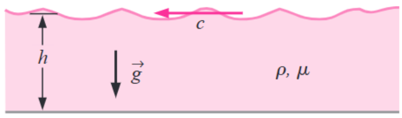
AKM 204-DÖNEM SONU-UYGULAMA SORU VE ÇÖZÜMLERİ

1. Use dimensional analysis to show that in a problem involving shallow water waves, both the Froude number and the Reynolds number are relevant dimensionless parameters. The wave speed c of waves on the surface of a liquid is a function of depth h, gravitational acceleration g, fluid density ρ, and fluid viscosity μ. Manipulate your Π's to get the parameters into the following form:

$$\text{Fr} = \frac{c}{\sqrt{gh}} = f(\text{Re}) \qquad \text{where } \text{Re} = \frac{\rho c h}{\mu}$$



Solution We are to show that Froude number and Reynolds number are the dimensionless parameters that appear in a problem involving shallow water waves.

Assumptions 1 Wave speed c is a function only of depth h, gravitational acceleration g, fluid density ρ, and fluid viscosity μ.

Analysis We perform a dimensional analysis using the method of repeating variables.

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters: $c = f(h, \rho, \mu, g) \qquad n = 5$

Step 2 The primary dimensions of each parameter are listed,

c	h	ρ	μ	g
$\{L^1t^{-1}\}$	$\{L^1\}$	$\{m^1L^{-3}\}$	$\{m^1L^{-1}t^{-1}\}$	$\{L^1t^{-2}\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since $j = 3$. We pick length scale h, density difference ρ, and gravitational constant g.

Repeating parameters: h, ρ, and g

Step 5 The Πs are generated. Note that for the first Π we do the algebra in our heads since the relationship is very simple. The dependent Π is

Π_1 = *Froude number*: $$\Pi_1 = \text{Fr} = \frac{c}{\sqrt{gh}} \qquad (1)$$

This Π is the *Froude number*. Similarly, the Π formed with viscosity is generated,

$$\Pi_2 = \mu h^a \rho^b g^c \qquad \{\Pi_2\} = \left\{ (m^1L^{-1}t^{-1})(L^1)^a (m^1L^{-3})^b (L^1t^{-2})^c \right\}$$

mass: $\{m^0\} = \{m^1m^b\} \qquad 0 = 1 + b \qquad b = -1$

time: $\{t^0\} = \{t^{-1}t^{-2c}\} \qquad 0 = -1 - 2c \qquad c = -\frac{1}{2}$

length: $\{L^0\} = \{L^{-1}L^aL^{-3b}L^c\} \qquad \begin{matrix} 0 = -1 + a - 3b + c \\ 0 = -1 + a + 3 - \frac{1}{2} \end{matrix} \qquad a = -\frac{3}{2}$

which yields

Π_2: $$\Pi_2 = \frac{\mu}{\rho h^{\frac{3}{2}}\sqrt{g}}$$

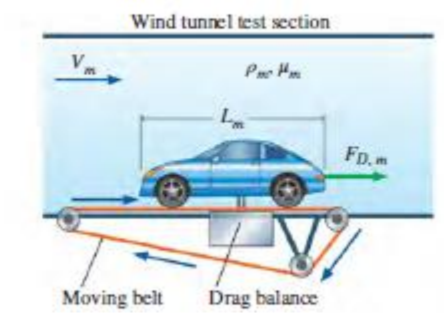
We can manipulate this Π into the Reynolds number if we invert it and then multiply by Fr (Eq. 1) The final form is

Modified Π_2 = *Reynolds number*: $$\Pi_2 = \text{Re} = \frac{\rho c h}{\mu}$$

Step 6 We write the final functional relationship as

Relationship between Πs: $$\boxed{\text{Fr} = \frac{c}{\sqrt{gh}} = f(\text{Re}) \quad \text{where} \quad \text{Re} = \frac{\rho c h}{\mu}}$$

2. The aerodynamic drag of a new sports car is to be predicted at a speed of 96 km/h (60 mph) at an air temperature of 25°C. Automotive engineers build a one-third scale model of the car to test in a wind tunnel. The temperature of the wind tunnel air is also 25°C. The drag force is measured with a drag balance, and the moving belt is used to simulate the moving ground (from the car 's frame of reference). Determine how fast the engineers should run the wind tunnel to achieve similarity between the model and the prototype. For air at T = 25°C and atmospheric pressure, ρ= 1.184 kg/m^3 and μ = 1.849 x 10^{-5} kg/ms.



