MATHEMATICS 101E CRN:............. FINAL 2 June 2010

QUESTION 1 The blanks below will be filled by students. (Except the score)

Surname:	Name:	Group Number:	List Number:	**Score**
Signature:	Electronic Post(e-mail) address:	Student Number:		

For the solution of this question please use only the front face and if necessary the back face of this page.

Evaluate the following integrals.

[13pts] **a)** $\displaystyle\int \frac{x+2}{x^2(x^2-1)}\,dx$ [12pts] **b)** $\displaystyle\int_0^{\ln\sqrt{3}} e^{-x}\tan^{-1}(e^x)\,dx$

a) $\dfrac{x+2}{x^2(x+1)(x-1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1} + \dfrac{D}{x-1}$.

$x+2 = Ax(x^2-1) + B(x^2-1) + Cx^2(x-1) + Dx^2(x+1)$

$x=0 \quad 2=-B \quad B=-2$ $\qquad x=-1 \quad 1=-2C \quad C=-\frac{1}{2}$.

$x=1 \quad 3=2D \quad D=\frac{3}{2}$ $\qquad x=2. \quad 4=6A+3B+4C+12D$.

$4 = 6A-6-2+18$. $\quad A=-1$.

$$\int \frac{x+2}{x^2(x^2-1)}dx = -\int\frac{dx}{x} - 2\int\frac{dx}{x^2} - \frac{1}{2}\int\frac{dx}{x+1} + \frac{3}{2}\int\frac{dx}{x-1}.$$

$$= -\ln|x| + \frac{2}{x} - \frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + c.$$

b) $\displaystyle\int_0^{\ln\sqrt{3}} e^{-x}\tan^{-1}(e^x)dx$ $\left[\begin{array}{ll} u=\tan^{-1}e^x & dv=e^{-x}dx. \\ du=\dfrac{e^x}{1+e^{2x}}dx & v=-e^{-x} \end{array}\right]$

$$\int_0^{\ln\sqrt{3}} e^{-x}\tan^{-1}e^x\,dx = -e^{-x}\tan^{-1}e^x\Big|_0^{\ln\sqrt{3}} + \int_0^{\ln\sqrt{3}} \frac{1}{1+e^{2x}}\,\frac{e^{-2x}}{e^{-2x}}dx.$$

$$= -\frac{1}{\sqrt{3}}\tan^{-1}\sqrt{3} + 1\,\frac{\pi}{4} + \frac{\ln(1+e^{-2x})}{-2}\Big|_0^{\ln\sqrt{3}}.$$

$$= -\frac{1}{\sqrt{3}}\frac{\pi}{3} + \frac{\pi}{4} - \frac{\overbrace{\ln(1+e^{-2\ln\sqrt{3}})}^{\ln(1+\frac{1}{3})}}{2} + \frac{\ln 2}{2}.$$

$$\int_0^{\ln\sqrt{3}} e^{-x}\tan^{-1}e^x\,dx = \frac{-\pi}{3\sqrt{3}} + \frac{\pi}{4} + \frac{1}{2}\ln\frac{3}{2}.$$

QUESTION 2 The blanks below will be filled by students. (Except the score)

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[13pts] **a)** Evaluate the following limit

$$\lim_{x\to\frac{\pi}{2}} \left(1-\cotan x\right)^{\sec x} = ?$$

[12pts] **b)** Find the area of the region that lies inside the circle $\mathbf{r=-2\sin\theta}$ and outside the circle $\mathbf{r=1}$.

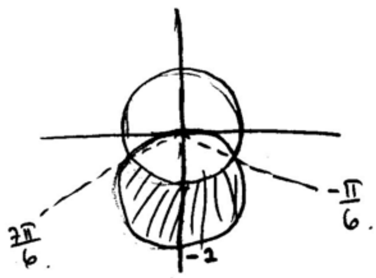
a) $\lim_{x\to\frac{\pi}{2}} (1-\cotan x)^{\sec x} \quad 1^\infty \quad = e^{\lim_{x\to\pi/2} \sec x \ln(1-\cotan x)}$

$$= e^{\lim_{x\to\pi/2} \frac{\ln(1-\cotan x)}{\cos x}} \quad \left(\frac{0}{0}\right)$$

$$= e^{\lim_{x\to\pi/2} \frac{\frac{\operatorname{cosec}^2 x}{1-\cotan x}}{-\sin x}}$$

$$= e^{\lim_{x\to\pi/2} \frac{-\operatorname{cosec}^2 x}{\sin x - \cos x}} = e^{\lim_{x\to\pi/2} \frac{-1}{\sin^3 x - \sin x\cos x}} = e^{-1}.$$

b)



$r=1 \quad r=-2\sin\theta$

$1=-2\sin\theta$

$-\frac{1}{2}=\sin\theta \quad \theta=-\frac{\pi}{6}, \frac{7\pi}{6}$

$$\text{Area}= 2\,\frac{1}{2}\int_{-\frac{\pi}{2}}^{-\pi/6} (-2\sin\theta)^2-1^2\, d\theta = \int_{-\frac{\pi}{2}}^{-\pi/6} 4\sin^2\theta - 1\, d\theta$$

$$\text{Area}= \int_{-\frac{\pi}{2}}^{-\pi/6} (2-2\cos 2\theta - 1)\, d\theta = \theta - \sin 2\theta \Big|_{-\pi/2}^{-\pi/6}$$

$$\text{Area}= -\frac{\pi}{6} - \sin\frac{-\pi}{3} + \frac{\pi}{2} + \sin -\pi = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$

