\leq 6 \quad (2)\\
x_2 - x_1 &\leq 3 \quad (3)\\
x_1, x_2 &\geq 0
\end{aligned}
$$

a) Shade in the feasible region corresponding to the constraints.



b) Name an objective function for which the optimal solution would be $(x_1, x_2) = (1, 4)$.
c) Name an objective function for which $(x_1, x_2) = (2,1)$ would be an optimal solution.

58. Consider the LP Model and solution of Oranj Juice example in the Lecture notes.

a. Write down the dual of the given LP.

b. Find the optimal solution of the dual model (values of the decision variables and objective function) using the complementary slackness theorem.

Oranj Juice example, LP Model:

Let x_1 and x_2 be the quantity of ounces of orange soda and orange juice (respectively) in a bottle of Oranj.

min $z = 2x_1 + 3x_2$

s.t. $0.5\,x_1 + 0.25\,x_2 \le 4$ (sugar const.)

$x_1 + 3\,x_2 \ge 20$ (vit. C const.)

$x_1 + x_2 = 10$ (10 oz in bottle)

$x_1, x_2 \ge 0$

Optimal Solution:

x_1=5, x_2=5, Z = 25.

59. ATK Beyaz produces three products using two resources, namely labor and a special machine tool. Data related to the production process are given in the following table.

	Labor (hours)	Machine tool (hours)	Unit profit (TL)
Product 1	2	1	8
Product 2	2	2	14
Product 3	4	1	7
Daily capacity	90	60	

The following LP is formulated to maximize company's total daily profit.

Let x_1, x_2 and x_3 be the production quantity of products 1, 2, and 3 (respectively) in a day.

max $z = 8x_1 + 14x_2 + 7x_3$

Subject to;

$2x_1 + 2x_2 + 4x_3 \le 90$

$x_1 + 2x_2 + x_3 \le 60$

$x_1, x_2, x_3 \ge 0$

Consider the problem and the LP to answer the following questions:

a) Prove that the basic feasible solution where x_1 and x_2 are basic variables (i.e., BV = { x_1 , x_2 }) is an optimal solution (Hint; $\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ -0,5 & 1 \end{bmatrix}$).

b) Form the simplex tableau where x_1 and x_2 are basic variables (Show the details of your calculations).

Z	x_1	x_2	x_3	s_1	s_2	RHS
1						
0						
0						

c) What would be the optimal solution if the profit of Product 1 is changed to 10 TL? (Use simplex method if necessary)

d) What would be the optimal solution if the daily capacity of the special machine tool is increased to 100 hours? (Use dual simplex method if necessary)

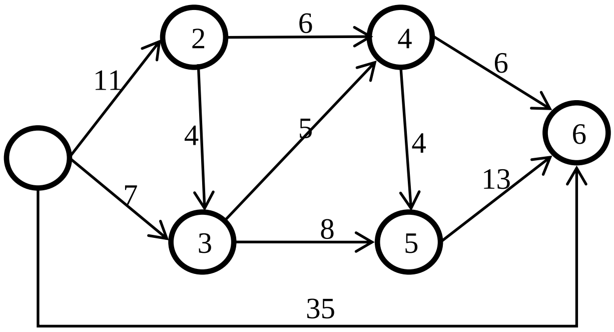
60. Three electric power plants with capacities of 25, 40, and 30 million kWh supply electricity to three cities. The maximum demands at the three cities are estimated at 30, 35, and 25 million kWh. The price per million kWh at the three cities is given in Table, below. During the month of August, there is a 20% increase in demand at each of the three cities, which can be met by purchasing electricity from another network at a premium rate of $1000 per million kWh. The network is not linked to city 3, however. The utility company wishes to determine the most economical plan for the distribution and purchase of additional energy.

		City		
		1	2	3
	1	\$600	\$700	\$400
Plant	2	\$320	\$300	\$350
	3	\$500	\$480	\$450

a) Formulate the problem as a transportation model (Show LP formulation and form the transportation tableau).

b) Determine an optimal distribution plan for the utility company using transportation simplex method (Use Vogel to find initial solution).

61. Consider the shortest path problem from node 1 to node 6 defined in the following network (related costs - cij - are given on each arc) . Solve the problem using Dijkstra Algorithm (show your calculations in details for each iteration).