Analysis Since there is only one independent Π in this problem, similarity is achieved if $\Pi_{2,m} = \Pi_{2,p}$, where Π_2 is the Reynolds number. Thus, we can write

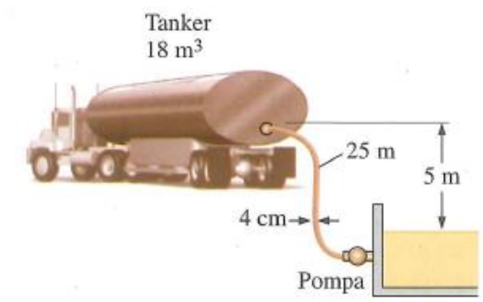
$$\Pi_{2,m} = \text{Re}_m = \frac{\rho_m V_m L_m}{\mu_m} = \Pi_{2,p} = \text{Re}_p = \frac{\rho_p V_p L_p}{\mu_p}$$

which can be solved for the unknown wind tunnel speed for the model tests, V_m,

$$V_m = V_p\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right) = (60.0 \text{ mph})(1)(1)(3) = \mathbf{180\ mph}$$

Thus, to ensure similarity, **the wind tunnel should be run at 180 miles per hour** (to three significant digits).

3. A vented tanker is to be filled with fuel oil with ρ = 920 kg/m^3 and μ = 0.045 kg/m·s from an underground reservoir using a 25-m-long, 4-cm-diameter plastic hose with a slightly rounded entrance and two 90° smooth bends. The elevation difference between the oil level in the reservoir and the top of the tanker where the hose is discharged is 5 m. The capacity of the tanker is 18 m^3 and the filling time is 30 min. Taking the kinetic energy correction factor at the hose discharge to be 1.05 and assuming an overall pump efficiency of 82 percent, determine the required power input to the pump. The loss coefficient is K_L = 0.12 for a slightly-rounded entrance and K_L = 0.3 for a 90° smooth bend (flanged). The plastic pipe is smooth and thus ε = 0. The kinetic energy correction factor at hose discharge is given to be α = 1.05.



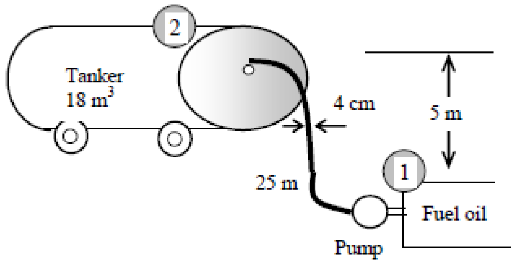
Analysis We choose point 1 at the free surface of oil in the reservoir and point 2 at the exit of the hose in the tanker. We note the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and the fluid velocity at point 1 is zero ($V_1 = 0$). We take the free surface of the reservoir as the reference level ($z_1 = 0$). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{pump,u}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{turbine,e}+h_L \quad\rightarrow\quad h_{pump,u}=\alpha_2\frac{V_2^2}{2g}+z_2+h_L$$

where

$$h_L=h_{L,total}=h_{L,major}+h_{L,minor}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$

since the diameter of the piping system is constant. The flow rate is determined from the requirement that the tanker must be filled in 30 min,



$$\dot{V}=\frac{V_{tanker}}{\Delta t}=\frac{18\,\text{m}^3}{(30\times 60\,\text{s})}=0.01\,\text{m}^3/\text{s}$$

Then the average velocity in the pipe and the Reynolds number become

$$V_2=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{0.01\,\text{m}^3/\text{s}}{\pi(0.04\,\text{m})^2/4}=7.958\,\text{m/s}$$

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(920\,\text{kg/m}^3)(7.958\,\text{m/s})(0.04\,\text{m})}{0.045\,\text{kg/m}\cdot\text{s}}=6508$$

which is greater than 4000. Therefore, the flow is turbulent. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\rightarrow\frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{6508\sqrt{f}}\right)$$

It gives f = 0.0347. The sum of the loss coefficients is

$$\sum K_L=K_{L,entrance}+2K_{L,bend}=0.12+2\times 0.3=0.72$$

Açık Haaland denklemş kullanılrsa f = 0.032716 bulunur. Yukarıdaki hesapta sıkıntı var.!

