QUESTION 1 The blanks below will be filled by students. (Except the score)

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For the solution of this question please use only the front face and if necessary the back face of this page.

Evaluate the following integrals

[10 pt] a)

$$\int x\tan^2 x \, dx = ?$$

[15 pt] b)

$$\int_{-\ln 2}^{0} \frac{dx}{e^x - 2e^{-x} - 1}$$

a) $I = \int x\tan^2 x \, dx = \int x.(\sec^2 x - 1) dx = \underbrace{\int x\sec^2 x dx}_{I_1} - \int x dx$

$\sec^2 x = \tan^2 x + 1$

$I_1 = \int x\sec^2 x dx \quad \begin{bmatrix} x = u, & dx = du \\ \sec^2 x dx = dv, & v = \tan x \end{bmatrix}$

$I_1 = x\tan x - \underbrace{\int \tan x dx}_{\int \frac{\sin x}{\cos x} dx} = x\tan x + \ln|\cos x| + C_1$

$I = x\tan x + \ln|\cos x| - \frac{x^2}{2} + C$

b) $I = \int_{-\ln 2}^{0} \frac{dx}{e^x - \frac{2}{e^x} - 1} = \int_{-\ln 2}^{0} \frac{e^x dx}{(e^x)^2 - e^x - 2} \quad \begin{bmatrix} e^x = u \\ e^x dx = du \end{bmatrix}$

$I = \int_{u_0}^{u_1} \frac{du}{u^2 - u - 2}$

$u^2 - u - 2 = (u-2)(u+1)$

$\frac{1}{(u-2)(u+1)} = \frac{A}{u-2} + \frac{B}{u+1} \qquad A = -B = \frac{1}{3}$

$I = \frac{1}{3}\left(\int_{u_0}^{u_1} \frac{du}{u-2} - \int_{u_0}^{u_1} \frac{du}{u+1}\right) = \frac{1}{3}\left(\ln|e^x - 2| - \ln|e^x + 1|\right)\Big|_{-\ln 2}^{0}$

$I = \frac{1}{3} \ln\left|\frac{e^x - 2}{e^x + 1}\right| \Big|_{-\ln 2}^{0} = \boxed{-\frac{1}{3}\ln 2}$

QUESTION 2 The blanks below will be filled by students. (Except the score)

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[10 pt] a) Find $\mathbf{f'(1)}$ if $\displaystyle\int_1^x \mathbf{f(t)dt = x + \ln(f(x))}$, $\mathbf{f(x) > 0}$ for all $\mathbf{x}$

[15 pt] b) The region bounded by the curve $\mathbf{y = \ln x}$ and the lines $\mathbf{x = 1}$ and $\mathbf{y = 1}$ is revolved about

i) the y-axis to genarate a solid. Find the volume of the this solid by using Disk/Washer Method

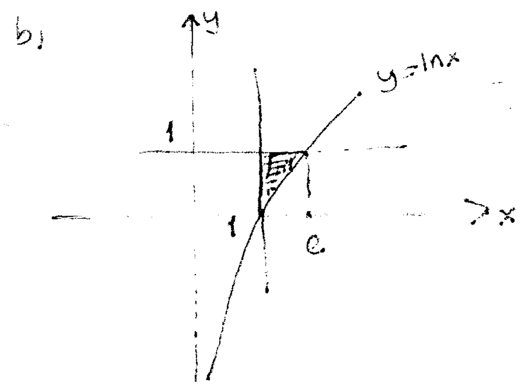
ii) the x-axis to genarate a solid. Find the volume of the this solid by using Shell Method

a) $\int_1^x f(t)dt = x + \ln f(x) \Rightarrow f(x) = 1 + \frac{f'(x)}{f(x)}$, $f(1) = 1 + \frac{f'(1)}{f(1)}$

$\underbrace{\int_1^1 f(t)dt}_{0} = 1 + \ln f(1) \Rightarrow \ln f(1) = -1 \quad f(1) = e^{-1} = \frac{1}{e}$

