END332E OPERATIONS RESEARCH II
PREVIUOS EXAM QUESTIONS

1. Which assumptions of LP are satisfied in IP, which of them are not satisfied? (Only give the names of assumptions. No comment please!)

2. Formulate a mixed binary IP having 2 decision variables and 2 constraints.

3. Write a LINDO program that can solve the IP given below.

   max $z = 5x + 8y$
   s.t. $x + y \leq 12$
   $3x \geq 9$
   $x \geq 0$ and integer; $y = 0$ or 1

4. The Smalltown Fire Department currently has seven conventional ladder companies and seven alarm boxes. The two closest ladder companies to each alarm box are given in the following table. The city council wants to maximize the number of conventional ladder companies that can be replaced with tower ladder companies. Unfortunately, political considerations dictate that a conventional company can be replaced only if, after replacement, at least one of the two closest companies to each alarm box is still a conventional company.

Alarm box	1	2	3	4	5	6	7
Two closest ladder companies	2,3	3,4	1,5	2,6	3,6	4,7	5,7

   a. Formulate an IP that can be used to maximize the number of conventional companies that can be replaced by tower companies.
   b. Suppose $y_k = 1$ if conventional company k is replaced. Show that if we let $z_k = 1 - y_k$, the answer in part (a) is equivalent to a set-covering problem.

5. Use Branch and Bound method to find the optimal solution to the following IP:

   max $z = 11x_1 + 2x_2$
   s.t. $2x_1 + x_2 \leq 1$
   $x_1 + 3x_2 \leq 2$
   $x_i = 0$ or 1

6. Answer the following questions:
   a. For what values of a, b, and c will $ax_1^2 + bx_1x_2 + cx_2^2$ be a convex function on R^2?
   b. For what values of a, b, and c will $ax_1^2 + bx_1x_2 + cx_2^2$ be a concave function on R^2?
   c. On the given set $S=(0,\infty)$, determine whether x^a $(0 \leq a \leq 1)$ is convex, concave, or neither

7. Ford has four automobile plants. Each is capable of producing the Focus, Fusion, or Mondeo, but it can only produce one of these cars. The fixed cost of operating each plant for a year and the variable cost of producing a car of each type at each plant are given in the table given below.

	Fixed cost	Variable Cost ($)		
Plant	(million $)	Focus	Mondeo	Fusion
1	7,000	12,000	16,000	9,000
2	6,000	15,000	18,000	11,000
3	4,000	17,000	19,000	12,000
4	2,000	19,000	22,000	14,000

   Ford faces the following restrictions:

i. Each plant can produce only one type of car
ii. The total production of each type of car must be at a single plant; that is, for example, if any Focuses are made at Plant 1, then all Focuses must be made there
iii. If plants 3 and 4 are used, then Plant 1 must also be used.

Each year Ford must produce 500,000 of each type of car. Formulate (do not solve) an IP whose solution will tell Ford how to minimize the annual cost of producing cars.

8. Consider the following IP:

$$\min z = 6x_1 + 8x_2$$
$$\text{s.t.} \quad 3x_1 + x_2 \geq 4$$
$$x_1 + 2x_2 \geq 4$$
$$x_1, x_2 \geq 0 \text{ and integer}$$

The optimal tableau for this IP's LR is given in the following table (M=100).

z	x_1	x_2	e_1	e_2	a_1	a_2	RHS
1	0	0	-0.8	-3.6	-99	-96	17.6
0	1	0	-0.4	0.2	0.4	-0.2	0.8
0	0	1	0.2	-0.6	-0.2	0.6	1.6

Use the cutting plane algorithm to find the optimal solution.

9. COLLUSIVE DUOPOLY MODEL: There are two firms producing "hobalays". It costs the first firm q_1 dollars to produce q_1 hobalays and the second firm $0.5q_2^2$ dollars to produce q_2 hobalays. If a total of q hobalays are produced, consumers will pay \$200–q for each hobalay. If the two manufacturers want to collude in an attempt to maximize the sum of their profits, how many hobalays should each company produce?

Collusion is an agreement between parties to refrain in participating in an activity that they normally would in order to reduce competition and gain higher profits (collude: combine together secretly)

10. Please describe geometrically "Finding the golden section point on a line"

11. Before a new product can be introduced, the activities in the following table must be completed (all times are in weeks)

Activity	Description	Pred.	Duration	a	b	m
A	Design the product	-	6	2	10	6
B	Survey the market	-	5	4	6	5
C	Place orders for RM	A	3	2	4	3
D	Receive RM	C	2	1	3	2
E	Build prototype of product	A, D	3	1	5	3
F	Develop ad campaign	B	2	3	5	4
G	Set up plan for mass production	E	4	2	6	4
H	Deliver product to stores	G, F	2	0	4	2

Pred.: Predecessors,

a. Draw the project diagram. Determine all critical paths and critical activities (please utilize CPM)
b. Set up an LP that can be used to determine the critical path
c. It is now 12 weeks before New Year. What is the probability that the product will be in the stores before New Year?
d. The duration of each activity can be reduced by up to 2 weeks at the following cost per week: A, 80 NTL; B, 60 NTL; C, 30 NTL; D, 60 NTL; E, 40 NTL; F, 30 NTL; G, 20 NTL. Assuming that the duration of each activity is known with certainty; formulate an LP that will minimize the cost of getting the product into the stores by New Year.

12. There will be four professors in the Fenerbahce College Business School. Each semester, 200 students will take each of the following courses: marketing, finance, OR, and statistics. The effectiveness of each professor in teaching each class is given at the following table. Each professor can teach a total of 200 students during the semester. The dean has set a goal of obtaining an average teaching effectiveness level of about 6 in each course. Deviations from this goal in any course are considered equally important. Formulate a goal programming model that can be used to determine the semester's teaching assignments.

Professor	Marketing	Finance	OR	Statistics
1	7	5	8	2
2	7	8	9	4
3	3	5	7	9
4	5	5	6	7

13. Each day, Oahu manufactures four types of gasoline: lead-free regular (LFR), lead-free premium (LFP), low-sulphur premium (LSP), lead-free ultimate (LFU). Because of resetting of machinery, the time required to produce a batch of gasoline depends on the type of gasoline last produced. The time (in minutes) required to manufacture each day's gasoline requirements are shown in the following table. Use the Cheapest-Insertion Heuristic to determine the order in which the gasolines should be produced each day. Please only begin with the subtour LFR-LFP-LFR.

Last produced	Gas to be next produced			
gasoline	LFR	LFP	LSP	LFU
LFR	-	50	120	140
LFP	60	-	140	110
LSP	90	130	-	60
LFU	130	120	80	-

14. COURNOT DUOPOLY MODEL: Let's consider "collusive duopoly model" problem. The Cournot solution to this problem is obtained as follows: Firm *i* will produce q_i*, where if firm 1 changes its production level from q_1*(and firm 2 still produces q_2*), then firm 1's profit will decrease. Also if firm 2 changes its production level from q_2*(and firm 1 still produces q_1*), then firm 2's profit will decrease. Firm *i* will produce q_i*, this solution is stable, because if either firm changes its production level, it will do worse. Find q_1* and q_2*

A Cournot game is a game between two firms. Both produce a certain good. No other firms do. The price they receive is a decreasing function of the total quantity of goods that the firms produce. That function is known to both firms. Each chooses a quantity to produce without knowing how much the other will produce.

15. Describe the difference between Hamiltonian cycle and Euler cycle (only one sentence)

16. Let's assume that there are five cities in an ancient country: Fertilis, Pium, Munitus, Carus, and Unique. Also assume that there are ten roads connecting each pair of cities (R1: F-P, R2: F-M, R3: F-U, R4: F-C, R5: P-C, R6: C-U, R7: U-M, R8: M-P, R9: P-U, R10: C-M).

a. Draw a graph representing cities and roads of this country.
b. Reveal a feasible solution for a TSP problem on the network.
c. Reveal a feasible solution for a Chinese postman problem on the network.

17. Solve the knapsack problem below if the maximum possible total weight is 40. Interpret the results.

Item	Weight	Value
1	15	20
2	12	24
3	9	15
4	8	14

18. Expected length of a project is 37.16 days and the variance of its length is 6.14 days. What is the probability that the project is completed within 40 days?

19. Please use B&B to determine a way if any exists to place four queens on a 4×4 chessboard so that no queen can capture another queen (*Hint*: Let x_{ij} = 1 if a queen is placed in row *i* and column *j* of the chessboard and x_{ij} =0 otherwise. A queen can move in a straight line vertically, horizontally, or diagonally, any number of unoccupied squares. She captures in the same way that she moves, replacing the opposing piece that got in her way).

