SOLUTIONS TO MIDTERM EXAM I

Answer 1

Decision variables (10 pts):

x_{ij}: number of games team i played with a result of j (i=1,2,3; j=1,2,3,4)

i=1: Fenerbahçe Medicana, i=2: Eczacıbaşı Dynavit, i=3: Vakıfbank;

j=1: standard win (3-0 or 3-1), j=2: win in the deciding set (3-2),

j=3: loss in the deciding set (2-3), j=4: standard loss (1-3 or 0-3).

IP MODEL:

Objective function:

Since we are only interested in feasible solutions that satisfy the constraints, we can use a dummy (null) objective function. This means we are not trying to maximize or minimize any particular value.

Constraints (20 pts):

Number of wins/losses

$$\sum_{i=1}^{3} x_{ij} = r_j \text{ for all } j$$

r_1 = 10 (number of standard wins), r_2 = 2 (number of wins in the deciding set),

r_3 = 2 (number of losses in the deciding set), r_4 = 1 (number of standard losses)

Number of games played

$$\sum_{j=1}^{4} x_{ij} = 5 \text{ for all } i \text{ (number of games played by team } i\text{)}$$

Equal total points

$3\,x_{i1} + 2\,x_{i2} + x_{i3} = 3\,x_{(i+1)1} + 2\,x_{(i+1)2} + x_{(i+1)3}$ for $i = 1$ *and* 2

Based on the first constraint (i=1), points won by the 1st team and the 2nd team are equalized

Based on the second constraint (i=2), points won by the 2nd team and the 3rd team are equalized

In this case, there is no need to equalize points won by the 1st team and the 3rd team

Equal total points (alternative representation)

$3\,x_{i1} + 2\,x_{i2} + x_{i3} = 12$ for all i (points won by team i)

As three favorite teams won ten games 3-0 or 3-1, two games 3-2, and lost two games 2-3, total points they won would be 36. So each team should have 12 points.

Sign restriction (5 pts):

All $x_{ij} \geq 0$ and integer

Answer 1 Open-form

Decision variables:

x_{ij}: number of games team i played with a result of j (i=1,2,3; j=1,2,3,4)

i=1: Fenerbahçe Medicana, i=2: Eczacıbaşı Dynavit, i=3: Vakıfbank;
j=1: standard win (3-0 or 3-1), j=2: win in the deciding set (3-2),
j=3: loss in the deciding set (2-3), j=4: standard loss (1-3 or 0-3).

IP MODEL:

Objective function:

Since we are only interested in feasible solutions that satisfy the constraints, we can use a dummy (null) objective function. This means we are not trying to maximize or minimize any particular value.

Constraints:

Number of wins/losses

$x_{11} + x_{21} + x_{31} = 10$ (number of standard wins)
$x_{12} + x_{22} + x_{32} = 2$ (number of wins in the deciding set)
$x_{13} + x_{23} + x_{33} = 2$ (number of losses in the deciding set)
$x_{14} + x_{24} + x_{34} = 1$ (number of standard losses)

Number of games played

$x_{11} + x_{12} + x_{13} + x_{14} = 5$ (number of games played by Fenerbahçe Medicana)
$x_{21} + x_{22} + x_{23} + x_{24} = 5$ (number of games played by Eczacıbaşı Dynavit)
$x_{31} + x_{32} + x_{33} + x_{34} = 5$ (number of games played by Vakıfbank)

Equal total points

$3\,x_{11} + 2\,x_{12} + x_{13} = 3\,x_{21} + 2\,x_{22} + x_{23}$ (points won by Fenerbahçe = points won by Eczacıbaşı)
$3\,x_{11} + 2\,x_{12} + x_{13} = 3\,x_{31} + 2\,x_{32} + x_{33}$ (points won by Fenerbahçe = points won by Vakıfbank)

Based on these two constraints, there is no need to equalize points won by Eczacıbaşı and points won by Vakıfbank

Equal total points (alternative representation)

$3\,x_{11} + 2\,x_{12} + x_{13} = 12$ (points won by Fenerbahçe Medicana)
$3\,x_{21} + 2\,x_{22} + x_{23} = 12$ (points won by Eczacıbaşı Dynavit)
$3\,x_{31} + 2\,x_{32} + x_{33} = 12$ (points won by Vakıfbank)

As three favorite teams won ten games 3-0 or 3-1, two games 3-2, and lost two games 2-3, total points they won would be 36. So each team should have 12 points.

Sign restriction:

All $x_{ij} \geq 0$ and integer

Answer 1 Alternative modeling: Binary variables

Modeling in this way using binary variables is not recommended. It would be inefficient and impractical due to the complexity introduced by the increased number of variables and constraints.

