WORKSHEET 1

Course: Mat101E
Content: Preliminaries

1. Write an equation for the line that passes through
   (a) $(-1,4)$ and $(0,2)$; (b) $(1,1)$ and $(0,1)$; (c) $(1,1)$ and $(1,0)$.
2. Write an equation for the line that passes through
   (a) $(-2,2)$ with slope -1; (b) $(-3,-1)$ with slope 0; (c) $(5,-6)$ and has no slope.
3. Write an equation for the line with
   (a) y-intercept 2 and slope $-2/3$; (b) x-intercept 2 and y-intercept -2;
   (c) x-intercept 3 and no slope; (d) x-intercept 3 and slope 0.
4. Write an equation for the line passing through the given point P that are (i) parallel and (ii) perpendicular to the given line:
   (a) $P(-1,2)$, $2x+y=4$; (b) $P(-1,1/2)$, $y=5$; (c) $P(2,-1)$, $x=1$.
5. Find the value of c for the lines passing through the given points with the given slope:
   (a) $(-2,4)$, $(2c,1)$, $m=1/2$; (b) $(-2,c^2)$, $(1,c)$, $m=0$;
   (c) $(c+1,-2)$, $(c^2-3c+5,5)$, no slope.
6. Find the domain and the range of the following functions:
   (a) $f(x)=\sqrt{1-x^2}$; (b) $f(x)=\dfrac{1}{\sqrt{x^2-1}}$; (c) $f(x)=2-\mid 3-x \mid$;
   (d) $f(x)=\dfrac{1}{1-3^{-x}}$; (e) $f(x)=\dfrac{1}{\ln^2(x+1)}$; (f) $f(x)=1+\cos(x+\pi)$;
   (g) $f(x)=\begin{cases} 4-x^2 & x\le 1 \\ (3/2)x+(3/2) & 1<x\le 3 \\ x+3 & x>3 \end{cases}$.
7. Compare the domain and the range of the functions $y=\sqrt{x^2}$ and $y=(\sqrt{x})^2$.
8. Find the domain and the range of the following functions:
   (a) $f(x)=\sqrt{\ln(x^2-8)}$; (b) $f(x)=\cos^{-1}(\ln x)$; (c) $f(x)=\ln(\sin 3x)$.
9. Graph the following equations. Determine whether the curve is a graph of a function. If so, is it a 1-1 function and is it possible to define the inverse function? What symmetries, if any, do the graphs have?
   (a) $\mid x+y \mid \ =1$; (b) $\mid x \mid + \mid y \mid \ =1$; (c) $y=-\dfrac{1}{x}$; (d) $y=-\dfrac{1}{x^2}$.

10. Determine whether the following functions are even, odd, or neither?

(a) $f(x) = \dfrac{x}{x^2-1}$; (b) $f(x) = x\sin x - \cos x$; (c) $f(x) = 2 \mid x \mid -1$;

(d) $f(x) = 2^x(1-\sec x)$; (e) $f(x) = \ln\dfrac{x-2}{x+2}$; (f) $f(x) = \ln(\sqrt{4x^2+1}-2x)$.

11. Assume that f is an even and g is an odd function, and both f and g are defined on the entire real line. Determine whether the following functions are even, odd, or neither?

(a) $f^2 = f \cdot f$; (b) f/g ; (c) gog .

12. Graph the following functions by using shifting and reflection:

(a) $y = \sqrt{x}$,
(b) $y =\mid \sqrt{x} \mid$,
(c) $y = \sqrt{\mid x \mid}$,
(d) $y = x^3$,
(e) $y =\mid x^3 \mid$,
(f) $y = \sin x$,
(g) $y =\mid \sin x \mid$,
(h) $y = \sin(x + \dfrac{\pi}{2}) - 1$,
(i) $y = \dfrac{1}{x}$,
(j) $y =\mid \dfrac{1}{x} \mid$,
(k) $y = 1 - 3^{x+1}$,
(l) $y = 1 - \ln x$,
(m) $y = -(x+1)^2$,
(n) $y = (-x+1)^2$,
(o) $y = \cos^{-1}(x+1) + \dfrac{\pi}{2}$.