20. We have 60 meters of fence and want to fence a triangular shaped area. Please formulate an NLP (*do not try to solve*) that will enable us to maximize the fenced area (*Hint*: The area of a triangle with sides of length *a*, *b*, and *c* is $(s (s - a) (s - b) (s - c))^{1/2}$, where *s* is half the parameter of the triangle).

21. Please use the method of steepest ascent to approximate the optimal solution to the following problem: max $z = - (x_1 - 2)^2 - x_1 - x_2^2$. Begin at point (6, 0).

22. Please answer the following questions:

a. What is the simple definition of the golden ratio Phi (Φ) and closely related value phi (ϕ)?
b. Describe the relation between Phi and Fibonacci numbers.

23. Fenerium must determine how many LCD TVs and DVD players should be stocked. It costs Fenerium $300 to purchase an LCD TV and $200 to purchase a DVD player. An LCD TV requires 3 units of storage space, and a DVD player requires 1 unit of storage space. The sale of an LCD TV earns Fenerium a profit of $150, and the sale of a DVD player earns Fenerium a profit of $100. Fenerium has set the following goals (listed in order of importance):

Goal 1: A maximum of $20,000 can be spent on purchasing LCD TVs and DVD players
Goal 2: Fenerium should earn at least $11,000 in profits from the sale of LCD TVs and DVD players.
Goal 3: LCD TVs and DVD players should use no more than 200 units of storage space.

Please formulate (*do not solve*) a preemptive goal programming model that Fenerium could use to determine how many LCD TVs and DVD players to order.

24. When an accounting firm audits a corporation, the first phase of the audit involves obtaining "knowledge of the business". This phase of the audit requires the activities in the table given below.

a. Please draw the project network and determine the critical path for the network.
b. Suppose that the cost of crashing each activity is as follows:

$A, 100A^2; B, 80B^2; C, 60C^2 ; D, 70D^2; E, 30E^2; F, 20F^2 ; G, 50G^2$

and that each activity may be crashed to a duration of 1 days, if desired.
Please formulate an NLP that will minimize the cost of finishing the project in 30 days or less.

Activity	Description	Immediate predecessors	Duration (Days)
A	Determining terms of engagement	-	3
B	Appraisal of auditability risk and materiality	A	6
C	Identification of types of transactions and possible errors	A	14
D	Systems description	C	8
E	Verification of systems description	D	4
F	Evaluation of internal controls	B, E	8
G	Design of audit approach	F	9

25. Find the KKT conditions for Giapetto problem and discuss their relation to the dual of the Giapetto LP

$$\text{Max } z = 3x_1 + 2x_2$$
$$\text{s.t.} \quad 2x_1 + x_2 \le 100$$
$$x_1 + x_2 \le 80$$
$$x_1 \le 40$$
$$x_1 \ge 0$$
$$x_2 \ge 0$$

26. Please formulate and solve the following problem by the DP backward recursive equation

$$\text{Max } z = 2x_1 + 3x_2 + 4x_3$$
$$\text{s.t.} \quad 2x_1 + 2x_2 + 3x_3 \le 4$$
$$x_1, x_2, x_3 \ge 0 \text{ and integers}$$

27. The Father Domino Company sells copying machines. A major factor in making a sale is Domino's quick service. Domino sells copiers in six cities: Boston, New York, Philadelphia, Washington, Providence, and Atlantic City. The annual sales of copiers projected depend on whether a service representative is within 150 miles of a city (see table).

Representative Within 150 Miles?	Sales					
	Boston	New York	Philadelphia	Washington	Providence	Atlantic City
Yes	700	1000	900	800	400	450
No	500	750	700	450	200	300

Each copier costs $500 to produce and sells for $1000. The annual cost per service representative is $80000. Domino must determine in which of its markets to base a service representative. Only Boston, New York, Philadelphia, and Washington are under consideration as bases for service representative. The distance (in miles) between the cities is shown in the second table. Formulate an IP that will help Domino maximize annual profits.

	Boston	New York	Philadelphia	Washington
Boston	0	222	310	441
New York	222	0	89	241
Philadelphia	310	89	0	146
Washington	441	241	146	0
Providence	47	186	255	376
Atlantic City	350	123	82	178

28. Fenerium can sell product 1 at a profit of 20 YTL/unit and product 2 at a profit of 5O YTL/unit. Three units of raw material are needed to manufacture 1 unit of product 1, and 6 units of raw material are needed to manufacture 1 unit of product 2. A total of 120 units of raw material are available. If any of product 1 is produced, a set up cost of $10 is incurred, and if any of product 2 is produced, a set up cost of $20 is incurred. Formulate an IP to maximize profits.

29. Suppose that a mathematical model fits linear programming except for the restrictions that at least two of the following four inequalities holds:

$$5x_1 + 3x_2 + 3x_3 - x_4 \le 10$$
$$2x_1 + 5x_2 - x_3 + 3x_4 \le 10$$
$$-x_1 + 3x_2 + 5x_3 + 3x_4 \le 10$$
$$3x_1 - x_2 + 3x_3 + 5x_4 \le 7$$

Show how to reformulate these restrictions to fit a Mixed Integer Programming model

30. Four jobs must be processed on a single machine. The time required to perform each job, the due date, and the penalty (in YTL) per day the job is late are given in Table below. Please determine the order of performing the jobs that will minimize the total penalty costs due to delayed jobs (*For simplicity, assume Job 4 as the last job*).

Job	Time (Days)	Due Date	Penalty
1	4	Day 4	4
2	5	Day 2	5
3	2	Day 13	7
4	3	Day 8	2

31. Please find the stationary points for the functions given below. Indicate whether these points are local optima, local maxima, or saddle points.

a. $f(\mathbf{x}) = x_1^2 + x_1x_2 + 5x_2^2 + 9(x_3-2)^2$

b. $f(\mathbf{x}) = x_1^2 - 2x_1 - x_2^2$

32. Taking into consideration the golden section numbers Phi (Φ) and phi (ϕ), please calculate the following three values:

Phi × phi = ; Phi – phi = ; Phi + phi =

33. Fenerbahce SK is trying to determine which of the following players should be signed for contract extension: Alexsandro de Souza, Semih Şentürk, Gökhan Gönül, Uğur Boral, Diego Alfredo Moreno Lugano. The cost of signing each player and the value each player will add to the team are shown in Table below. Fenerbahce SK wants to sign the players who will add the most value to the team concerning the $17 million budget limit. Which of the players should be signed?

Player	Cost of signing player ($ millions)	Value added to the team
Alex	7	8
Semih	5	7
Gökhan	4	6
Uğur	3	4
Lugano	3	3

34. The Işıl Electric Company, located in Istanbul's Old Town area, produces two products popular with home renovators: old-fashioned chandeliers and ceiling fans. Both the chandeliers and fans require a two-step production process involving wiring and assembly. It takes about 2 hours to wire each chandelier and 3 hours to wire a ceiling fan. Final assembly of the chandeliers and fans requires 6 and 5 hours, respectively. The production capability is such that only 12 hours of wiring time and 30 hours of assembly time are available. Each chandelier produced nets (yields as net profit) the firm $7 and each fan $6.

a. Formulate an IP that Işıl can use for production mix decision.
b. Find the optimal tableau for the IP's LR.
c. Generate a Gomory Cut using the optimal tableau for the LR (*only find the cut constraint*). Show the first iteration for the dual simplex algorithm (*only indicate the entering variable and leaving variable, do not find the optimal solution to IP*)

35. Reformulate (*do not solve*) Işıl Electric Problem as a goal programming model with the following goals (in order of importance):

Goal 1: Produce at least 4 chandeliers and 3 ceiling fans.
Goal 2: Limit overtime in the assembly department to 10 hrs. and in the wiring department to 6 hrs.
Goal 3: Maximize profit

36. Dr. Caligari has just retired and wants to invest his pension in one or more investment alternatives. Suppose that there are N alternative means of investment and he will earn $ $G_i(s)$ net present value if he invests $\$s$ in alternative i (i=1,2,..N). Develop a dynamic programming recursion that can be used to maximize Dr. Cagligari's total net present value of all investments (suppose that Dr. Cagliari has $\$D$ to invest and he may prefer not to invest some of his money).

37. The owner of a shipyard has to satisfy the demand of his customers for the following 4 years (d_1= 3, d_2= 4, d_3=5, d_4=2). The fixed cost of producing one or more ship on a shipyard is 30 million dollars per year. The variable cost of producing a ship can be considered to be 10 million dollars. If the holding cost of a ship is equal to %20 of its total cost (fixed + variable cost) per year, find production strategy (how much to produce each year) to meet the demands with minimum cost by using Wagner-Whithin Algorithm.