$\frac{1}{e} = 1 + \frac{f'(1)}{1/e} \to \frac{1}{e} - 1 = e f'(1) \Rightarrow \boxed{f'(1) = \frac{1-e}{e^2}}$

b)



i) $V_y = \pi \int_{y=0}^{1} (e^{2y} - 1) dy$

$V_y = \pi . \left(\frac{e^{2y}}{2} - y\right)\Big|_{y=0}^{1} = \pi\left(\frac{e^2}{2} - 1 - \frac{1}{2}\right)$

$V_y = \pi \frac{(e^2 - 3)}{2}$

ii) $V_x = 2\pi \int x y dy$

$V_x = 2\pi \int_{y=0}^{1} (e^y - 1) y dy$

$V_x = 2\pi \int_{y=0}^{1} e^y y dy - 2\pi \underbrace{\int_{y=0}^{1} y dy}_{\frac{y^2}{2}\Big|_0^1} = 2\pi \int_0^1 y e^y dy - \pi$

$\int_0^1 y e^y dy \quad \left[y = u \quad dy = du \quad e^y dy = dv, \; v = e^y \right]$ $\qquad V_x = \pi$

$= y.e^y\Big|_0^1 - \int_0^1 e^y dy = e - e^y\Big|_0^1 = e - e + 1 = 1$

QUESTION 3 The blanks below will be filled by students. (Except the score)

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[13 pt] a) For a certain number **A** the integral

$$\int_2^\infty \left(\frac{Ax}{x^2+1} - \frac{1}{2x+1}\right)dx$$

converges. Find that value of A and evaluate the integral.

[12 pt] b)

$$\lim_{x\to\infty}\left[(x+3)e^{\frac{1}{x-2}} - x\right] = ?$$

a) $I = \lim_{b\to\infty} \int_2^b \left(\frac{Ax}{x^2+1} - \frac{1}{2x+1}\right)dx = \lim_{b\to\infty}\left(\int_2^b \frac{Ax}{x^2+1}dx - \int_2^b \frac{dx}{2x+1}\right)$

$= \lim_{b\to\infty}\left(\int_2^b \frac{Ax\,dx}{x^2+1} \left[\begin{matrix} x^2+1=u \\ x\,dx = \frac{du}{2}\end{matrix}\right] - \frac{1}{2}\ln|2x+1| \Big|_2^b\right)$

$= \lim_{b\to\infty} \frac{1}{2}\left(A\ln(x^2+1) - \ln|2x+1|\right)\Big|_2^b = \lim_{b\to\infty}\frac{1}{2}\left(\ln\frac{(x^2+1)^A}{|2x+1|}\right)\Big|_2^b$

$= \lim_{b\to\infty}\frac{1}{2}\left(\ln\frac{(b^2+1)^A}{|2b+1|} - \ln\frac{5^A}{5}\right)$

If $A = \frac{1}{2}$ $\quad I = \lim_{b\to\infty}\frac{1}{2}\left(\ln\left(\frac{\sqrt{1+\frac{1}{b^2}}}{(2+\frac{1}{b})}\right) - \ln\frac{\sqrt{5}}{5}\right)$

$I = \frac{1}{2}\left(\ln\frac{1}{2} - \ln\frac{1}{\sqrt{5}}\right) = \frac{1}{2}\left(\ln\frac{1}{2} + \frac{1}{2}\ln 5\right) = \frac{1}{2}\left(-\ln 2 + \frac{1}{2}\ln 5\right)$

$\underline{I = \frac{1}{2}\ln\left(\frac{\sqrt{5}}{2}\right)}$

b) $\lim_{x\to\infty}\left[x\left(e^{\frac{1}{x-2}} - 1\right) + 3e^{\frac{1}{x-2}}\right] = \lim_{x\to\infty}\underbrace{x\left(e^{\frac{1}{x-2}} - 1\right)}_{0.\infty} + \lim_{x\to\infty} 3e^{\frac{1}{x-2}} = \boxed{4}$