62. Find the values of A, B, C, D, E, F, G and H by examining optimal tableau and Lindo output.

max $z = -3x_1 + x_2 + 2x_3$

subject to

$x_2 + 2x_3 \leq 6$

$-x_1 + 3x_3 \leq 4$

$-2x_1 + 3x_2 - x_3 \leq 9$

$x_1, x_2, x_3 \geq 0$

Optimal tableau:

z	x_1	x_2	x_3	s_1	s_2	s_3	ST
1	3	0	0	1	0	0	6
0	2/3	1	0	1	-2/3	0	10/3
0	-1/3	0	1	0	1/3	0	4/3
0	-13/3	0	0	-3	7/3	1	1/3

Lindo output:

```
 LP OPTIMUM FOUND AT STEP       2

        OBJECTIVE FUNCTION VALUE

        1)       6.000000

  VARIABLE          VALUE          REDUCED COST
        X1          0.000000           A
        X2          3.333333           0.000000
        X3          B                  0.000000

       ROW    SLACK OR SURPLUS     DUAL PRICES
        2)          0.000000           1.000000
        3)          0.000000           0.000000
        4)          C                  D

 NO. ITERATIONS=        2
```

```
RANGES IN WHICH THE BASIS IS UNCHANGED:

                          OBJ COEFFICIENT RANGES
VARIABLE          CURRENT        ALLOWABLE         ALLOWABLE
                   COEF          INCREASE          DECREASE
      X1        -3.000000           E                 F
      X2         1.000000         0.000000          1.000000
      X3         2.000000         9.000000          0.000000

                          RIGHTHAND SIDE RANGES
     ROW          CURRENT        ALLOWABLE         ALLOWABLE
                   RHS           INCREASE          DECREASE
       2         6.000000         0.111111          3.333333
       3         4.000000         5.000000          0.142857
       4         9.000000           G                 H
```

A = ………… B = ………… C = ………… D = …………

E = ………… F = ………… G = ………… H = …………

63. Currently we own 100 shares each of stocks 1 through 6. The original price we paid for these stocks, today's price, and the expected price in one year for each stock is shown in the following table. We need money today and are going to sell some of our stocks.

The tax rate on capital gains is 30%. If we sell 50 shares of stock 1, then we must pay tax of 0.3 x 50 x (30 - 20) = $150. We must also pay transaction costs of 1% on each transaction. Thus, our sale of 50 shares of stock 1 would incur transaction costs of 0.01 x 50 x 30 = $15.

After taxes and transaction costs, we must be left with $12,000 from our stock sales. Our goal is to maximize the expected (before-tax) value in one year of our remaining stock. Formulate a linear program to find what stocks we should sell. Assume it is all right to sell a fractional share of stock.

Stock	Shares Owned	Purchase Price	Current Price	Price in one year
1	100	20	30	36
2	100	25	34	39
3	100	30	43	42
4	100	35	47	45
5	100	40	49	51
6	100	45	53	55

64. Find the optimal solution of the following LP by using graphical solution technique (Hint: use complementary slackness theorem).

max $z = 3x_1 + 4x_2 + x_3 + 5x_4$

s.t. $x_1 + 2x_2 + x_3 + 2x_4 \le 5$

$2x_1 + 3x_2 + x_3 + 3x_4 \le 8$

$x_1, x_2, x_3, x_4 \ge 0$

65. A company has five jobs to be done. The following matrix shows the return in USD of assigning i^{th} machine (i=1,2,3,4,5) to the j^{th} job (j=1,2,3,4,5). Assign jobs to the machines (utilize Hungarian method) so as to <u>maximize the expected profit</u>

		Jobs				
		1	2	3	4	5
Machines	1	5	11	10	12	4
	2	2	4	6	3	5
	3	3	12	5	14	6
	4	6	14	4	11	7
	5	7	9	8	12	8

66. Türker already saved up 300 TL cash. Additionally, at the beginning of each month Türker earns salary and pays bills as shown in the table below. Any money left over will be invested for one month at the interest rate of 0.1% per month; for two months at 1% per two months; for three months at 3% per three months; or for four months at 8% per four months. Develop an LP model to determine an investment strategy that maximizes the cash on hand at the beginning of month 5.