Information for question 38

Sudoku

The Sudoku puzzle was invented in Indianapolis in 1979 but reached widespread international popularity just in 2005 after being launched at the end of 2004 by one of the leading British newspapers, "The Times".
Sudoku can be described as a logic-based placement puzzle. The aim of the classic version of the puzzle is to enter a numerical digit from 1 through 9 in each cell of a 9×9 grid made up of 3×3 blocks, starting with various digits given in some cells; each row, column, and block must contain only one instance of each numeral. Below are reported a classic Sudoku proper puzzle and the corresponding solution.

				6		1	7	3
	2		5		3			
7						2		
4			6					9
2								8
5					4			2
		6						5
			8		6		4	
8	4	7		9				

9	8	5	4	6	2	1	7	3
6	2	1	5	7	3	8	9	4
7	3	4	1	8	9	2	5	6
4	7	3	6	2	8	5	1	9
2	6	9	7	5	1	4	3	8
5	1	8	9	3	4	7	6	2
1	9	6	2	4	7	3	8	5
3	5	2	8	1	6	9	4	7
8	4	7	3	9	5	6	2	1

38. Given any initial grid where some elements have already been filled by digits, please express an IP formulation of a Sudoku puzzle.

Information for question 39-41

Fenerium Truck

Fenerbahçe's authentic licensed products are sold by Fenerbahçe Sportif Inc.'s "Fenerium" stores across the country, as well as the mobile "Fenerium Trucks". One of the Fenerium Trucks will visit Muğla, Nevşehir, Sinop, and Van. The distances between these cities are as follows:

km.	Muğla	Nevşehir	Sinop	Van
İstanbul	784	729	701	1640
Muğla		780	1063	1773
Nevşehir			603	1023
Sinop				1156

Fenerium Truck should visit each city once and then return to İstanbul (make a round-trip route that minimizes the total distance traveled).

39. Please formulate the TSP as an IP model by using Lindo syntax.

40. Please use the Nearest-Neighbor Heuristic method to solve the TSP applying this procedure beginning at İstanbul only

41. Assume that Fenerium truck will only visit Muğla, Nevşehir, and Sinop (Van will not be visited). Please treat this new problem as a combinatorial optimization problem. By using Branch and Bound approach that utilizes Hungarian method, please find the initial solution and propose two new branches for this initial solution

42. Solve the IP problem given below using the cutting plane algorithm.

$\max z = 8\,x_1 + 5\,x_2$
st $\quad 2\,x_1 + 3\,x_2 \leq 5$
$x_1, x_2 \geq 0$ and integer

43. Minimize $3x^2 + 4xy + 5y^2$ subject to $x + y \geq 4$ where $x \geq 0$ and y is urs

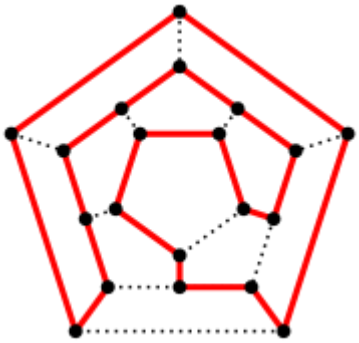
44. An IE department of a university is acquiring mathematical programming software (code) for use in OR classes. The department might be interested in three types of optimization problems. There are four types of codes. An "X" in the entry of the following table indicates that the code in the column is capable of solving the problem in the row.

	Code			
Problem	1	2	3	4
LP	X	X	X	X
IP	-	X	-	X
NLP	-	-	X	X
Objective	3	4	6	14

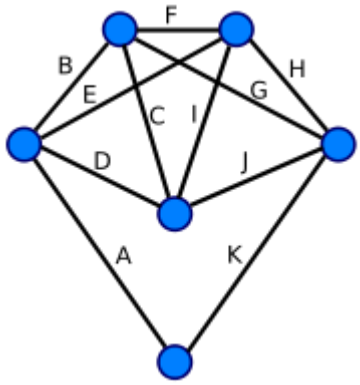
a. Taking objective function coefficients as indications of code quality, formulate a model to acquire a maximum quality collection of codes with at most one providing LP, at most one providing IP and at most one providing NLP.
b. Taking objective function coefficients as code costs, formulate a model to acquire a minimum cost collection of codes providing LP, IP and NLP capability.
c. Taking objective function coefficients as code costs, formulate a model to acquire a minimum cost collection of codes with exactly one providing LP, one providing IP and one providing NLP.

45. The path in the graph given below is a …… cycle.

This is a feasible solution to a …… problem.



46. A …… cycle is given following the edges of the graph given below in alphabetical order.
The …… problem is to find a shortest cycle (circuit) that visits every edge of a graph.



47. If the sequence of numbers given below are Fibonacci numbers, fill in the blanks
0, 1, ……, ……, ……, ……, ……, ……, ……, 34

48. Dorian makes luxury cars and jeeps for high-income men and women. It wishes to advertise with 1 minute spots in comedy shows and football games. Each comedy spot costs $50K and is seen by 7M high-income women and 2M high-income men. Each football spot costs $100K and is seen by 2M high-income women and 12M high-income men.
Dorian has three goals (in order of importance):

- Its ads should be seen by at least 24M high-income men (HIM)
- Its ads should be seen by at least 28M high-income women (HIW)
- At most $250K can be spent on ads (Budget)

When the problem is solved using goal programming simplex, we come up with the following unique optimal table (x_1: *the number of comedy spots*, x_2: *the number of football spots*):

	x_1	x_2	s_1^+	s_2^+	s_3^+	s_1^-	s_2^-	s_3^-	RHS
Row 0 (HIM)	0	0	0	0	0	$-P_1$	0	0	0
Row 0 (HIW)	0	0	0	0	0	0	$-P_2$	0	0
Row 0 (Budget)	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
HIM	1	0	-0.15	0.025	0	0.15	-0.025	0	3.6
HIW	0	0	-5	-7.5	1	5	7.5	-1	70
Budget	0	1	0.025	-0.088	0	-0.025	0.0875	0	1.4

a) HIM goal is met ❍ TRUE ❍ FALSE
b) HIW goal is met ❍ TRUE ❍ FALSE
c) Budget goal is met ❍ TRUE ❍ FALSE
d) State conditions (either exact value or range) on A, B, C, D, E, F, G, H, and I

49. …… million HIM and …… million HIW see Dorian's ads. Why?

50. The money spent by Dorian would be …… thousand dollars. Why?

51. Determine an initial interior point for the LP model of the Advertisement Example:

$$\min 50\,x_1 + 100\,x_2$$
$$\text{s.t. } 7\,x_1 + 2\,x_2 \geq 28$$
$$2\,x_1 + 12\,x_2 \geq 24$$
$$x_1, x_2 \geq 0$$

52. Find the critical points for the function: $f(x,y) = 40 + x^3(x-4) + 3(y-5)^2$. Are these points local maxima, local minima, or saddle points?

53. Kadıköy municipality wants to add a minimum of one thousand new parking spaces in the downtown area. The following table shows the estimated cost (in millions TL) of the four proposed projects and the number of spaces each would yield (in hundreds).

	Project			
	1	2	3	4
Cost	16	9	11	13
Spaces	8	3	6	6

The municipality wants to meet its goal at minimum total cost. Assume that every project is available only on an all-or-nothing basis.

a. Formulate an IP model to choose an optimal parking program.
b. Solve this IP model by utilizing Branch and Bound method.
c. Give an executive summary for your solution.

54. Fenerbahçe's authentic licensed products are sold by Fenerbahçe Sportif Inc.'s "Fenerium" stores across the country. Fenerium must deal constantly with such decision problems in choosing how to divide its limited budgets among many competing projects. The following table shows a list of the projects and the related information.

Budget requirements in millions TL span the following five years in the table. Each project can be chosen once and must be completed (i.e. if project 5 is chosen, Fenerium has to pay associated costs which are 5 and 8 in 2012, 2013 respectively).

Values of the projects are included as a column in the table. Fenerium is trying to maximize the total value. Projects 4 and 5 are mutually exclusive. That is, at most one of them can be included in the program plan. Similarly, projects 8 and 11 are mutually exclusive. On the other hand, projects 9 and 14 are also mutually exclusive. Project 11 depends on project 2. That is, project 2 must be chosen if project 11 is. Similarly project 3 must be chosen if any of projects 4, 5, 6, 7 is.

Fenerium must decide which of the 14 indicated projects to include in program plan.

Formulate an IP model for Fenerium.