13. For the following functions find f^{-1} and verify that $(fof^{-1})(x) = (f^{-1}of)(x) = x$. Graph f and f^{-1}.

(a) $f(x) = x^2 + 2x + 1,\ x \geq -1$; (b) $f(x) = \dfrac{x+5}{x-3},\ x \neq 3$;

(c) $f(x) = (2-x)(x-8),\ x \leq 5$; (d) $f(x) = \sqrt{x-1} + 2$.

14. Let $f(x) = \ln\left(\dfrac{5x-x^2}{4}\right)$ and $g(x) = \sqrt{x}$. Find the domain of the composition $(gof)(x)$.

15. Find the functions $f(x)$ and $g(x)$ which satisfy the equations $(gof)(x) =\mid \sin x \mid$ and $(fog)(x) = (\sin\sqrt{x})^2$.

16. Find the values of the following trigonometric expressions:

(a) $\cos\dfrac{3\pi}{4}$ (b) $\csc\dfrac{3\pi}{4}$ (c) $\sin\dfrac{4\pi}{3}$ (d) $\cot\dfrac{4\pi}{3}$ (e) $\sec\dfrac{11\pi}{6}$ (f) $\tan\dfrac{11\pi}{6}$

17. Express the given quantity in terms of $\sin x$ and $\cos x$.

(a) $\cos(\dfrac{\pi}{2} \mp x)$ (b) $\sin(\pi \mp x)$ (c) $\tan(\dfrac{3\pi}{4} \mp x)$

18. By using the sum formulas, find the values of the following trigonometric expressions:

(a) $\sin\dfrac{5\pi}{12} + \sin\dfrac{\pi}{12}$ (b) $\cos\dfrac{7\pi}{12} - \cos\dfrac{\pi}{12}$

19. Let $a = x^2 + x + 1$, $b = 2x + 1$ and $c = x^2 - 1$ be the sides of a triangle ABC. Find the angle A.

20. Evaluate the following:

(a) $\sin^{-1}\frac{1}{2}$ (b) $\sin^{-1}(-\frac{1}{2})$ (c) $\cos^{-1}\frac{1}{\sqrt{2}}$ (d) $\cos^{-1}(-\frac{1}{\sqrt{2}})$

21. Prove the following identities.

(a) $\sin(\cos^{-1}x) = \sqrt{1-x^2}$

(b) $\cos(\sin^{-1}x) = \sqrt{1-x^2}$

(c) $\cos^{-1}x + \sin^{-1}x = \pi/2$

(d) $\cos^{-1}x + \cos^{-1}(-x) = \pi$

(e) $\cot^{-1}\frac{1}{x} - \tan^{-1}x = \pi,\ x < 0$

(f) $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) - \sin^{-1}x = 0$

22. Find the parametric equations and a parameter interval for the motion of a particle that starts at $(a, 0)$ and traces the circle $x^2 + y^2 = a^2$

(a) once clockwise (b) twice counterclockwise.

23. Find a Cartesian equation for each of the following paths. Graph the Cartesian equation.

(a) $x = \sec^2 t - 1, \quad y = \tan t, \quad -\pi/2 < t < \pi/2$

(b) $x = 4\sin t, \quad y = 4\cos t, \quad 0 \le t \le 2\pi$

(c) $x = 4\cos t, \quad y = 2\sin t, \quad 0 \le t \le \pi$

(d) $x = 4\cos t, \quad y = 2\sin t, \quad -\pi/2 \le t \le \pi/2$

(e) $x = 2t + 3, \quad y = t^2 - 1, \quad -2 \le t \le 2$

24. Find parametrizations for

(a) the line segment with the end points $(-1, 3)$, $(2, 3)$

(b) the upper half of the parabola $x - 1 = y^2$

(c) the ray (half line) with the initial point $(-1, 2)$ that passes through the point $(0, 0)$