Decision variables:

x_{ijk}: a binary variable that equals 1 if team i has a result of j in game k, and 0 otherwise (i=1,2,3,4; j=1,2,3; k=1,2,3,4,5)

i=1: Fenerbahçe Medicana, i=2: Eczacıbaşı Dynavit, i=3: Vakıfbank;
j=1: standard win (3-0 or 3-1), j=2: win in the deciding set (3-2),
j=3: loss in the deciding set (2-3), j=4: standard loss (1-3 or 0-3);
k: game (round/week) number.

Variables:

y_{ij}: number of games team i played with a result of j

BIP MODEL:

Objective function:

Since we are only interested in feasible solutions that satisfy the constraints, we can use a dummy (null) objective function. This means we are not trying to maximize or minimize any particular value.

Constraints:

One outcome per game

$$\sum_{j=1}^{4} x_{ijk} = 1 \text{ for all } i \text{ and } k$$

Number of wins/losses

$$\sum_{i=1}^{3} \sum_{k=1}^{5} x_{ijk} = r_j \text{ for all } j$$

r_1 = 10 (number of standard wins), r_2 = 2 (number of wins in the deciding set),
r_3 = 2 (number of losses in the deciding set), r_4 = 1 (number of standard losses)

Equal total points

$$3\sum_{k=1}^{5} x_{11k} + 2\sum_{k=1}^{5} x_{12k} + \sum_{k=1}^{5} x_{13k} = 3\sum_{k=1}^{5} x_{21k} + 2\sum_{k=1}^{5} x_{22k} + \sum_{k=1}^{5} x_{23k}$$

(points won by Fenerbahçe Medicana = points won by Eczacıbaşı Dynavit)

$$3\sum_{k=1}^{5} x_{11k} + 2\sum_{k=1}^{5} x_{12k} + \sum_{k=1}^{5} x_{13k} = 3\sum_{k=1}^{5} x_{31k} + 2\sum_{k=1}^{5} x_{32k} + \sum_{k=1}^{5} x_{33k}$$

(points won by Fenerbahçe Medicana = points won by Vakıfbank)

Based on these two constraints, there is no need to equalize points won by Eczacıbaşı Dynavit and points won by Vakıfbank

Equal total points (alternative representation)

$$3\sum_{k=1}^{5} x_{11k} + 2\sum_{k=1}^{5} x_{12k} + \sum_{k=1}^{5} x_{13k} = 12 \text{ (points won by Fenerbahçe Medicana)}$$

$$3\sum_{k=1}^{5} x_{21k} + 2\sum_{k=1}^{5} x_{22k} + \sum_{k=1}^{5} x_{23k} = 12 \text{ (points won by Eczacıbaşı Dynavit)}$$

$$3\sum_{k=1}^{5} x_{31k} + 2\sum_{k=1}^{5} x_{32k} + \sum_{k=1}^{5} x_{33k} = 12 \text{ (points won by Vakıfbank)}$$

As three favorite teams won ten games 3-0 or 3-1, two games 3-2, and lost two games 2-3, total points they won would be 36. So each team should have 12 points.

Team performance

The aim is to determine the number of standard wins, wins in the deciding set, losses in the deciding set, and the standard losses for each team; therefore, we need additional variables to represent the team performances.

$$y_{i1} = \sum_{k=1}^{5} x_{i1k} \text{ for all } i \text{ (standard wins for team } i\text{)}$$

$$y_{i2} = \sum_{k=1}^{5} x_{i2k} \text{ for all } i \text{ (wins in the deciding set for team } i\text{)}$$

$$y_{i3} = \sum_{k=1}^{5} x_{i3k} \text{ for all } i \text{ (losses in the deciding set for team } i\text{)}$$

$$y_{i4} = \sum_{k=1}^{5} x_{i4k} \text{ for all } i \text{ (standard losses for team } i\text{)}$$

Sign restriction:

All x_{ijk} = 0 or 1

Answer 2

$b_4 = 300 \le x = 310 \le b_5 = 400$ (2 pts.)

→ $y_4 = 1$ (4 pts.);

$y_1=y_2=y_3=y_5=0$ (2 pts.);

$z_1=z_2=z_3=z_6=0$ (2 pts.);

$z_4 + z_5 = 1$ (2 pts.)

$x = 310 = 300z_4 + 400z_5$ (2 pts.)

→ $z_4 = 0.9$ (3 pts.), $z_5 = 0.1$ (3 pts.)

Answer 3

MODEL (5 pts.)

If x_j is 1 then item j will be in the knapsack, 0 otherwise

$$\begin{aligned} \max \quad & 9x_1 + 16x_2 + 14x_3 + 5x_4 \\ \text{st} \quad & 3x_1 + 4x_2 + 4x_3 + 2x_4 \le 10 \\ & x_j = 0 \text{ or } 1 \qquad j = 1, \dots 4 \end{aligned}$$

SOLUTION (20 pts.)