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QUESTION 3 The blanks below will be filled by students. (Except the score)

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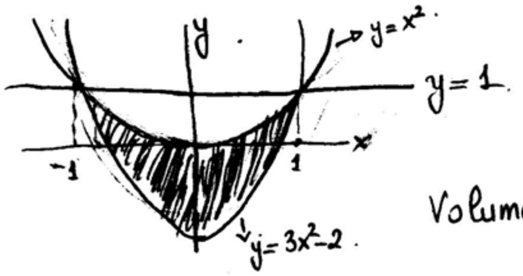
[13pts] **a** Find the volume of solid generated by revolving the region **R** enclosed by the curves $\mathbf{y = 3x^2 - 2}$ $\mathbf{y = x^2}$ about the line $\mathbf{y = 1}$.

[12pts] **b)** Find the volume of solid generated by revolving the region **R** enclosed by the curves $\mathbf{y = \sin x}$ y=cos **x** and the lines $\mathbf{x} = \frac{\pi}{4}$ and $\mathbf{x} = \frac{\pi}{2}$ about **y** axis by using Shell method.

(No credit will be given to any other method)

a)



Volume $=\pi\int_{-1}^{1}(1-3x^2+2)^2-(1-x^2)^2\,dx$.

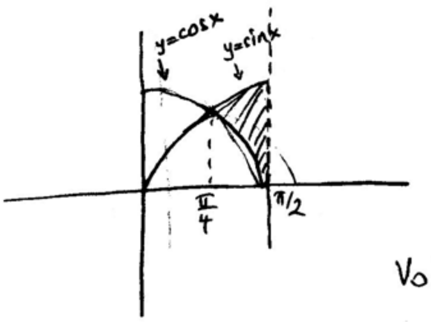
Volume $=\pi\int_{-1}^{1}(9-18x^2+9x^4-1+2x^2-x^4)\,dx$.

Volume $=\pi\int_{-1}^{1}(8x^4-16x^2+8)\,dx=\pi\left(8\frac{x^5}{5}-\frac{16}{3}x^3+8x\right)\Big|_{-1}^{1}$

Volume $=\pi\left(\frac{8}{5}-\frac{16}{3}+8\right)-\pi\left(-\frac{8}{5}+\frac{16}{3}-8\right)=\left(\frac{16}{5}-\frac{32}{3}+16\right)\pi$

Volume $=\frac{128\pi}{15}$

b)



Volume $=2\pi\int_{\pi/4}^{\pi/2}x(\sin x-\cos x)\,dx$. $\left[\begin{array}{ll} x=u & (\sin x-\cos x)dx=dv \\ dx=du. & +\cos x-\sin x=v \end{array}\right]$

Volume $=2\pi\left[-x\cos x-x\sin x\Big|_{\pi/4}^{\pi/2}+\int_{\pi/4}^{\pi/2}(\cos x+\sin x)\,dx\right]$

Volume $=2\pi\left[-\frac{\pi}{2}+\frac{\pi}{4}\frac{\sqrt{2}}{2}+\frac{\pi}{4}\frac{\sqrt{2}}{2}+(\sin x-\cos x)\Big|_{\pi/4}^{\pi/2}\right]$

Volume $=2\pi\left[-\frac{\pi}{2}+\frac{\pi}{2}\frac{\sqrt{2}}{2}+1-0\right]=-\pi^2+\pi^2\frac{\sqrt{2}}{2}+2\pi$

Volume $=\pi^2\left(\frac{\sqrt{2}}{2}-1\right)+2\pi$

QUESTION 4 The blanks below will be filled by students. (Except the score)

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[13pts] a) Let $\int_3^\infty \frac{\ln x}{\sqrt{x}+1}\,dx$. Investigate convergence or divergence of the improper integral.

[12pts] b) Find the length of the curve $x = t^3$, $y = \frac{3t^2}{2}$ $0 \le t \le \sqrt{3}$.

a) let $g(x) = \frac{1}{\sqrt{x}}$ $p = \frac{1}{2} < 1$, $\int_3^\infty g(x)\,dx$ is divergent.

Let us apply Limit Comparison Test.

$$\lim_{x\to\infty} \frac{\frac{\ln x}{\sqrt{x}+1}}{\frac{1}{\sqrt{x}}} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x}+1} \ln x = \infty$$

since $\int_3^\infty \frac{1}{\sqrt{x}}\,dx$ is divergent.

$\int_3^\infty \frac{\ln x}{\sqrt{x}+1}\,dx$ is divergent.

b) $L = \int_{t_0}^{t_1} \sqrt{x'^2(t) + y'^2(t)}\,dt$

$x'^2 = (3t^2)^2$

$y'^2 = (3t)^2$

$$L = \int_0^{\sqrt{3}} \sqrt{9t^4 + 9t^2}\,dt = 3\int_0^{\sqrt{3}} t\sqrt{t^2+1}\,dt = \frac{3}{2}(t^2+1)^{3/2}\frac{2}{3}\Big|_0^{\sqrt{3}}$$

$$L = 4^{3/2} - 1 = 7$$