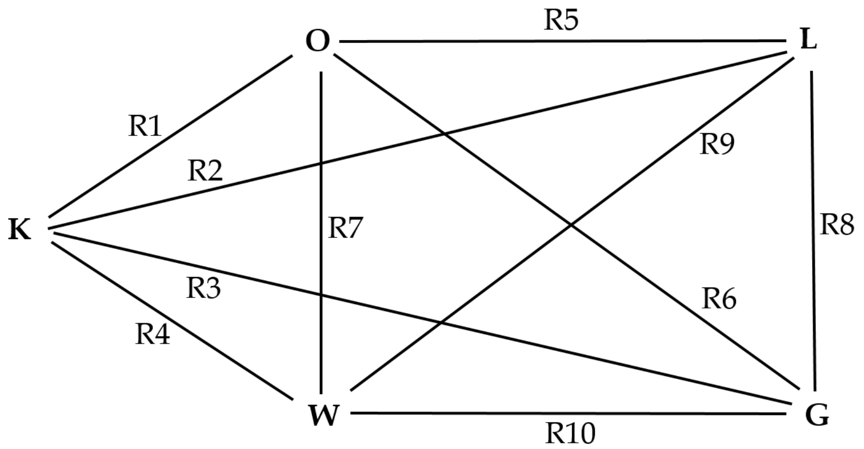
END 332E FINAL EXAM SOLUTIONS

1.

(R1: K-O, R2: K-L, R3: K-G, R4: K-W, R5: O-L, R6: O-G, R7: O-W, R8: L-G, R9: L-W, R10: G-W).



a. (10 pts.)

Start from K, go to O, then to L, then to G, and then to W. Return to K from W:

R1, R5, R8, R10, R4

This cycle visits each city exactly once and returns to home city.

There are other feasible solutions:

b. (10 pts.)

Start from K, use R1 to O, then continue with using roads between L, G, W, K, L, W, O, G, K:

R1, R5, R8, R10, R4, R2, R9, R7, R6, R3

This cycle includes each rode exactly once and returns to home city.

There are other feasible solutions:

c. (5 pts.)

Hamiltonian cycle is a cycle which visits each vertex (node) exactly once and returns to the starting vertex whereas Euler cycle is a cycle which visits every edge (connection) exactly once and also returns to the starting vertex.

OR

Hamiltonian cycle is a path through a graph that starts and ends at the same vertex (node) but includes every other vertex exactly once while Euler cycle is also a path through a graph which starts and ends at the same vertex and includes every edge (connection) exactly once.

2.

POSSIBLE PAIRINGS AND PAIRING COSTS

Pairing 1: SJ101 – SJ203 – SJ406 – SJ308 Duration: 13h

Pairing 2: SJ101 – SJ204 – SJ305 - SJ407 Duration: 12h (1.5 pts.)

Pairing 3: SJ402 – SJ204 – SJ310 – SJ211 Duration: 17h (1.5 pts.)

Pairing 4: SJ203 – SJ406 – SJ308 – SJ109 Duration: 12.5h (1.5 pts.)

Pairing 5: SJ204 – SJ305 – SJ407 – SJ109 Duration: 10.5h (1.5 pts.)

Pairing 6: SJ305 – SJ407 – SJ109 – SJ212 Duration: 11h (1.5 pts.)

Pairing 7: SJ406 – SJ308 – SJ109 – SJ211 Duration: 10h (1.5 pts.)

$$X_j = \begin{cases} 1 & \text{if pairing } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

OBJECTIVE FUNCTION (5 pts.)

min $13\, x_1 + 12\, x_2 + 17\, x_3 + 12.5\, x_4 + 10.5\, x_5 + 11\, x_6 + 10\, x_7$

CONSTRAINTS (12 constraints: 12 pts.)

$x_1 + x_2 = 1$ (flight 101)

$x_3 = 1$ (flight 402)

$x_1 + x_4 = 1$ (flight 203)

$x_2 + x_3 + x_5 = 1$ (flight 204)

$x_2 + x_5 + x_6 = 1$ (flight 305)

$x_1 + x_4 + x_7 = 1$ (flight 406)

$x_2 + x_5 + x_6 = 1$ (flight 407)

$x_1 + x_4 + x_7 = 1$ (flight 308) *redundant constraint (same as flight 406 constraint)*

$x_4 + x_5 + x_6 + x_7 = 1$ (flight 109)

$x_3 = 1$ (flight 310) *redundant constraint (same as flight 402 constraint)*

$x_3 + x_7 = 1$ (flight 211)

$x_6 = 1$ (flight 212)

SIGN RESTRICTIONS (4 pts.)

$x_j = 0$ or 1 $j = 1, \ldots 7$

3. In addition to x_t $(t = 1, \dots 7)$;
Define y_t $(t = 1, \dots 7)$ as a binary variable

2 points

$$Min \sum_{t=1}^{7} x_t^2 + 5 \sum_{t=1}^{7} y_t$$

8 points

Subject to

$$\prod_{t=1}^{7} (x_t + 1) = 1000$$

8 points

$$1000 y_t \geq x_t \qquad t = 1,2, \dots 7.$$

5 points

$$x_t \geq 0 \text{ and integer} \quad t = 1,2, \dots 7.$$
$$y_t \in \{0,1\} \quad t = 1,2, \dots 7.$$

2 points

4.

Min f(x) ⇒Steepest descent method
Search direction is $-\nabla f(v)$

$$f(\mathbf{X}) = [2,1]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [x_1, x_2]\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$f(\mathbf{X}) = 2x_1 + x_2 + x_1^2 + 2x_2^2 + x_1 x_2$$

$$\frac{df}{dx_1} = 2 + 2x_1 + x_2$$

$$\frac{df}{dx_2} = 1 + 4x_2 + x_1$$

$\nabla f(\text{X}) = (2 + 2x_1 + x_2, 1 + 4x_2 + x_1)$

$v_0 = (0,1) \to \nabla f(\boldsymbol{v}_0) = \nabla f(0,1) = (3,5)$

$-\nabla f(0,1) = (-3,-5)$

$\boldsymbol{v}_1 = \boldsymbol{v}_0 + t\nabla f(\boldsymbol{v}_0)$

$\boldsymbol{v}_1 = (0,1) + t(-3,-5) = (-3t, 1-5t)$

$Solve\ min\ f(v_1)\ subject\ to\ t \geq 0;$

$f(-3t, 1-5t) = 74t^2 - 34t + 3$

$f'(-3t, 1-5t) = 148t - 34 = 0$

$t = 0.2297$

$\boldsymbol{v}_1 = (-3t, 1-5t) = (-0.6891, -0.1485)$

$\nabla f(\text{X}) = (2 + 2x_1 + x_2, 1 + 4x_2 + x_1)$

$\nabla f(-0.6891, -0.1485) = (0.4733, -0.2831)$

The length of the gradient vector at new point is:

$\| \nabla f(-0.6891, -0.1485) \| = 0.5515$