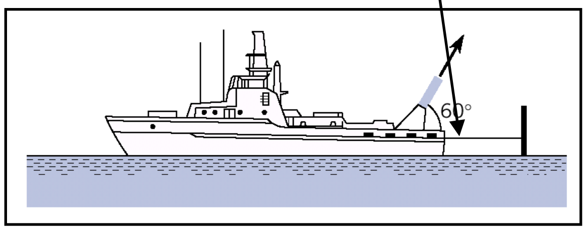
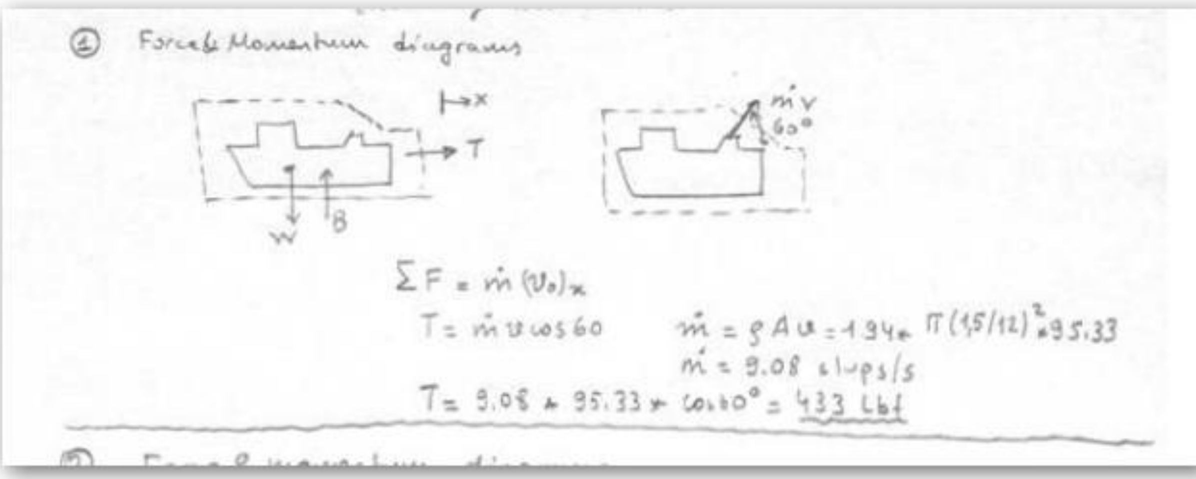
ADDITIONAL TUTORIAL FOR 5 AND 6TH CHAPTERS

1. A firehose on a boat is producing a 3-in diameter water jet with a speed of v = 65 mph. The boat is held stationary by a cable attached to a pier, and the water temperature is 50 °F calculate the tension in the cable. (Text Book Prob. 6.9)





$\Sigma F = \dot{m}(v_o)_x$

$T = \dot{m}\, v \cos 60$

$\dot{m} = \rho A v = 1.94 \times \pi(1.5/12)^2 \times 95.33$

$\dot{m} = 9.08$ slugs/s

$T = 9.08 \times 95.33 \times \cos 60° = \underline{433 \text{ Lbf}}$

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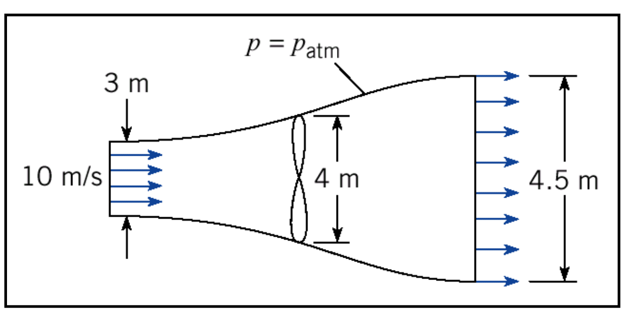
2. A windmill is operating in a 10-m/s wind that has a density of air 1.2 kg/m^3. The diameter of the windmill is 4 m. The constant pressure (atmospheric) streamline has a diameter of 3 m upstream of the windmill and 4.5 downstream. Assume that the velocity distributions are uniform and the air is incompressible. Determine the thrust on the windmill. (Text Book Prob. 6.66).