	Budget Requirements (millions TL)						Not	Depends
Project	2011	2012	2013	2014	2015	Value	With	On
1	6					200		
2	2	3				3		
3	3	5				20		
4					10	50	5	3
5		5	8			70	4	3
6			1	8	4	20		3
7	1	8				5		3
8				5		10	11	
9	4	5				200	14	
10		8	4			150		
11			2	7		18	8	2
12	5	7				8		
13		1	4	1	1	300		
14		4	5	3	3	185	9	
Budget	10	12	14	14	14			

55. Suppose we are given n potential facility locations and a list of m clients who need to be serviced from exactly one of these locations. There is a fixed cost c_j of opening a facility at location j, while there is a cost d_{ij} of serving client i from facility j.

a. What is the decision problem here?
b. What is the objective?
c. Formulate an IP model that finds the optimal decision with respect to the purpose you have defined.

56. For the IP problem given below,

$\max z = 6x_1 + 5x_2$
$\text{s.t.} \quad 2x_1 + 3x_2 \leq 9$
$x_1, x_2 \geq 0$ and integer

a. Find the optimal tableau for the LR of the IP.
b. Generate a Gomory Cut using the optimal tableau for the LR.
c. Show the first iteration for the dual simplex algorithm (only indicate the entering variable and leaving variable, no need to find the optimal solution to IP)

57. Ask yourself a Set Packing problem question (easy stories are welcome). Formulate an IP model for your problem.

58. An industrial engineer is planning the production over the next T months and the number of workers he/she needs on every month are d_t (t=1,…,T). Monthly fee of a worker is C_1 TL. An unwanted worker will not be fired, he/she will cost C_2 TL, which is equivalent to a pay without a productive work. Hiring a new worker will incur an additional training cost of C_3 TL. Identify and explain decision variables, stages, states and formulate a recursion to make a plan over the next T months with minimum cost.

59. Use the method of steepest ascent to approximate the optimal solution to the following problem. Begin at point (1,0).

$$\min z = x_1^2 + x_2^2 - 2x_1x_2.$$

60. Solve the following optimization problem by using KKT optimality conditions. (*Hint*: draw the feasible region and identify the active (binding, satisfied with equality) constraint(s) at the optimal point)

$$\begin{aligned} \min \quad & x^2 + y^2 \\ \text{s.t.} \quad & x + y \leq 4 \\ & 3x + y \geq 3 \\ & -x + y \geq -2 \\ & y \leq 2 \\ & y \geq 0 \end{aligned}$$

61. Use Branch and Bound method to find the optimal solution to the following IP:

$\max z = 5x_1 + 2x_2$
$\text{s.t.} \quad x_1 + x_2 \leq 3$
$2x_1 + 4x_2 \leq 5$
$x_i \geq 0$ and integer

62. Fenerbahçe SK is trying to determine which of the following players of Fenerbahçe Ülker team roster should be signed for contract extension: Roko Leni Ukic, Tarence Anthony Kinsey, Mirsad Türkcan, Oğuz Savaş, and Ömer Onan. The cost of signing each player and the value each player will add to the team are shown in the table below. Fenerbahçe SK wants to sign the players who will add the most value to the team concerning the $ 8 million budget limit.

Player	Cost of signing player ($ millions)	Value added to the team
Ukic	4	8
Kinsey	3	7
Mirsad	2	5
Oğuz	1.5	4
Ömer	1.5	3

a. Formulate an IP that could be used to determine which players should be signed (*just formulate the problem, do not solve*)
b. Please determine the type of this IP problem.

63. If Golden Section Search (GSS) algorithm is utilized to solve the following model with a final interval of uncertainty having a length less than 0.3, indicate the number of iterations that must be performed to converge the result? (*do not utilize GSS, just find the number*):

min $x^2 + 3$
s.t. $-3 \leq x \leq 3$

64. A company has *n* factories. Factory *i* is located at point (x_i, y_i), in the *x*-*y* plane. The company wants to locate a warehouse at point (x, y) that minimizes

$$\sum_{i=1}^{i=n}(\text{euclidean distance from factory } i \text{ to warehouse})^2$$

Where should the warehouse be located? Prove that your solution is optimal.

65. Suppose that you are interested in choosing a set of investments {1, … , 7} using binary variables. Model the following constraints separately.

a) You cannot invest in all of them
b) You must choose at least one of them
c) Investment 4 can be chosen only if investment 2 is also chosen
d) You must choose either both investments 1 and 5 or neither of them
e) You must chose either at least one of the investments 1, 2, 3 or at least two investments from 2, 4, 5, 6.

66. Suppose that the cost of producing a product according to the production amount is given with the following function (*x* is the production amount, the maximum value of *x* is 50)

$$f(x) = \begin{cases} 0 & x = 0 \\ 10 + 4x & 0 < x \leq 15 \\ 60 + 2x & 15 < x \leq 30 \\ 30 + 3x & 30 < x \leq 50 \end{cases}$$

If the sales price of the product is 4 TL and 5 TL to the markets, formulate (do not solve) the problem to find the sales amounts of the products to the markets that maximizes the total profit.

67. ABC Company knows that the demand for its product during each of the next three months will be as follows: month 1, 2 units; month 2, 3 units; month 3, 2 units. At the beginning of each month, the company must determine how many units should be produced during the current month. During a month in which any units are produced, a setup cost of $2.5 is incurred. In addition, there are variable costs of $1.5, $1 and $2 for the month 1, month 2 and month 3 respectively. The inventory cost is $ 1.5 for each product in stock at the end of a month. Products produced during a month may be used to meet demand for that month or any future month. Given that the initial inventory level is 1 unit. Use an appropriate dynamic programming method to determine a production schedule that will meet all demands on time and will minimize the sum of production and holding costs during the three months.

68. At the beginning of each year, Barnes Carr Oil sets the world oil price. If a price p is set, then $D(p)$ barrels of oil will be demanded by world customers. We assume that during any year, each oil company sells the same number of barrels of oil. It costs Barnes Carr Oil c dollars to extract and refine each barrel of oil. Barnes Carr cannot set too high a price, however, because if a price p is set and there are currently N oil companies, then $g(p, N)$ oil companies will enter the oil business [$g(p, N)$ could be negative]. Setting too high a price will dilute future profits because of the entrance of new companies. Barnes Carr wants to maximize the discounted profit the company will earn over the next 20 years. Formulate a recursion that will aid Barnes Carr in meeting its goal. Initially, there are 10 oil companies.

69. A city parks department has been given a grant of €600million to expand its public recreational facilities. Four different types of facilities have been requested by the city council members: gymnasiums, athletic fields, tennis courts, and swimming pools. The total demand by various neighborhoods has been estimated to be 7 gyms, 10 athletic fields, 8 tennis courts and 12 swimming pools. Each facility costs a certain amount, requires a certain number of hectares and has an expected usage as shown in the following table:

Facility	Cost (€million)	Required hectares	Expected usage (people/week)
Gymnasium	80	4	1500
Athletic field	24	8	3000
Tennis court	15	3	500
Swimming pool	40	5	1000

The parks department has located a total of 50 hectares of land for construction (although more land could be located if necessary).

The council has established the following list of prioritised goals:

1. The parks department must spend at least the total grant (otherwise the amount not spent will be returned to the government).
2. The facilities should be used by 20,000 or more people weekly.
3. If more land is required, the additional amount should be limited to ten hectares.
4. The parks department would like to meet the demands of the members of the city council. However, this priority should be weighted according to the number of people expected to use each facility.

Formulate (do not solve) this Goal Programming problem.

70. Consider a binary IP formulation for a TSP. Suppose there are 10 cities.
How many constraints shall we have in this model?
Why (*how are you finding the number you are indicating*)?

71. The optimal tableau for an IP's LR is given in the following optimal tableau:

z	x_1	x_2	x_3	s_1	s_2	s_3	RHS	BV
1	8	0	0	0	10	10	280	z=280
0	1.6	0	0	1	1.2	-5.6	27.2	s_1=27.2
0	1.6	0	1	0	1.2	-1.6	11.2	x_3=11.2
0	0.8	1	0	0	-0.4	1.2	1.6	x_2=1.6

a. Generate a Gomory Cut using the table (*only find the cut constraint*).
b. Please show the first iteration for the dual simplex algorithm after adding the cut (*only indicate the entering variable and leaving variable, do not find the optimal sol'n to IP*)

72. Hallco runs a day shift and a night shift. No matter how many units are produced, the only production cost during a shift is a setup cost. It costs $8,000 to run the day shift and $4,500 to run the night

shift. Demand for the next two days is as follows: day 1, 2,000; night 1, 3,000; day 2, 2,000; night 2, 3,000. It costs $1 per unit to hold a unit in inventory for a shift.
Please formulate (*do not solve*) an IP that can be used by Hallco to determine a production schedule that minimizes the sum of setup and inventory costs. All demand must be met on time.