Ratios: 9/3, 16/4, 14/4, 5/2

Priorities of the items: 2 - 3 - 1 - 4

LR sol'n: x2 = 1, x3 = 1, x1 = 2/3; z = 36

P1: **x1 = 0**; x2 = 1, x3 = 1, x4 = 1; z = 35 (Candidate)

P2: **x1 = 1**; x2 = 1, x3 = 3/4; z = 35.5

P3: **x1 = 1, x3 = 0**; x2 = 1, x4 = 1; z = 30 < 35 ---

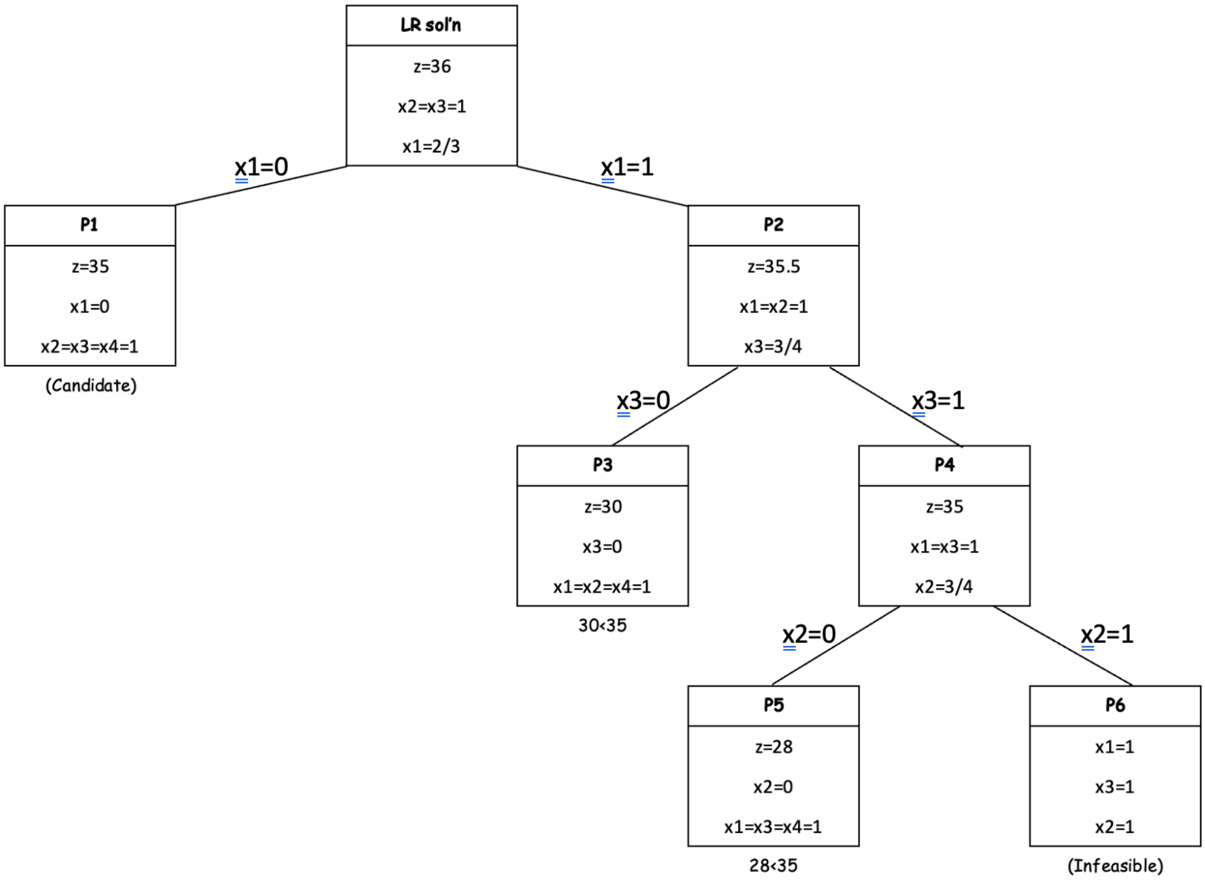
P4: **x1 = 1, x3 = 1**; x2 = 3/4, x4 = 0; z = 35

P5: **x1 = 1, x3 = 1, x2 = 0**; x4 = 1; z = 28 < 35 ---

P6: **x1 = 1, x3 = 1, x2 = 1**; infeasible

INTERPRETATION (5 pts.)

Items 2, 3, and 4 should be in the knapsack, in this case total value would be 35



Answer 4

A-B-C-D-A (2 pts.): 50+100+70+130=350 km (2 pts.)

B-A-C-D-B (2 pts.): 60+100+70+120=350 km (2 pts.) -> A-C-D-B-A (1 pts.)

C-D-B-A-C (2 pts.): 70+120+60+100=350 km (2 pts.) -> A-C-D-B-A (1 pts.)

D-C-A-B-D (2 pts.): 70+90+50+110=320 km (2 pts.) -> A-B-D-C-A (1 pts.)

The NNH method identifies the best tour as A-B-D-C-A, with a total distance of 320 km.

The best tour (2 pts.); A-B-D-C-A (2 pts.); total distance 320 km (2 pts.)