Continuity $V_2 = 10 \times (3/4.5)^2 = 4.44$ m/s

x-Momentum $\sum F_x = \dot{m}(V_2 - V_1) = 1.2\,(\pi/4 \times 3^2)(10)(4.44 - 10)$

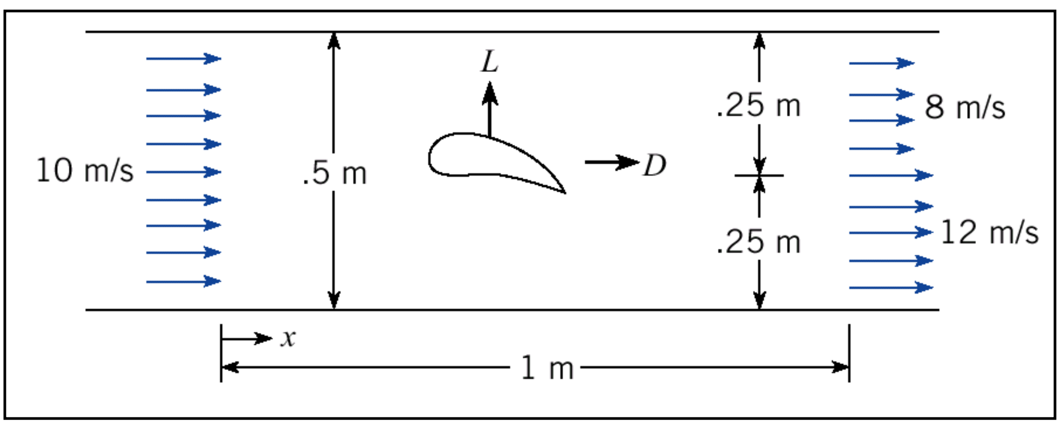
$F_x = -472$ N (acting to the left)

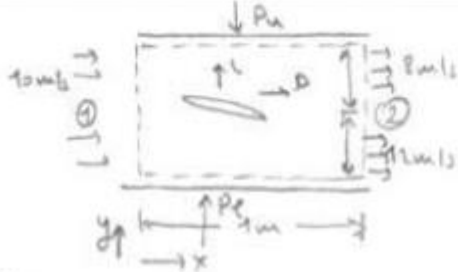
T = 472 N (Acting to the right)

3. An engineer is measuring the lift and drag on an airfoil section mounted in a two-dimensional wind tunnel. The wind tunnel is 0.5 m high and 0.5 deep (into the paper). The upstream wind velocity is uniform at 10 m/s, and downstream velocity is 12 m/s and 8 m/s as shown. The vertical component of velocity is zero at both stations. The test section is 1m long. The engineer measures the pressure distribution in the tunnel along the upper and lower walls and finds

$$p_u = 100 - 10x - 20x(1-x) \;\; \text{(Pa, gage)}$$
$$p_l = 100 - 10x + 20x(1-x) \;\; \text{(Pa, gage)}$$

where x is the distance in meters measured from the beginning of the test section. The gas density is homogeneous throughout and equal to 1.2 kg/m^3. The lift and drag are the vectors indicated on the figure. The forces acting on the fluid are in the opposite direction to these vectors. Find the lift and drag forces on the airfoil section. (Text Book Prob. 6.69).





x-Momentum

$$\sum F_x = \sum_{cs} \dot{m} v_2 - \dot{m}_1 v_1$$

$$-D + p_1 A_1 - p_2 A_2 = v_1(-\rho v_1 A_1) + v_a(\rho v_a A/2) + v_b(\rho v_b A/2)$$

$$-D/A = p_2 - p_1 - \rho v_1^2 + \rho v_a^2/2 + \rho v_b^2/2$$

$p_1 = p_u(x=0) = p_\ell(x=0) = 100$ kP gage

$p_2 = p_u(x=1) = p_\ell(x=1) = 90$ kP gage

$$-D/A = 90 - 100 + 1.2(-100 + 32 + 72) = -5.2$$

$$D = 5.2 \times 0.5^2 = \underline{1.3}\ N$$

y-momentum

$$\sum F_y = 0$$

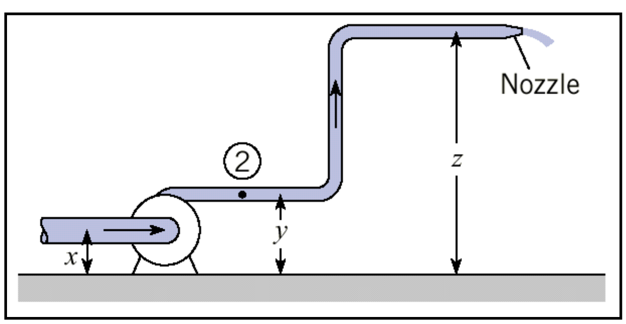
$$-L + \int_0^1 p_\ell B\,dx - \int_0^1 p_u B\,dx = 0 \qquad B: \text{depth of the tunnel}$$