73. Houseco Developers is considering erecting three office buildings. The time required completing each and the number of workers required to be on the job at all times are shown in the table below. Once a building is completed, it brings in the following amount of rent per year: building 1, $50,000; building 2, $30,000; building 3, $40,000. Houseco faces the following constraints:

a. During each year, 60 workers are available.
b. At most, one building can be started during any year.
c. Building 2 must be completed until the end of year 4.

Please formulate an IP (*do not solve*) that will maximize the total rent earned by Houseco through the end of year 4.

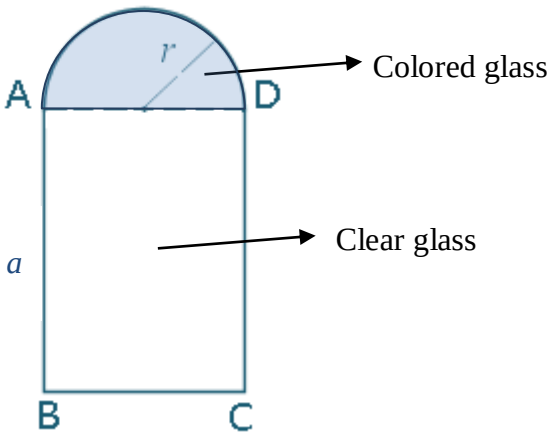
Building	Duration of Project (Years)	Number of Workers Required
1	2	30
2	2	20
3	3	20

74. Four jobs must be processed on a single machine. The time required to perform each job, the due date, and the penalty (in TL) per day the job is late are given in the table below. Use branch and bound to determine the order of performing the jobs that will minimize the total penalty costs due to delayed jobs if Job 3 is the last job.

Job	Time (Days)	Due Date	Penalty
1	4	Day 4	4
2	5	Day 2	5
3	2	Day 13	7
4	3	Day 8	2

75. Expected length of a project is 60 days and the variance of its length is 10.2 days. What is the probability that the project is completed within 55 days?

76. A window is in the form of a rectangle surmounted by a semicircle. If the rectangle is of clear glass while the semicircle is of colored glass which transmits only half as much light per square foot as clear glass does, and the total perimeter is fixed, find the proportions of the window that will admit the most light.



77. Consider the following nonlinear programming model;

$$\max 4x_1^2 + x_2^2$$
$$\text{st}$$
$$x_2 \geq 3x_1$$
$$x_1^2 + 3x_2 \leq 3$$
$$x_1 + x_2 \leq 1$$
$$4x_1 - x_2 \leq 2$$
$$x_1 \geq 0$$

(a) Write the Karush-Khun-Tucker(KKT) Optimality Conditions.
(b) Is this KKT Conditions are sufficient for optimality of the given problem?
(c) Analyze the existence of a local maximum given that the first and the third constraints are binding (use KKT conditions).

78. Consider the multiobjective LP given below

$$\mathbf{max}\, \boldsymbol{6x_1 + 4x_2}$$
$$\mathbf{max}\, \boldsymbol{x_2}$$
$$\text{st}$$
$$\boldsymbol{3x_1 + 2x_2 \leq 12}$$
$$\boldsymbol{x_1 + 2x_2 \leq 10}$$
$$\boldsymbol{x_1, x_2 \geq 0}$$

with the targets of minimum 20 for the first objective and 4 for the second objective. It is given that satisfying the first objective is more important than satisfying the second objective. Solve the problem using the graphical method.

79. ATK-Blue manufactures and sells two types of handmade carpets: Milas Style and Uşak style. The company makes a profit of 700 TL from a Milas carpet and a profit of 1500 TL from an Uşak carpet. Three units of raw material and 10 hours of work time are needed to manufacture a Milas carpet, and 6 units of raw material and 15 hours of work time are needed to manufacture an Uşak carpet. A total of 120 units of raw material and 360 hours of work time are available.

a) If any of Milas Carpet is produced, a setup cost of 5000 TL is incurred, and if any of Uşak Carpet is produced, a setup cost of 10000 TL is incurred. Formulate an IP to maximize profits. Define decision variables, an objective function, and constraints.

b) Suppose that the setup cost is changed as follows: 7000 TL is incurred if one of the styles is produced and 12000 TL is incurred if both of the styles are produced. How will you modify the IP?

80. Given the function

$$\min y = -x^2 + 5x - 4x \cdot \sin(4x)$$
$$\text{st } -1 \leq x \leq 1$$

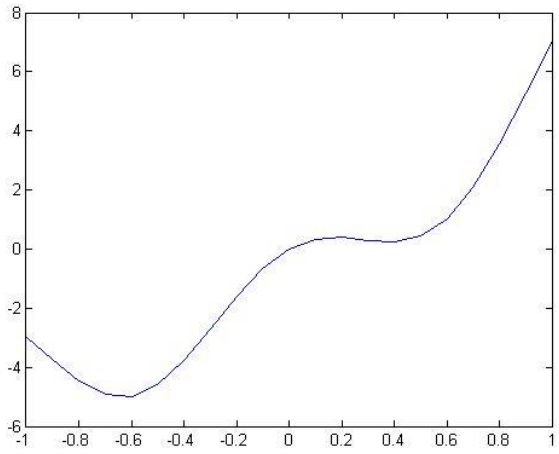
decide which of the two optimization methods given below is more appropriate for solving the problem and solve the problem using the selected method.

a) Golden Section Search with an interval of certainty level of $\frac{1}{2}$.

b) Newton's Method with a starting point of -0.6 and a stopping tolerance of 0.1.

Note: The angles denoted in the sin(.) function is in radians, not in degrees (π radians = 180°).

The graph of the function is given below:



81. Maçka Spor is leading the 5th Amateur Football League by one point difference over its successor, Maslak Spor. Both teams will play two more matches until the end of the season. The two teams will play against each other at the last match.
For the next two matches Maçka Spor can play in offensive or defensive formations. If they play offensive, they have 70% chance of winning and 30% chance of losing. If they play defensive, they have 30% chance of winning, 60% chances of drawing and 10% chance of losing. Maslak Spor is assumed to win its next match with 80% percent, and draw with 20% percent (notice that the result of the match against each other solely depends on the strategy of Maçka Spor).
Winning a game scores 3 points and drawing scores 1 point. If the teams tie the league (i.e. end the league with the same point), the team that wins in the last match will win the title. If the teams tie the league and they draw on the last match, Maslak Spor will win the title. Assume other teams do not have any chance of winning the title.
Use Dynamic programming to find a strategy for Maçka Spor for the next two matches to win the league title. What is probability of winning the title for Maçka Spor under that strategy? (*Hint*: determine the point difference as the state)

82. A bakery has 120 trained bakers. Each month new bakers are hired but none are fired. Training of a new baker takes 1 month and each trainee needs 1 trained baker to spend 1 month acting as a supervisor rather than making bread.
A trainee made bread cannot be sold. Each month some d_i number of trained bakers are needed (i = 1, 2,…,T). 8% of trained bakers leave the job every month but none of the trainees leave. Set up a dynamic programming formulation to minimize payroll cost in months 1, 2,…,T (Assume trainees are paid the same as trained bakers). Identify the states/stages and define your variables.

83. A significant budget reduction at the Istanbul Technical University left the Athletic Department short on operating funds compared to the previous year.
The alternatives capable of realizing the proposed budget cuts included dropping an entire sport from the university's intercollegiate athletic family: the elimination of basketball program (A1), American football program (A2), curling program (A3). The Athletic Department decided on the following attributes to evaluate the alternatives: the number of people directly affected negatively (C1), money saved by the department after the sport was dropped (C2), miscellaneous (C3). The miscellaneous attribute considered the aggregated evaluation of the indigenous factors (how natural it was to have the sport), facility proximity, fan support, past success, and the facility required to maintain each sport. This category utilized a five-point cost scale ranging from very low to very high, with very low being the best for dropping.
The decision matrix for the budget problem is given below:
C1 was given a weight of 20%. C2 was weighted as 70% and C3 was weighted at 10%. Analyze the problem to find the preference order of alternatives by SAW method.

Programs	Attributes C1	C2	C3
A1: Drop basketball	30	174,140	Average (3)
A2: Drop A. football	29	74,683	Low (2)
A3: Drop curling	12	22,496	Very low (1)

84. Consider a knapsack problem with three items. Assume that
benefits obtained if item i is chosen (c_i) are 4, 5, 7;
amounts of the available resource used by item i (a_i) are 6, 7, 8; and
total amount of an available resource (b) is 13

a. How many possible solutions can be generated?
b. Write your own story for this problem (easy stories are welcome)
c. Formulate an IP model for your problem. Please define decision variables, the objective function, constraints, and sign restrictions (if any).
d. Solve your knapsack problem with Branch and Bound method. Draw the B&B tree.
e. Interpret the optimal solution you have found (submit an executive summary).