Month	1	2	3	4
Earnings (TL)	400	800	300	300
Bills (TL)	600	500	500	250

67. Reformulate the following optimization problem as an equivalent LP problem.
(x_j are the decision variables and; a_{ij} and b_{ij} are the given coefficients)

$$min \frac{1}{m}\sum_{i=1}^{m} max\{0, -\sum_{j=1}^{n} a_{ij}x_j + x_{n+1} + 1\} + \frac{1}{k}\sum_{i=1}^{m} max\{0, \sum_{j=1}^{n} b_{ij}x_j - x_{n+1} + 1\}$$

68. min z = $4x1 + x2$

s.t. $3x_1+x_2 \geq 6$

$3x_1+5x_2 \geq 15$

$x_1+x_2 \geq 4$

$x_1,x_2 \geq 0$

a) Optimal tableau for the Phase I of two phase simplex solution for the above LP model is given below. Solve Phase II to find the optimal solution of the model.

x_1	x_2	e_1	e_2	e_3	a_1	a_2	a_3	RHS
0	0	0	0	0	-1	-1	-1	0
1	0	0	1/2	-5/2	0	-1/2	5/2	5/2
0	1	0	-1/2	3/2	0	1/2	-3/2	3/2
0	0	1	1	-6	-1	-1	6	3

b) Illustrate the path that solutions follow at each iteration of Phase II on a graph.

69. Compare degeneracy vs. alternative optima.

70. Consider the following LP and its optimal tableau.

Min z = $-3x_1 - 2x_2 - 5x_3$

s.t. $x_1 + 2x_2 + x_3 \leq b_1$

$3x_1 + 2x_3 \leq b_2$

$x_1 + 4x_2 \leq b_3$

x_1 , x_2 , $x_3 \geq 0$

Note that b_1, b_2 and b_3 are constants.

Optimal tableau:

z	x_1	x_2	x_3	s_1	s_2	s_3	RHS	BV
1	a	b	0	c	d	0	-1350	z
0	g	e	0	1/2	-1/4	0	100	x_2
0	3/2	0	1	0	1/2	0	230	x_3
0	2	0	0	-2	1	f	20	s_3

a) Find a,b,c,d,e,f,g values by usingrevised simplex formulations (Please fill the following table)

a	b	c	d	e	f	g

b) Find b_1, b_2 and$b_3$values using revised simplex formulations (Please fill the following table and show your work in details)

b_1	b_2	b_3

c) Write down the dual of the given LP and find the solution of the dual model by using optimal solution of the primal model. (Do not solve the dual model!)

d) Find ranges for the objective function coefficient of x_2 in which the current basis remain optimal. Find the new optimal solution if the coefficient of x_2 is equal to -1 (use revised simplex or dual simplex if necessary).

e) Find the ranges for b_1 in which the current basis remain optimal. Find the new optimal solution if b_1 = 200 (use revised simplex or dual simplex if necessary).

71. The Airline Company builds commercial airplanes and the last stage in the production process is to produce the jet engines and then to install them in the completed airplane frame. The company has been working under some contracts to deliver airplanes in the near future, and the production of the jet engines for these planes must now be scheduled for the next 4 months. To meet the contracted dates for delivery, the company must supply engines for installation in the quantities indicated in the second column of the Table. The facilities that will be available for producing the engines vary according to works scheduled during this period. The resulting monthly differences in the maximum number that can be produced and the cost (in millions of dollars) of producing each one are given in the third and fourth columns of Table. Because of the variations in production costs, it may well be worthwhile to produce some of the engines a month or more before they are scheduled for installation. The drawback is that such engines must be stored until the scheduled installation (the airplane frames will not be ready early) at storage cost of $100,000 per month for each engine.

Month	Scheduled Installations	Maximum Production	Unit Cost of Production (million $)
1	10	25	2.08
2	15	35	2.11
3	25	30	2.10
4	20	10	2.13

The production manager wants a schedule developed for the number of engines to be produced in each of the 4 months so that the total of the production and storage costs will be minimized.

a) Formulate a balanced transportation model for the problem (Form the transportation tableau).
<u>Answer one of the following questions</u> (<u>Indicate which one you choose</u>);

b) Find a basic feasible solution to the transportation problem in part a using minimum cost method.

c) Find a basic feasible solution to the following transportation problem using Least-Cost method.

