Analyzing the Problem (SAW, WP, TOPSIS)

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MADM Methods

- Elementary Methods
- **Value Based Methods**
  - Multi Attribute Value Theory
  - **Simple Additive Weighting**
  - **Weighted Product**
  - **TOPSIS**
- Outranking Methods
- AHP/ANP

SAW

- Simple Additive Weighting – Weighted Average – Weighted Sum (Yoon & Hwang, 1995; Vincke, 1992…)
- A global (total) score in the SAW is obtained by adding contributions from each attribute.
- A common numerical scaling system such as normalization (instead of single dimensional value functions) is required to permit addition among attribute values.
- Value (global score) of an alternative can be expressed as:

$$V(a_i) = V_i = \sum_{j=1}^{n} w_j r_{ij}$$

Example for SAW

Normalized (Linear) Decision Matrix and Global Scores

	Price	Comfort	Perf.	Design	V_i
Norm. w	0.3333	0.2667	0.2	0.2	
a_1	0.3333	1	1	1	**.7778**
a_2	0.4	1	0.6667	1	.7334
a_3	0.4	0.6667	1	1	.7111
a_4	0.5	0.6667	1	0.6667	.6778
a_5	0.5	0.6667	0.6667	1	.6778
a_6	0.5	0.3333	1	1	.6555
a_7	1	0.3333	0.6667	0.6667	.6889

Sensitivity Analysis

DM7xSensitivity

WP

- Weighted Product (Yoon & Hwang, 1995)
- Normalization is not necessary!
- When WP is used weights become exponents associated with each attribute value;
  - a positive power for benefit attributes
  - a negative power for cost attributes
- Because of the exponent property, this method requires that all ratings be greater than 1. When an attribute has fractional ratings, all ratings in that attribute are multiplied by 10^m to meet this requirement

$$V_i = \prod_j (x_{ij})^{w_j}$$

Example for WP

Quantitative Decision Matrix and Global Scores

	Price	Comfort	Perf.	Design	V_i
Norm. w	.3333	.2667	.2	.2	
$\boldsymbol{a_1}$	300	3	3	3	**.3108**
a_2	250	3	2	3	.3045
a_3	250	2	3	3	.2964
a_4	200	2	3	2	.2944
a_5	200	2	2	3	.2944
a_6	200	1	3	3	.2654
a_7	100	1	2	2	.2843

TOPSIS

- Technique for Order Preference by Similarity to Ideal Solution (Yoon & Hwang, 1995; Hwang & Lin, 1987)
- Concept:

  Chosen alternative should have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution
- Steps:
  - Calculate normalized ratings
  - Calculate weighted normalized ratings
  - Identify positive-ideal and negative-ideal solutions
  - Calculate separation measures
  - Calculate similarities to positive-ideal solution
  - Rank preference order

Steps

- Calculate normalized ratings
  - Vector normalization (Euclidean) is used
  - Do not take the inverse rating for cost attributes!
- Calculate weighted normalized ratings
  - $v_{ij} = w_j * r_{ij}$
- Identify positive-ideal and negative-ideal solutions

$$a^* = \{v_1^*, v_2^*, \ldots, v_j^*, \ldots, v_n^*\} = \left\{ \left(\max_i v_{ij} \middle| j \in J_1 \right), \left(\min_i v_{ij} \middle| j \in J_2 \right) \middle| i = 1, \ldots, m \right\}$$

$$a^- = \{v_1^-, v_2^-, \ldots, v_j^-, \ldots, v_n^-\} = \left\{ \left(\min_i v_{ij} \middle| j \in J_1 \right), \left(\max_i v_{ij} \middle| j \in J_2 \right) \middle| i = 1, \ldots, m \right\}$$

where J_1 is a set of benefit attributes and J_2 is a set of cost attributes

Steps

- Calculate separation measures
  - Euclidean distance (separation) of each alternative from the ideal solutions are measured:

$$S_i^* = \sqrt{\sum_j (v_{ij} - v_j^*)^2} \qquad S_i^- = \sqrt{\sum_j (v_{ij} - v_j^-)^2}$$

- Calculate similarities to positive-ideal solution

$$C_i^* = S_i^- / (S_i^* + S_i^-)$$

- Rank preference order
  - Rank the alternatives according to similarities in descending order.
  - Recommend the alternative with the maximum similarity

Example for TOPSIS

- Normalized (Vector) Decision Matrix

	Price	Comfort	Perf.	Design
Norm. w	0.3333	0.2667	0.2	0.2
a_1	0.5108	0.5303	0.433	0.4121
a_2	0.4256	0.5303	0.2887	0.4121
a_3	0.4256	0.3536	0.433	0.4121
a_4	0.3405	0.3536	0.433	0.2747
a_5	0.3405	0.3536	0.2887	0.4121
a_6	0.3405	0.1768	0.433	0.4121
a_7	0.1703	0.1768	0.2887	0.2747

Weighted Normalized Ratings & Positive–Negative Ideal

	Price	Comfort	Perf.	Design
a_1	0.1703	0.1414	0.0866	0.0824
a_2	0.1419	0.1414	0.0577	0.0824
a_3	0.1419	0.0943	0.0866	0.0824
a_4	0.1135	0.0943	0.0866	0.0549
a_5	0.1135	0.0943	0.0577	0.0824
a_6	0.1135	0.0471	0.0866	0.0824
a_7	0.0568	0.0471	0.0577	0.0549
a^*	.0568	.1414	.0866	.0824
a^-	.1703	.0471	.0577	.0549

Separation Measures & Similarities to Positive Ideal Solution

	S^*	S^-	C^*	Rank
a_1	0.1135	0.1024	0.4742	5
$\boldsymbol{a_2}$	0.0899	0.1022	**0.5321**	1
a_3	0.0973	0.0679	0.4111	6
a_4	0.0787	0.0792	0.5016	3
a_5	0.0792	0.0787	0.4984	4
a_6	0.11	0.0693	0.3866	7
a_7	0.1024	0.1135	0.5258	2

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