85. A young couple, Aslı and Kerem, want to divide their main household chores (marketing, cooking, dishwashing, and laundering) between them so that each has two tasks but the total time they spend on household duties is kept to a minimum. Their efficiencies on these tasks differ, where the time each would need to perform the task is given by the following table. Formulate a binary IP model for this problem (just formulate the problem, do not solve). Please define decision variables, an objective function, constraints, and sign restrictions (if any).

	Time needed per week (hours)			
	Marketing	Cooking	Dishwashing	Laundry
Aslı	4.5	7.8	3.6	2.9
Kerem	4.9	7.2	4.3	3.1

86. Prove that the intersection of two convex sets is a convex set.

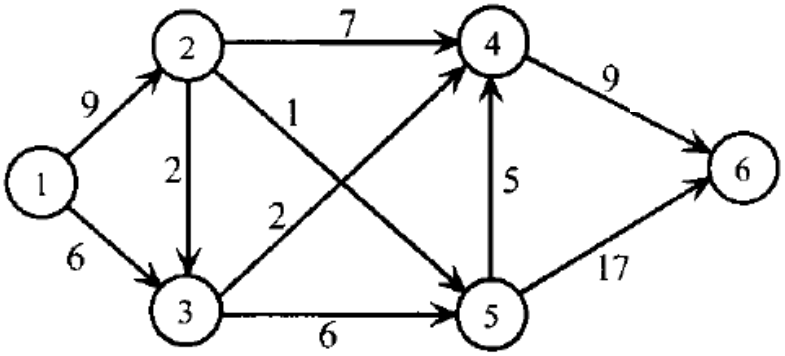
87. Considers the following function;

$$f(x; y; z) = x + y + 2z$$

on the surface $x^2 + 2y^2 + z^2 = 3$

(a) Use Lagrange multipliers to find all the critical values (stationary points) of f on the given surface.
(b) Determine the maxima and minima of f on the given surface by evaluating f at the critical values.

88. Find the shortest path from every node to node 6 in the following network using dynamic programming (distance between the nodes are shown on the arcs.). Show your calculations in details.



89. Arda has 10 grams of gold in his hand. At the beginning of day t, he has the opportunity to sell gold at a price st TL per gram and can buy gold at pt TL per gram (Assume pt > st, t = 1,2,…,T). Because of the

security reasons, Arda restricts himself to hold at most 25 grams of gold at the end of each day. Formulate a recursion that can be used to maximize Arda's total profit earned during the next T days. Define stages, states, decisions, recursion formula, objective and formula for initializing the solution procedure.

90. Solve the following integer non-linear programming model using dynamic programming.

$$\text{Max } Z = (0.8 + \sqrt{x_1}) \cdot \ln(x_2 + 2) \cdot (0.5 + x_3)$$

$$\text{Subject to} \quad x_1 + x_2 + x_3 = 3$$

$$x_1, x_2, x_3 \geq 0, and\ x_1, x_2, x_3 are\ integers$$

91. Solve the IP problem given below using the cutting plane (Gomory cut) algorithm (*do not utilize other methods such as graphical solution technique or branch and bound algorithm*)

max $z = 2\ x_1 + 3\ x_2$
s.t. $2\ x_1 - x_2 \leq 0$
$2\ x_2 \leq 21$
$x_1, x_2 \geq 0$ and integer

92. Four jobs must be processed on a single machine. The time required to perform each job, the due date, and the penalty (in TL) per day the job is late are given in the table below. Use branch and bound approach to determine the order of performing the jobs that will minimize the total penalty costs due to delayed jobs if Job 4 is the last job. Indicate the total penalty cost for the determined order.

Job	Time (Days)	Due Date	Penalty
1	3	Day 11	8
2	1	Day 2	3
3	6	Day 7	6
4	9	Day 2	8

93. Consider *n* points in $\mathbf{R}^2$ such that $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. Formulate an NLP whose solution will yield the circle of smallest radius enclosing these *n* points

94. Consider the following problem.

$$Min\ Z = \frac{50}{x} + \frac{20}{y} + xy$$

$$s.t.\ x \geq 1, y \geq 1$$

a) Write down the Kuhn–Tucker conditions for the problem.
b) Is there any KKT point where both of the constraints are not binding? (hint: if both constraints are not binding then $x > 1\ and\ y > 1$)

95. A backgammon player will be playing three consecutive matches with friends tonight. For each match, he will have the opportunity to place an even bet that he will win; the amount bet can be any quantity of his choice between zero and the amount of money he still has left after the bets on the preceding matches. For each match, the probability is $\frac{1}{2}$ that he will win the match and thus win the amount bet, whereas the probability is $\frac{1}{2}$ that he will lose the match and thus lose the amount bet. He will begin with \$75, and his goal is to have \$100 at the end. (Because these are friendly matches, he does not want to end up with more than \$100.) Therefore, he wants to find the optimal betting policy (including all ties) that maximizes the probability that he will have exactly \$100 after the three matches (assume that the play will bet 0 or a multiple of 25\$).
Use dynamic programming to solve this problem.

(Definition of *even bet*: odds offering an equal chance of winning or losing, with the amount won being the same as the stake)

96. Giapetto produces wooden soldiers and trains. Each soldier sells for $27, uses $10 of raw materials and takes $14 of labor & overhead costs. Each train sells for $21, uses $9 of raw materials, and takes $10 of overhead costs. Each soldier needs 2 hours finishing and 1 hour carpentry; each train needs 1 hour finishing and 1 hour carpentry. Giapetto has he following goals in order of importance:

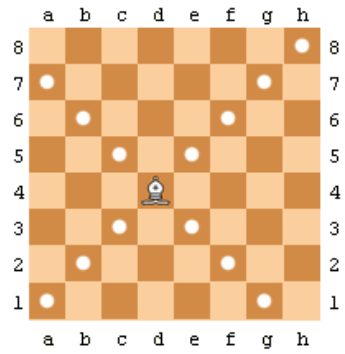
- would like to achieve a satisfactory profit level of $210
- does not want to use more than 100 hours of finishing
- does not want to use more than 80 hours of carpentry

a. Formulate a Goal Programming Model for Giapetto. (Please define decision variables and indicate an objective function, constraints, and sign restrictions).

b. Solve your model graphically.

c. Interpret your results (How many toys should be produced? Which goals are met, which goals are not met? Indicate deviations for unmet goals).

97. Formulate an IP model to determine the minimum number of bishops that can be placed on an empty 8x8 chessboard so that each square contains a bishop or is attacked by one. (*Hint*: A bishop can move in a straight line diagonally, any number of unoccupied squares as it can be seen in the following figure. It captures in the same way that it moves, replacing another bishop that got in its way).



98. The optimal tableau for an IP's LR is given in the following table:

z	x1	x2	s1	s2	RHS
1	5,5	0	0	2,5	7,5
0	-0,5	0	1	-0,5	0,5
0	1,5	1	0	0,5	1,5

Use Gomory Cut algorithm to find the optimal solution.

99.

a. Why may decision making be defined as intentional and reflective choice in response to perceived needs? To answer this question please fill in the blanks below:
   Intentional choice is a choice which is done ……
   Reflective choice is a choice which affects ……
   Perceived needs are based on the definition of ……

b. Consider a multi attribute decision making problem is being solved and the data is given at a decision matrix. Please define x_{ij}, r_{ij}, v_{ij}, w_j

c. If you are using Tchebycheff distance based (linear) normalization method,
   What is the highest normalized value? ……
   What is the lowest normalized value? ……

d. For a decision problem of which movies to go, give an example for each of the attribute type:
   Benefit attribute: ……
   Cost attribute: ……

Nonmonotonic attribute: ……

e. Assume that the cost of four alternatives are 11, 2, 12, and 6 respectively. Normalize these costs by Manhattan distance based normalization.