$$-L + \int_0^1 (100 - 10x + 20x(1-x))\,0.5\,dx - \int_0^1 (100 - 10x - 20x(1-x))\,0.5\,dx = 0$$

$$-L + 0.5(100x - 5x^2 + 10x^2 - (20/3)x^3 \big|_0^1 - 0.5(100x - 5x^2 - 10x^2 + 20/3\,x^3 \big|_0^1 = 0$$

$$-L + 49.167 - 45.833 = 0 \rightarrow \underline{L = 3.334\ N}$$

4. For this system, the discharge of water is 0.20 m^3/s, x = 1.0 m, y = 2.0 m, z = 7.0 m, and the pipe diameter is 30 cm. Neglecting head losses, what is the pressure head at point 2 if the jet from the nozzle is 15 cm in diameter? (Text Book Prob. 7.32)



$V_n = Q/A_n = 0.20/((\pi/4)\times 0.15^2) = 11.32$ m/s   vel. of jet from nozzle

$V_n^2/2g = 6.53$ m

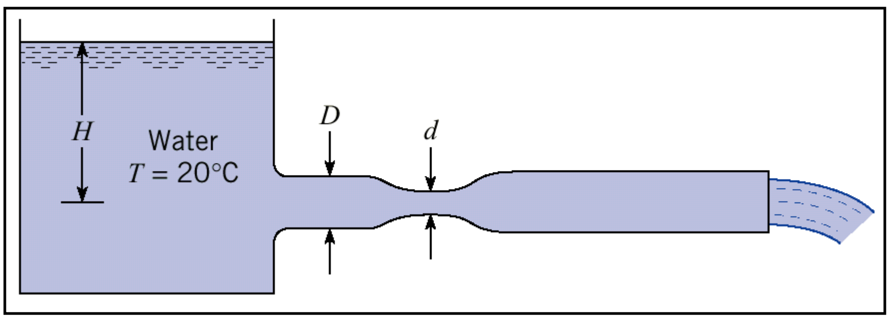
$V_2 = Q/A_2 = 0.20/((\pi/4 \times 0.30^2) = 2.83$ m/s

$V_2^2/2g = 0.408$ m

$p_2/\gamma + 0.408 + 2 = 0 + 6.53 + 7$

$p_2/\gamma = 11.1$ m

5. In this system d = 25 cm, D = 40 cm, and the head loss from the venturi meter to the end of the pipe is given by $h_l = 0.9V^2/2g$, where V is the velocity in the pipe. Neglecting all other head losses, determine what head H will first initiate cavitation if the atmospheric pressure is 100 kPa absolute. What will be the discharge at incipient cavitation? (Text Book Prob. 7.57).



First write en. eq. from the Venturi section to the end of the pipe.

From table A.5 $p_v = 2340$ Pa, abs.

$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_L$

$P_{vapour}/\gamma + V_1^2/2g = 0 + V_2^2/2g + 0.9 V_2^2/2g$

$P_{vapour} = 2.340$ Pa Abs $= -97660$ Pa gage

$V_1A_1 = V_2A_2$; $V_1 = V_2 A_2/A_1 = 2.56 V_2$; $V_1^2/2g = 6.55 V_2^2/2g$

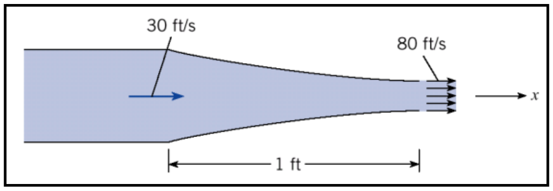
$-97660/9790 + 6.55 V_2^2/2g = 1.9 V_2^2/2g$; $\underline{V_2 = 6.49 \text{ m/s}}$