Haaland Denklemi: $\frac{1}{\sqrt{f}}\cong-1.8\log\left[\frac{6.9}{\text{Re}}+\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$

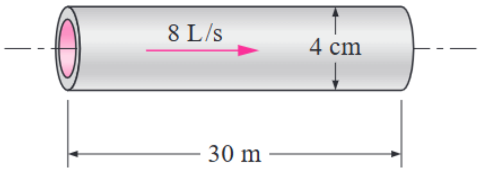
Note that we do not consider the exit loss unless the exit velocity is dissipated within the system (in this case it is not). Then the total head loss, the useful pump head, and the required pumping power become

$$h_L = \left(f\frac{L}{D} + \sum K_L \right)\frac{V_2^2}{2g} = \left((0.0347)\frac{25\,\text{m}}{0.04\,\text{m}} + 0.72 \right)\frac{(7.958\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 72.33\,\text{m}$$

$$h_{\text{pump,u}} = \frac{V_2^2}{2g} + z_2 + h_L = 1.05\frac{(7.958\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} + 5\,\text{m} + 73.33\,\text{m} = 80.72\,\text{m}$$

$$\dot{W}_{\text{pump}} = \frac{\dot{V}\rho g h_{\text{pump,u}}}{\eta_{\text{pump}}} = \frac{(0.01\,\text{m}^3/\text{s})(920\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(80.72\,\text{m})}{0.82}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2} \right)\left(\frac{1\,\text{kW}}{1\,\text{kN}\cdot\text{m/s}} \right) = \mathbf{8.88\,kW}$$

4. Water at 15°C (ρ= 999.1 kg/m^3 and μ=1.138 x 10^{-3} kg/m*s) is flowing steadily in a 30-m-long and 4-cm-diameter horizontal pipe made of stainless steel at a rate of 8 L/s. Determine (a) the pressure drop, (b) the head loss, and (c) the pumping power requirement to overcome this pressure drop.

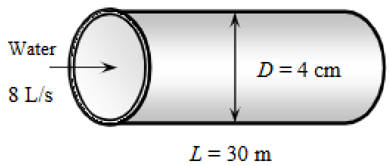


Properties The density and dynamic viscosity of water are given to be ρ = 999.1 kg/m^3 and μ = 1.138×10^{-3} kg/m·s, respectively. The roughness of stainless steel is 0.002 mm.

Analysis First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.008\,\text{m}^3/s}{\pi(0.04\,\text{m})^2/4} = 6.366\,\text{m}/s$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.1\,\text{kg/m}^3)(6.366\,\text{m/s})(0.04\,\text{m})}{1.138\times10^{-3}\,\text{kg/m}\cdot\text{s}} = 2.236\times10^5$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{2\times10^{-6}\,\text{m}}{0.04\,\text{m}} = 5\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{5\times10^{-5}}{3.7} + \frac{2.51}{2.236\times10^5\sqrt{f}} \right)$$

It gives f = 0.01573. Then the pressure drop, head loss, and the required power input become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.01573\frac{30\,\text{m}}{0.04\,\text{m}}\frac{(999.1\,\text{kg/m}^3)(6.366\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}} \right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2} \right) = \mathbf{239\ kPa}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.01573\frac{30\,\text{m}}{0.04\,\text{m}}\frac{(6.366\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \mathbf{24.4\,m}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.008\,\text{m}^3/s)(239\,\text{kPa})\left(\frac{1\,\text{kW}}{1\,\text{kPa}\cdot\text{m}^3/\text{s}} \right) = \mathbf{1.91\,kW}$$

Therefore, useful power input in the amount of 1.91 kW is needed to overcome the frictional losses in the pipe.

Açık Haaland denklemi Haaland Denklemi: $\frac{1}{\sqrt{f}} \cong -1.8\log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7} \right)^{1.11} \right]$ kullanılırsa f = 0.015718 çıkar.

5. Consider the velocity field unsteady, two-dimensional velocity field given by $\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + [1.5 + 2.5\,sin(\omega t) - 0.8y]\vec{j}$, where angular frequency ω is equal to 2π rad/s (a physical frequency of 1 Hz). Verify that this flow field can be approximated as incompressible.

$$u = 0.5 + 0.8x \qquad \text{and} \qquad v = 1.5 + 2.5\sin(\omega t) - 0.8y$$

If the flow is incompressible, Eq. 9–16 must apply. More specifically, in Cartesian coordinates Eq. 9–17 must apply. Let's check:

$$\underbrace{\frac{\partial u}{\partial x}}_{0.8} + \underbrace{\frac{\partial v}{\partial y}}_{-0.8} + \underbrace{\frac{\partial w}{\partial z}}_{0 \text{ since 2-D}} = 0 \quad \rightarrow \quad 0.8 - 0.8 = 0$$

So we see that the incompressible continuity equation is indeed satisfied at any instant in time, and **this flow field may be approximated as incompressible**.