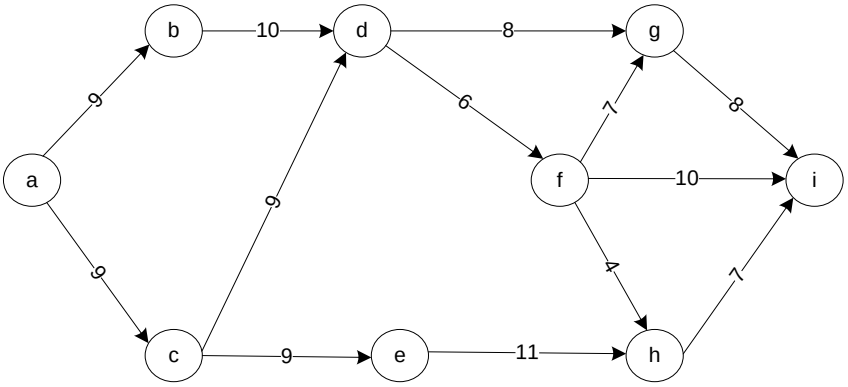
100. Prove that the intersection of two convex set is also convex. (Hint: use definition of convex set and formal definition of intersection of two sets: $x \in A \; and \; x \in B \; \leftrightarrow x \in (A \cap B)$)

101. A firm manufactures a commodity at two different factories. The total cost of manufacturing depends on the quantities, q1 and q2, supplied by each factory, and is expressed by the joint cost function,

$$C = f(q_1, q_2) = 2{q_1}^2 + q_1 q_2 + {q_2}^2 + 500$$

The company's objective is to produce 200 units, while minimizing production costs. Solve the related NLP to find how many units will be supplied by each factory.

102. Determine the <u>longest path</u> from node a to node i in the network given in the following figure using dynamic programming (you do not need formulate recursion for the problem). On each arc its distance is given.



103. Imagine that you have $5,000 to invest and that you will have an opportunity to invest that amount in either of two investments (*k*= A or B) at the beginning of each of the next 5 years. Both investments have uncertain returns. For investment k, if you invest x dollars you will win y dollars with a probability of $p_k(x, y)$ (*k* = A, B). You are allowed to make only one investment (A or B) each year, and you can invest at most available money at the beginning of the year each time. (Any additional money accumulated is left idle.)

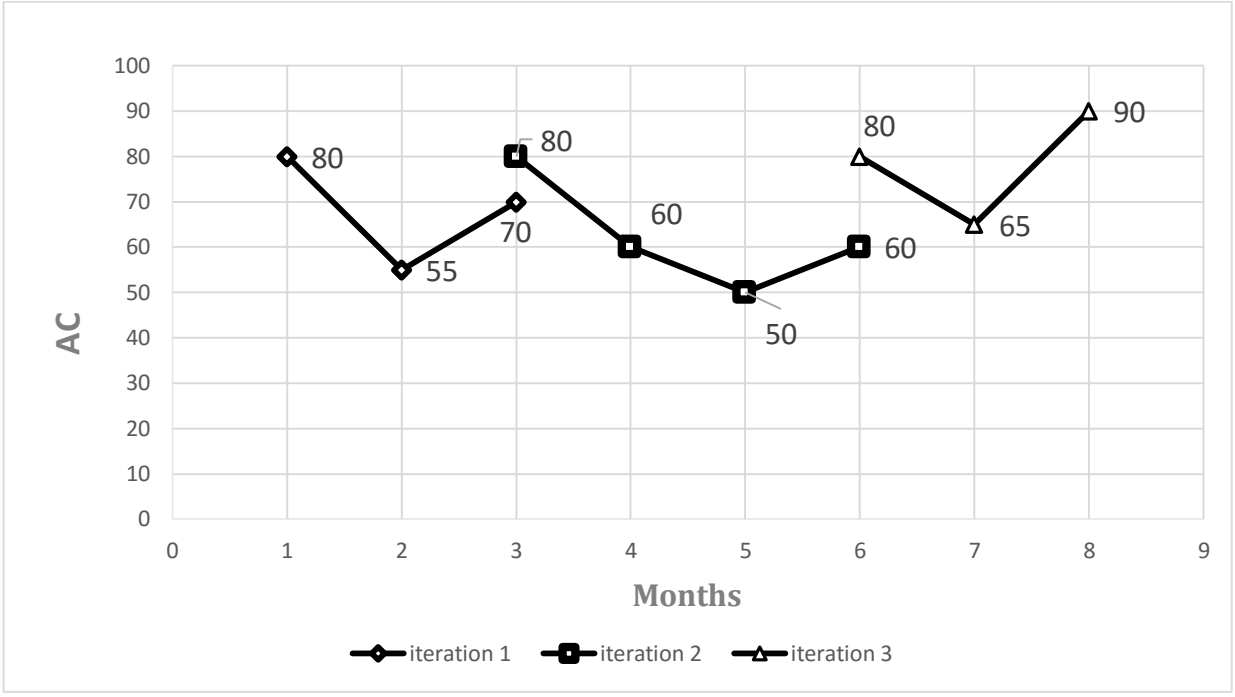
a) Formulate a dynamic programming recursion to find the investment policy that maximizes the expected amount of money you will have after 5 years.

b) Suppose that your aim is to have at least $10,000 at the end of year 5. How would you formulate a dynamic programming recursion for this new situation to find the investment policy that maximizes the probability that you will have at least $10,000 after 5 years.

104. A decision maker wants to find a production plan with minimum cost for the next 8 months using Silver-Meal Heuristic method. Through the application of the method, he obtains an average per-period costs are shown in the following graphic. Let inventory holding cost *h* = 1 and variable production cost *c* = 10, first month's demand is 60 units and assume there is no inventory at the beginning of month 1.

a. Find the resulting production plan of Silver-Meal Heuristic. What is the total cost of this production plan (calculate related data from the graphic)?

b. How would the total cost change if the decision maker uses Wagner-Whitin Algorithm instead of Silver-Meal Heuristic? Why? Discuss the possible result of Wagner-Whitin Algorithm compared to the result of Silver-Meal Heuristic (do not solve the problem using Wagner-Whitin Algorithm).



105. Consider a long roll of wallpaper that repeats its pattern every 1 meter. Four sheets of wall paper must be cut from the roll. With reference to the beginning (point 0) of the wallpaper, the beginning and end of each sheet are located as shown in the Table. Thus, sheet 1 begins 0.3 m from the beginning of the roll (or 1.3 m, 2.3 m, etc. from the beginning of the roll) and sheet 1 ends 0.7 m from the beginning of the roll (or 1.7 m, 2.7 m, etc. from the beginning of the roll). Assume we are at the beginning of the roll. A final cut is made to bring the roll back to the beginning of the pattern. In what order should the sheets be cut to minimize the total amount of wasted paper? Assume the problem as a Traveling Salesperson Problem and utilize Branch and Bound approach covered at "Solving Combinatorial Optimization Problems" class to find the optimal order.

Sheet	Beginning (meters)	End (meters)
1	0.3	0.7
2	0.4	0.8
3	0.2	0.5
4	0.7	0.9

106. Use Branch and Bound method to find the optimal solution to the following IP (do not utilize other methods such as graphical solution technique or cutting plane algorithm).

$$\max z = 10x_1 + x_2$$
$$\text{s.t.} \quad 10x_1 \le 15$$
$$x_1 + x_2 \le 10$$
$$x_1 \ge 0 \text{ and integer; } x_2 \ge 0$$

107. Jim Matthews, Vice-President for Marketing of the J.R. Nickel Company, is planning advertising campaigns for two unrelated products. These two campaigns need to use some of the same resources. Therefore, Jim knows that his decisions on the levels of the two campaigns need to be made jointly after considering these two resource constraints. In particular, letting x1 and x2 denote the levels of campaigns 1 and 2, respectively, these constraints are;

$4x_1 + x_2 \le 20$ and $x_1 + 4x_2 \le 21$.

In facing these decisions, Jim is well aware that there is a point of diminishing returns when raising the level of an advertising campaign too far. At that point, the cost of additional advertising becomes larger than the increase in net revenue (excluding advertising costs) generated by the advertising. After careful analysis, he and his staff estimate that the net profit from the first product (including advertising costs) when conducting the first campaign at level x_1 would be $3x_1 - (x_1 - 1)^2$ in millions of dollars. The corresponding estimate for the second product is $3x_2 - (x_2 - 2)^2$.

This analysis led to the following quadratic programming model for determining the levels of the two advertising campaigns:

Maximize $Z = 3x_1 - (x_1 - 1)^2 + 3x_2 - (x_2 - 2)^2$,

subject to

$$4x_1 + x_2 \leq 20$$
$$x_1 + 4x_2 \leq 21$$

and

$$x_1 \geq 0,\ x_2 \geq 0.$$

a. Give the KKT conditions for this problem.
b. You are given the information that the optimal solution does not lie on the boundary of the feasible region. Use this information to derive the optimal solution from the KKT conditions.
c. Give an interpretation of optimal values of Lagrange multipliers related to the resource constraints (λ_1 and λ_2) that might be useful to Jim Matthews.

108. Solve the following mathematical program using dynamic programming.

$$Max\ z = (x_1 - 1)^2 + (x_2 - 2)^3 + \sqrt{x_3 + 1}$$

st.