$Q = V_2 A_2 = 6.49 \times \pi/4 \times 0.4^2 = \underline{0.815}$ m³/s

$z_1 = V_2^2/2g + h_L$

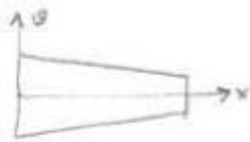
$H = 1.9 V_2^2/2g = 4.08$ m

6. If the velocity varies linearly with distance through this water nozzle, what is the pressure gradient, dp/dx , halfway through the nozzle? (Textbook Prob. 5.10)



Euler's equation $\frac{d}{dx}(p+\gamma z) = -\rho a_x$

But z = constant ; ∴ $\frac{dp}{dx} = -\rho a_x$ $\quad a_x = a_{convective}$



$a_{convective} = v \frac{dv}{dx}$

linear velocity variation has been given so

$v = ax+b \rightarrow$ $\quad x=0 \quad v = 30$ ft/s $\rightarrow b = 30$

$x = 1 \quad v = 80$ ft/s $\rightarrow 80 = a.1 + 30 \rightarrow a = 50$

$v = 50x + 30$ (Velocity function)

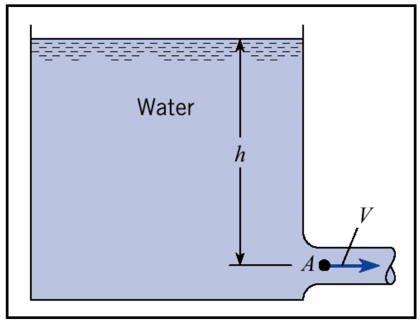
At the halfway through the nozzle $\rightarrow v = 50 \times 0.5 + 30 \rightarrow v = 55$ ft/s

$\frac{dv}{dx} = 50$ (ft/s/ft)

$a_{convective} = 55$ (ft/s) $*$ 50 (ft/s/ft) = $\underline{2750 \text{ ft/s}^2}$

Finally $\frac{dp}{dx} = -1.94 \left(\frac{\text{slug}}{\text{ft}^3}\right) 2750 \left(\frac{\text{ft}}{\text{s}^2}\right) = \underline{-5355 \text{ psf/ft}}$

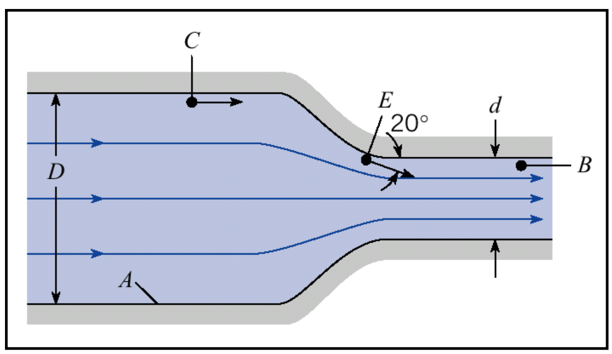
7. The velocity in the outlet pipe from this reservoir is 6 m/s and h = 15 m. Because of the rounded entrance to the pipe, the flow is assumed to be irrotational. Under these conditions, what is the pressure at A. (Textbook Prob. 5.47).



Bernoulli Equation $\quad p_2 = \gamma(z_1 - V_2^2/2g) = 9810\,(15 - 36/(2\times 9.81))$

$\underline{p_2 = 129.15 \text{ kPa}}$

8. The flow pattern through the pipe contraction is as indicated, and the discharge of water is 70 cfs. For d = 2 ft and D = 6 ft, what will be the pressure at point B if the pressure at point A is 3500 psf? (Textbook Prob. 5.57).



Continuity Eq. $\quad V_A = Q/A_A = 70/(\pi/4\times 6^2) = 2.476$ ft/s

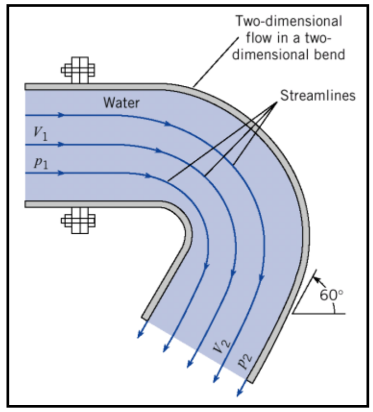
$V_B = Q/A_B = 70/(\pi/4\times 2^2) = 22.28$ ft/s

Bernoulli Eq. $\quad \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$

$$\frac{p_B}{\gamma} = \frac{3500}{62.4} - \frac{2.48^2}{64.4} - \frac{22.28^2}{64.4} - 4 = 2775 \text{ lbf/ft}^2$$

$\underline{p_B = 19.3 \text{ lbf/in}^2 \text{ (psi)}}$

9. As shown in the figure, water flows in the two-dimensional bend, which turns in a horizontal plane. Here $V_1 = V_2 = .13$ m/s, $p_1 = 10$ kPa gage, and, $p_2 = 0$ gage. Would you expect cavitation to occur anywhere within the bend if the atmospheric pressure is 100 kPa absolute? Hint: The streamlines are drawn to scale (Textbook Prob. 5.74).