Discussion Although there is an unsteady term in v, it has no y-derivative and drops out of the continuity equation.

6. Consider a steady, two-dimensional, incompressible, irrotational velocity field specified by its velocity potential function, $\phi = 3(x^2 - y^2) + 4xy - 2x - 5y + 2$.
(a) Calculate velocity components u and v.
(b) Verify that the velocity field is irrotational in the region in which ϕ applies.
(c) Generate an expression for the stream function in this region.

Analysis (*a*) The velocity components are found by taking the x and y partial derivatives of ϕ,

Velocity components:

$$\boxed{u = \frac{\partial \phi}{\partial x} = 6x + 4y - 2 \qquad v = \frac{\partial \phi}{\partial y} = -6y + 4x - 5} \tag{1}$$

(*b*) We plug in u and v from Eq. 1 into the z component of vorticity to get

z-component of vorticity:

$$\zeta_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 4 - 4 = 0 \tag{2}$$

Since $\zeta_z = 0$, and the only component of vorticity in a 2-D flow in the x-y plane is in the z direction, the vorticity is zero, and **the flow is irrotational in the region of interest**.

(*c*) The stream function is found by integration of the velocity components. We begin by integrating the x component, $\partial\psi/\partial y = u$, and then taking the x derivative to compare with the known value of v,

$$\psi = 6xy + 2y^2 - 2y + f(x) \quad \rightarrow \quad v = -\frac{\partial \psi}{\partial x} = -6y - f'(x) = -6y + 4x - 5 \tag{3}$$

From which we see that $f'(x) = 5 - 4x$. Integrating with respect to x,

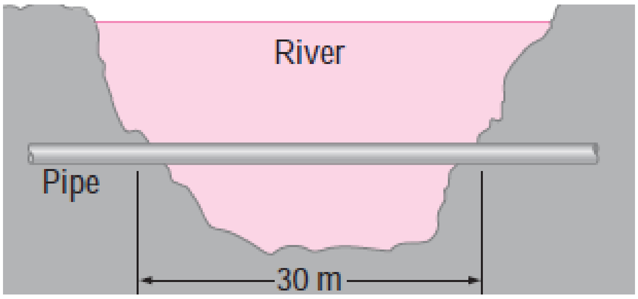
$$f(x) = 5x - 2x^2 + \text{constant} \tag{4}$$

The constant is arbitrary since velocity components are always derivatives of ψ. Thus,

Stream function:

$$\boxed{\psi = 6xy + 2y^2 - 2y + 5x - 2x^2 + \text{constant}} \tag{5}$$

7. A 2.2-cm-outer-diameter pipe is to span across a river at a 30-m-wide section while being completely immersed in water. The average flow velocity of water is 4 m/s and the water temperature is 15°C. Determine the drag force exerted on the pipe by the river.



Assumptions **1** The outer surface of the pipe is smooth so that Fig. 11–34 can be used to determine the drag coefficient. **2** Water flow in the river is steady. **3** The direction of water flow is normal to the pipe. **4** Turbulence in river flow is not considered.

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1\ \text{kg/m}^3$ and $\mu = 1.138 \times 10^{-3}\ \text{kg/m} \cdot \text{s}$.

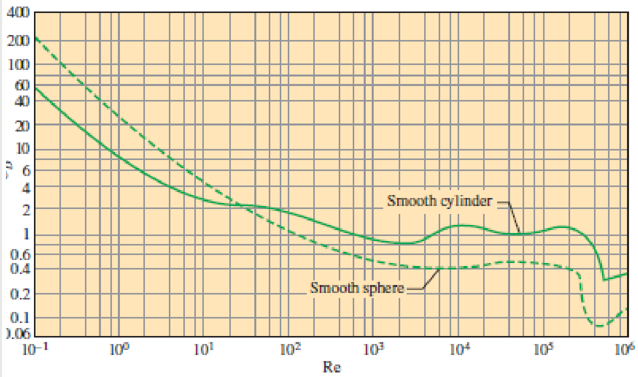
Analysis Noting that $D = 0.022$ m, the Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{\rho VD}{\mu} = \frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})(0.022\ \text{m})}{1.138 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} = 7.73 \times 10^4$$