$\Rightarrow \lim_{x\to\infty}\frac{e^{\frac{1}{x-2}} - 1}{\frac{1}{x}} \quad \left(\frac{0}{0}\right)$

$= \lim_{x\to\infty}\frac{\frac{-1}{(x-2)^2}e^{\frac{1}{x-2}}}{\frac{-1}{x^2}} = \lim_{x\to\infty}\frac{x^2}{(x-2)^2}e^{\frac{1}{x-2}} = \lim_{x\to\infty}\frac{x^2}{(x-2)^2}\cdot\lim_{x\to\infty}e^{\frac{1}{\infty}}$ $\left(\frac{\infty}{\infty}\right)$

$\left(\Rightarrow \lim_{x\to\infty}\frac{x^2}{(x-2)^2} = 1\right) \qquad = 1$

QUESTION 4 The blanks below will be filled by students. (Except the score)

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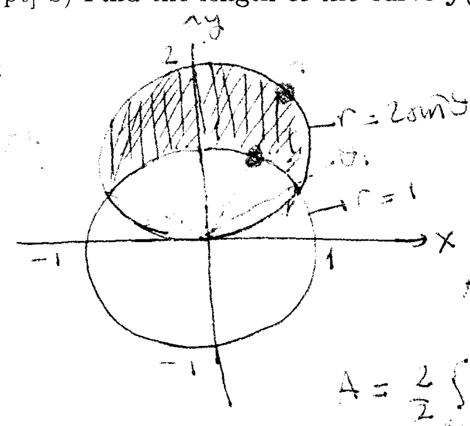
[12 pt] a) Find the area of the region which lies inside the curve $\mathbf{r = 2\sin\theta}$ and outside the curve $\mathbf{r = 1}$.

[13 pt] b) Find the length of the curve $\mathbf{y(x)} = \int_0^{\mathbf{x}} \sqrt{\cos \mathbf{2t}}\,\mathbf{dt}$ from $\mathbf{x = 0}$ to $\mathbf{x} = \pi/4$

a)



$r = 2\sin\theta \qquad r = 1$

$1 = 2\sin\theta, \quad \theta_1 = \frac{\pi}{6} \qquad \theta_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$$A = \frac{1}{2}\int_{\theta=\alpha}^{\theta=\beta} r^2(\theta)\,d\theta$$

$$A = \frac{2}{2}\int_{\theta=\pi/6}^{\theta=\pi/2} (4\sin^2\theta - 1)\,d\theta \qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$A = \frac{4}{2}\int_{\theta=\pi/6}^{\theta=\pi/2}(1-\cos 2\theta)\,d\theta - \int_{\pi/6}^{\pi/2} d\theta = \left[2.\left(\theta - \frac{\sin 2\theta}{2}\right) - \theta\right]\Big|_{\theta=\pi/6}^{\theta=\pi/2}$$

$$A = (\theta - \sin 2\theta)\Big|_{\theta=\pi/6}^{\theta=\pi/2} = \frac{\pi}{2} - \frac{\pi}{6} - \underbrace{\sin\pi}_{0} + \sin\frac{\pi}{3} = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$

b)

$$L = \int_{x=0}^{x=\pi/4} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \qquad \frac{dy}{dx} = \sqrt{\cos 2x}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\cos 2x \qquad 1+\cos 2x = 2\cos^2 x$$

$$\sqrt{1+\cos 2x} = \sqrt{2}.\sqrt{\cos^2 x} = \sqrt{2}.|\cos x|$$

$$L = \sqrt{2}\int_0^{\pi/4} |\cos x|\,dx$$

$$L = \sqrt{2}.\int_0^{\pi/4}\cos x\,dx = \sqrt{2}\sin x\Big|_{x=0}^{x=\pi/4} = \sqrt{2}\Big(\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}} - \underbrace{\sin 0}_{0}\Big)$$

$$\boxed{L = 1}$$