$x_1 + x_2 + x_3 = 4$

$x_2 \leq 3$

$x_1, x_2, x_3 \geq 0$ and all variables are integer.

109. Pizza Pizza wants to determine the locations of franchise partnerships in Şişli to be the recognized leader in the pizza category.

There are 10 sub-districts in Şişli and the populations of sub-districts are given in Table 1. Pizza Pizza wants to group these sub-districts to construct 4 main districts for their franchisees. The geographical center of franchisee will be one of the centers of sub-districts in each district. Each sub-district should be assigned one of the districts to deliver pizza for people in these sub-districts. The workload of each franchisee should be balanced so that the working conditions of each franchisee would be similar to each other. To balance the workload of franchisee in one district, the total population assigned in one of the districts should be between %85 and %115 of the average population of 10 sub-districts. 3 franchise partnerships have already built in Harbiye, Mecidiyeköy and Fulya. Pizza Pizza can close existing franchisees and open new one. However, opening a new franchisees cost 10 units and closing an existing one costs 5 units. Pizza Pizza tries to minimize the distance between the territory centers of franchisee and assigned sub-districts and opening and closing costs. In Table 2, the distance between the centers of each sub-districts are given.

Table 1: Population of sub-districts

Sub-District	1	2	3	4	5	6	7	8	9	10
	Harbiye	Mecidi-yeköy	Fulya	Feriköy	Kuştepe	Esentepe	İnönü	Merkez	Halaskar-gazi	İzzet-paşa
Popul'n	11256	10769	10485	10144	9223	8732	5764	3561	2769	2514

Table 2: Distance between sub-districts

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	0	1701	2161	1876	2394	3514	1101	2790	1529	1938
2	1701	0	2813	483	734	1864	1625	1249	2072	2828
3	2161	2813	0	2558	3110	3880	1263	3110	757	392
4	1876	483	2558	0	591	1650	1489	917	1856	2634
5	2394	734	3110	591	0	1133	2079	577	2430	3209
6	3514	1864	3880	1650	1133	0	3053	793	3303	4061
7	1101	1625	1263	1489	2079	3053	0	2263	514	1210
8	2790	1249	3110	917	577	793	2263	0	2513	3276
9	1529	2072	757	1856	2430	3303	514	2513	0	779
10	1938	2828	392	2634	3209	4061	1210	3276	779	0

a. Define your decision problem's objective, parameters and variables clearly.
b. Formulate a MIP model for the given problem using ∑, ∀ notations.

110.

a. Why attributes are multiple, conflicting, weighted, and incommensurable in MADM?
b. What is the difference between classification and sorting?

111. For the mixed IP model given below

$$\begin{aligned} \max\ & 2x_1 - 2x_2 \\ \text{s.t. } & x_1 + x_2 \le 5 \\ & -x_1 + 2x_2 \le 8 \\ & x_1 - x_2 \le 2 \\ & x_1 \ge 0 \\ & x_2 \ge 0 \text{ and integer} \end{aligned}$$

a) Show the feasible region of the problem graphically.
b) Solve the problem graphically; what is the optimum value and solution of the problem?
c) Show the convex hull of the problem graphically, and also write the constraints which define the convex hull.

112. Ask yourself a Set Partitioning problem question. Formulate an IP model for your problem.

113. Consider the following one dimensional minimization problem:

$$\begin{aligned} \max\ & -4x^3 + 4x^2 - x + 13 \\ \text{s.t.}\ & 0.4 \le x \le 2.4 \end{aligned}$$

a. Is the function given above is unimodal on $\mathbb{R}$? Is it possible to apply the Golden Section Search to this NLP? Why?
b. How many iterations does it take the Golden Section Search to stop for a final interval of uncertainty having length less than 0.1?
c. Apply the Golden Section Search to solve the problem with a final interval of uncertainty having length less than 1.

114. Consider the nonlinear program

$$\min\ \ 8(x_1 - 2)^2 + 2(x_2 - 1)^2$$

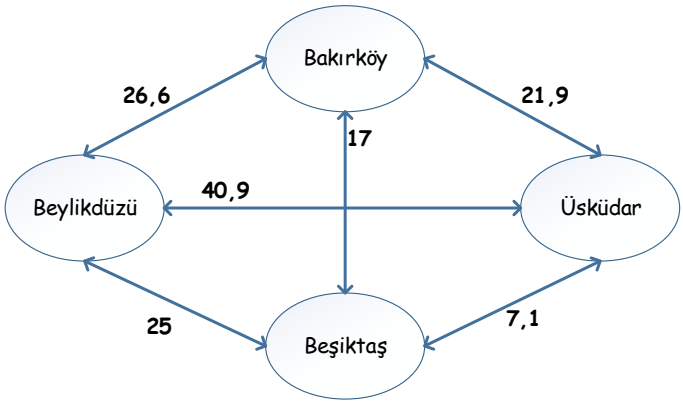
$$\text{s.t. } 16x_1 + 6x_2 = 63$$

a. Calculate the local minimum with objective value of this model.
b. Is the local minimum that you find in part (a) global minimum? Why?

115. A firm based in Beylikdüzü is the supplier of vending machines and it supplies coffee, snacks and refreshments. This firm has machines set up in firms in Beylikdüzü, Beşiktaş, Bakırköy and Üsküdar. The technician in charge of maintaining and refilling these machines has to visit these districts every day. This technician wants to find to shortest path leading from Beylikdüzü to every other node and then return back to Beylikdüzü. How can he/she minimize the distance travelled? The distances (in km) between the districts visited are provided in figure.

Consider and solve this problem with a dynamic programming model.

a. Define the stages, states, decisions and recursive function
b. Solve the problem.



116. For a device to work properly, three subsystems of the device must all function properly. To increase the reliability of the device, spare units may be added to each system. It costs 100 TL to add a spare unit to system 1, 300 TL to system 2, and 200 TL to system 3. As a function of the number of added spares (a maximum of two spares may be added to each system), the probability that each system will work is given in Table. Use dynamic programming to maximize the probability that the device will work properly, given that 600 TL is available for spare units. Identify and explain decision variables, stages, states and formulate a recursion to design the device with optimal reliability.

Number of spares	Probability that a system works		
	system 1	system 2	system 3
0	.85	.60	.70
1	.90	.85	.90
2	.95	.95	.98

117. You are planning on moving to a new house. You need to move n items of size a_j , j=1,2,…,n. You have bought m boxes (box i has size b_i , i= 1,2,…,m). You need to rent a truck that will help fit all your items in it. There are two sizes of trucks to choose from: the smaller truck has a size of Q_1 and the larger truck has a size more than Q_1. Since the rent of the smaller truck is cheaper, you would like to rent the smaller truck if the move is possible with it. Formulate an integer (or mixed integer) programming problem in order to decide whether the move is possible with a single trip of the smaller truck. Note that you can put multiple items in the same box and size is the only criterion determining if an item can be put into a box.

118. Consider the following problem.

$$\begin{aligned} &\min\ xy \\ &\text{s.t.} \\ &\quad x + y \geq 2 \\ &-x + y \geq 0 \\ &\quad x \quad\ \geq 0 \\ &\quad\quad y \geq 0 \end{aligned}$$

a) Write down the Kuhn–Tucker conditions for the problem.
b) Analyze the existence of a KKT point given that the third and the fourth constraints are not binding.
c) Determine the type of the KKT point that you found in part b (is it local minimum, global minimum, or saddle point).

119. Solve the following BIP model with Gomory Cut algorithm.

$$\begin{aligned} &\max\ z = 5x_1 + 4x_2 + 10x_3 \\ &s.t.\ 10x_1 + 8x_2 + 5x_3 \leq 13 \\ &x_i = 0\ or\ 1, \qquad i = 1,2,3 \end{aligned}$$

120. At the beginning of each year, an aircraft engine is in good, fair, or poor condition. It costs 1 million TL to run a good engine for a year, 2 million TL to run a fair engine for a year, and 5 million TL to run a poor engine for a year. An engine in fair or poor condition can be replaced and it immediately becomes a

good machine. Replacement cost is 3 million TL for a fair machine and 6 million TL for a poor machine. The transition probability matrix for an engine is as follows:

	Good	Fair	Poor
Good	0.6	0.3	0.1
Fair	0	0.8	0.2
Poor	0	0	1

According to this matrix, for instance, if an engine is in fair condition, it will be fair with 0.8 probability or poor with 0.2 probability in the next year.

a) Formulate a dynamic programming recursion to minimize expected cost over 10 years time given that the engine is in good condition at the beginning of year 1 (Define stages, states, decision, recursion formula, and solution procedure).

b) *solve part (a) to get extra points* at (*b*)
Solve the problem using dynamic programming and find a strategy over an infinite horizon.