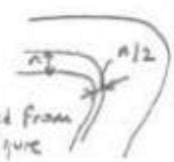


Check min. pressure value. Min pressure will occur where streamlines have smallest spacing (inside of bend). Thus

$$p_{min} + \rho V_{m}^2/2 = p_1 + \rho V_1^2/2 \Rightarrow V_{min} \times n_{min} = V_L \times n_L$$

where n is streamline spacing

$$V_{min} = V_1 \times n_1 / n_{min} = V_1 \times 2$$



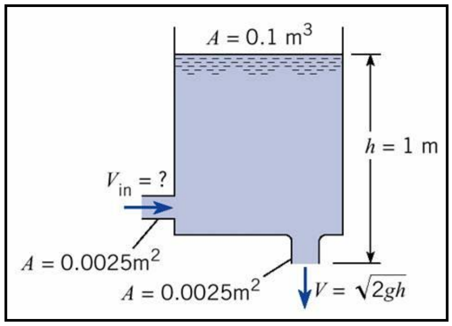
Scaled from figure

$$P_{min} = P_1 + (\rho/2)(-3V_1^2)$$

$$= 110 \times 10^3 + (1000/2)(-3 \times 13^2)$$

$P_{min} = -143$ kPa abs so $p_{min} < P_{vapor}$; ∴ Cavitation will occur

10. A tank has a hole in the bottom with a cross-sectional area of 0.0025 m^2 and an inlet line on the side with a cross-sectional area of 0.0025 m^2, as shown. The cross-sectional area of the tank is 0.1 m^2. The velocity of the liquid flowing out the bottom hole is $V = \sqrt{2gh}$ where h is the height of the water surface in the tank above the outlet. At a certain time the surface level in the tank is 1 m and raising at the rate of 0.1 cm/s. The liquid is incompressible. Find the velocity of the liquid through the inlet. (Textbook Prob. 4.57).



$$\sum_{c.s} \rho \cdot \vec{V} \cdot \vec{A} = -\frac{d}{dt}\int_{cv} \rho \, d\forall$$

$$-\rho V_{in} A_{in} + \rho V_{out} A_{out} = -\frac{d}{dt}(\rho A_{tank} h) = -A_{tank}\rho\,(dh/dt)$$

$$-V_{in}(0.0025) + \sqrt{2g(1)}\;0.0025 = -0.1 \times 0.1 \times 10^{2}$$

$$V_{in} = \sqrt{1962} + \frac{10^{-4}}{0.0025} = \underline{4.47\,m/s}$$

12. It is predicted that a flow field will have the following velocity components: (Textbook Prob. 4.101)

$$u = V(x^3 + xy^2) \qquad v = V(y^3 + yx^2) \qquad w = 0$$

V is a constant. Is such a flow field possible? (Does it satisfy continuity?)

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial t} = u(3x^2 + y^2) + u(3y^2 + x^2) + 0$$

Since $\neq 0$ Continuity is not satisfied

13. A tornado is modelled as a combined force and free vortex. The core of the tornado (forced vortex region) has a diameter of 10 miles. The wind velocity at a distance of 50 miles from the center of the tornado is 20 mph. What is the wind velocity at the edge of the core? What is the centrifugal acceleration of the air at this location? (Textbook Prob. 4.105).

Wind velocity at edge of core : V_{10}

Centrifugal acceleration at edge of core : a_c

The velocity variation in a free vortex is $Vr = \text{constant}$

Thus $V_{50}\,(50) = V_{10}\,(10)$ $\quad \therefore\ V_{10} = V_{50}\,\frac{50}{10} = 20\times 5 = 100$ mph

Acceleration (Eulerian Formulation)

$$V = 100 \times 5280 / 3600 = 147 \text{ ft/s}$$

$$a_c = \frac{V^2}{r} = \frac{147^2}{10 \times 5280} = 0.409 \text{ ft/s}^2$$