	1	**2**	**3**	
1	4	17	2	5
2	8	18	9	7
3	2	16	4	9
4	1	7	3	10
	8	10	13	

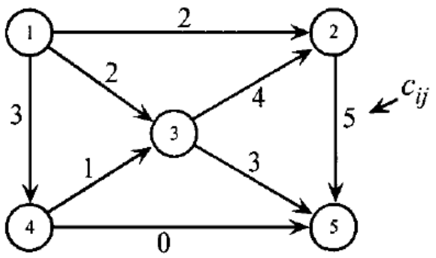
72. A company is involved in the production of two items (X and Y). The resources need to produce X and Y are twofold, namely machine time for automatic processing and craftsman time for hand finishing. The table below gives the number of minutes required for each item:

	Machine time	Craftsman time
Item X	13	20
Item Y	19	29

The company has 40 hours of machine time available in the next working week but only 35 hours of craftsman time. Machine time costs 10 TL per hour worked and craftsman time costs 2 TL per hour worked. Both machine and craftsman idle times incur no costs. The revenue received for each item

produced (all production is sold) is 20 TL for X and 30 TL for Y. The company has a specific contract to produce 10 items of X per week for a particular customer.
Formulate the problem of deciding how much to produce per week as an LP (*define decision variables, indicate objective function, constraints, and sign restrictions*).

73. Consider the following shortest path problem between nodes 1 and 5. cij parameters are given on each arc. Convert the problem to an assignment problem and solve it using the Hungarian Method.



74. Consider the following balanced transportation tableau for a minimization problem.

	1	2	3	
1	4	15	2	6
2	8	12	9	8
3	2	10	4	10
4	1	7	3	12
	8	13	15	

a) Show that BV = {x_{11}, x_{21}, x_{22}, x_{32}, x_{42}, x_{43}} is a basic solution but not a feasible solution.
b) Show that BV = {x_{13}, x_{22}, x_{31}, x_{33}, x_{41}, x_{42}} is the optimal solution.
c) Find the ranges for c_{42} for which the current solution remains optimal.
d) Find the optimal solution if c_{42} = 5.

75. Beerco manufactures ale and beer from corn, hops, and malt. Currently 40 lb. of corn, 30 lb. of hop, and 40 lb. of malt are available. A barrel of ale sells for \$40 and requires 1 lb. of corn, 1 lb. of hop, and 2 lb. of malt. A barrel of beer sells for \$50 and requires 2 lb. of corn, 1 lb. of hop, and 1 lb. of malt. Beerco wants to maximize total sales revenue. Examine the given optimal tableau for the LP model of problem and answer the following questions.
a) What are the optimal values of primal decision variables (ALE and BEER) and the optimal values of dual decision variables (y_1, y_2, y_3) ?
b) Suppose Beerco is considering manufacturing malt liquor. A barrel of malt liquor may be sold for \$50 and requires 0.5 lb. of corn, 3 lb. of hop, and 3 lb. of malt. Should Beerco manufacture and sell any malt liquor? (utilize "duality and sensitivity analysis")

Optimal tableau:

z	Ale	Beer	s_1	s_2	s_3	RHS
1	0	0	20	0	10	1200
0	0	1	2/3	0	-1/3	40/3
0	0	0	-1/3	1	-1/3	10/3
0	1	0	-1/3	0	2/3	40/3

Optimal value of ALE: …………… Optimal value of BEER: ……………
Optimal value of y_1: …………… Optimal value of y_2: …………… Optimal value of y_3: ……………

76. Find the values of A, B, C, D, E, F, G and H by examining optimal tableau and Lindo output.

max $3x_1 + 2x_2 + x_3$

s.t.

$x_2 + 2x_3 \le 6$

$x_1 - 3x_3 \le 4$

$x_1 + x_2 + x_3 \le 3$

$x_1, x_2, x_3 \ge 0$

Optimal tableau:

z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
1	0	1	2	0	0	3	9
0	0	1	2	1	0	0	6
0	0	-1	-4	0	1	-1	1
0	1	1	1	0	0	1	3

Lindo output:

```
        OBJECTIVE FUNCTION VALUE

        1)       9.000000

  VARIABLE        VALUE          REDUCED COST
        X1          A                0.000000
        X2         0.000000         B
        X3         0.000000         2.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         6.000000         0.000000
        3)         C                0.000000
        4)         0.000000         D

 NO. ITERATIONS=        2


 RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
       X1        3.000000        INFINITY          1.000000
       X2        2.000000         E                F
       X3        1.000000        2.000000          INFINITY

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
        2        6.000000        INFINITY          6.000000
        3        4.000000         G                H
        4        3.000000        1.000000          3.000000
```

A = ………… B = ………… C = ………… D = …………

E = ………… F = ………… G = ………… H = …………