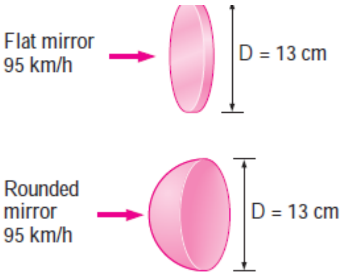
The drag coefficient corresponding to this value is, from Fig. 11–34, $C_D = 1.0$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force acting on the pipe becomes

$$F_D = C_D A \frac{\rho V^2}{2} = 1.0(30 \times 0.022\ \text{m}^2)\frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2}\right)$$

$$= 5275\ \text{N} \cong 5300\ \text{N}$$



8. As part of the continuing efforts to reduce the drag coefficient and thus to improve the fuel efficiency of cars, the design of side rearview mirrors has changed drastically from a simple circular plate to a streamlined shape. Determine the amount of fuel and money saved per year as a result of replacing a 13-cm-diameter flat mirror by one with a hemispherical back. Assume the car is driven 24,000 km a year at an average speed of 95 km/h. Take the density and price of gasoline to be 0.8 kg/L and $0.60/L, respectively; the heating value of gasoline to be 44,000 kJ/kg; and the overall efficiency of the engine to be 30 percent.



$C_D = 1.1$

$C_D = 0.4$

SOLUTION The flat mirror of a car is replaced by one with a hemispherical back. The amount of fuel and money saved per year as a result are to be determined.

Assumptions 1 The car is driven 24,000 km a year at an average speed of 95 km/h. **2** The effect of the car body on the flow around the mirror is negligible (no interference).

Properties The densities of air and gasoline are taken to be 1.20 kg/m^3 and 800 kg/m^3, respectively. The heating value of gasoline is given to be 44,000 kJ/kg. The drag coefficients C_D are 1.1 for a circular disk and 0.4 for a hemispherical body.

Analysis The drag force acting on a body is determined from

$$F_D = C_D A \frac{\rho V^2}{2}$$

where A is the frontal area of the body, which is $A = \pi D^2/4$ for both the flat and rounded mirrors. The drag force acting on the flat mirror is

$$F_D = 1.1 \frac{\pi(0.13\text{ m})^2}{4} \frac{(1.20\text{ kg/m}^3)(95\text{ km/h})^2}{2} \left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2 \left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 6.10\text{ N}$$

Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 24,000 km are

$$W_{drag} = F_D \times L = (6.10\text{ N})(24{,}000\text{ km/year}) = 146{,}400\text{ kJ/year}$$

$$E_{in} = \frac{W_{drag}}{\eta_{car}} = \frac{146{,}400\text{ kJ/year}}{0.3} = 488{,}000\text{ kJ/year}$$

Then the amount and costs of the fuel that supplies this much energy are

$$\text{Amount of fuel} = \frac{m_{fuel}}{\rho_{fuel}} = \frac{E_{in}/HV}{\rho_{fuel}} = \frac{(488{,}000\text{ kJ/year})/(44{,}000\text{ kJ/kg})}{0.8\text{ kg/L}}$$

$$= 13.9\text{ L/year}$$

$$\text{Cost} = (\text{Amount of fuel})(\text{Unit cost}) = (13.9\text{ L/year})(\$0.60/\text{L}) = \$8.32/\text{year}$$

That is, the car uses 13.9 L of gasoline at a cost of $8.32 per year to overcome the drag generated by a flat mirror extending out from the side of a car.

The drag force and the work done to overcome it are directly proportional to the drag coefficient. Then the percent reduction in the fuel consumption due to replacing the mirror is equal to the percent reduction in the drag coefficient:

$$\text{Reduction ratio} = \frac{C_{D,\text{ flat}} - C_{D,\text{ hemisp}}}{C_{D,\text{ flat}}} = \frac{1.1 - 0.4}{1.1} = 0.636$$

$$\text{Fuel reduction} = (\text{Reduction ratio})(\text{Amount of fuel})$$

$$= 0.636(13.9\text{ L/year}) = \mathbf{8.84\text{ L/year}}$$

$$\text{Cost reduction} = (\text{Reduction ratio})(\text{Cost}) = 0.636(\$8.32/\text{year}) = \mathbf{\$5.29/\text{year}}$$

Since a typical car has two side rearview mirrors, the driver saves more than $10 per year in gasoline by replacing the flat mirrors with hemispherical ones.

<u>Güç harcamaları ise şöyle bulunur</u>.

$P=F_D*V$ = 6.10 (N) * 95 (km/sa) / 3.6 (m/s) = 161 W (Düz Ayna)

$P=F_D*V$ = 0.4/1.1 * 161 = 58 W (Yuvarlatılmış Ayna)