WORKSHEET-6

Course title: MAT101E
Content: Transcendental Functions

1. Find dy/dx.

   (a) $y = \left(1 + \frac{1}{x}\right)^x$

   (b) $y = 2^x \ \lg_3(x^2 + 1)$

   (c) $y = x(\sin(\ln x) - \cos(\ln x))$

   (d) $y = (\sin x)^x$

   (e) $y = \int_0^{\ln x} \sin e^t \ dt$

   (f) $y = \int_{e^{4\sqrt{x}}}^{e^{2x}} \ln t \ dt$

   (g) $\ln y = e^y \sin x$

   (h) $y = \dfrac{\ln x}{1 + \ln x}$

   (i) $y = x^3(\sin^{-1} x)^2$

   (j) $\tan y = e^x + \ln x$

   (k) $y = (\cos x)^x$

   (l) $y = \sinh^{-1} \dfrac{1}{1 - x}$

   (m) $e^{2x} = \sin(x + 3y)$

   (n) $y = \ln(\cosh^{-1} x) - \cosh^{-1}(\ln x)$

   (o) $y = e^{\sinh x} + \tan^{-1}(\cosh x)$

   (p) $y = (\ln x)^{\ln x}$

   (q) $y = \sqrt[5]{\dfrac{(x^2 + 1)\,(x + 3)^{1/2}}{x - 1}}$

   (r) $y = \ln \sqrt[3]{\dfrac{x\,(x + 1)\,(x - 2)^{1/2}}{(x^2 + 1)\,(2x + 3)}}$

   (s) $y = \log_3 \left(\dfrac{x + 1}{x - 1}\right)^{\ln 3}$

   (t) $y = x^{\sin x}$

2. Graph following functions.

   (a) $f(x) = x(\ln x)^2$

   (b) $f(x) = (x + 2)e^{\frac{1}{x+2}}$

   (c) $f(x) = x \ e^x$

   (d) $y = xe^{1/x}$

   (e) $y = \ln(\sinh x)$

   (f) $y = \dfrac{\ln x}{x}$

   (g) $y = \sin^{-1} \dfrac{4x}{x^2 + 4}$

3. Find the area of the regions enclosed by the following curves.

   (a) $y = e^x, \ y = -e^{-x}, \ x = -1, \ x = 1$

   (b) $y = e^x, \ y = \ln x, \ x = 1, \ x = 2$

   (c) $y = \ln x, \ y = 0, \ x = 0, \ y = 1$

   (d) $y = \dfrac{(\ln x)^2}{x}, \ x = 1, \ x = e^3$

   (e) $y = \operatorname{sech} x, \ y = 0, \ x = -\ln\sqrt{3}, \ x = \ln\sqrt{3}$

4. Find the area between the curve $y = \tan x$ and the x-axis from $x = -\pi/4$ to $x = \pi/3$

5. Evaluate following integrals.

(a) $\displaystyle\int \frac{4}{2x+1}\,dx$

(b) $\displaystyle\int \frac{e^x\,dx}{e^x-1}$

(c) $\displaystyle\int \frac{e^x\,dx}{\sqrt{1+e^{2x}}}$

(d) $\displaystyle\int \frac{\cot x\,dx}{\sqrt{\ln(\sin x)}}$

(e) $\displaystyle\int \frac{\cot^{-1} x}{1+x^2}\,dx$

(f) $\displaystyle\int \frac{e^{\sin^{-1} x}\,dx}{\sqrt{1-x^2}}$

(g) $\displaystyle\int \frac{dx}{x\ln x\sqrt{\ln^2 x-1}}$

(h) $\displaystyle\int \frac{1+\sinh\sqrt{x}}{\sqrt{x}}\,dx$

(i) $\displaystyle\int \coth^2 x\,dx$

(j) $\displaystyle\int \frac{1+\cosh 2x}{\sinh^2 2x}\,dx$

(k) $\displaystyle\int \frac{dx}{x\sqrt{\ln^2 x-1}}\,dx$

(l) $\displaystyle\int \frac{\cos 2x}{\sqrt{1+\sin^2 2x}}\,dx$

(m) $\displaystyle\int \frac{\cosh\theta\,d\theta}{\sinh\theta+\cosh\theta}$

(n) $\displaystyle\int \frac{\tan x\,dx}{\sqrt{1+\cos^2 x}}$

(o) $\displaystyle\int \frac{dx}{1+\cosh^2 x}$

(p) $\displaystyle\int_0^{\pi/3} \frac{4\sin\theta}{1-4\cos\theta}\,d\theta$

(q) $\displaystyle\int_{\pi/4}^{\pi/2} (1+e^{\cot\theta})\csc^2\theta\,d\theta$

(r) $\displaystyle\int_0^{\pi/4} (1/3)^{\tan t}\sec^2 t\,dt$

6. Suppose that the differentiable function $y = f(x)$ has an inverse and that the graph of f passes through the point $(2, 4)$ and has a slope $1/3$ there. Find the value of df^{-1}/dx at $x = 4$.

7. Locate and identify the absolute extreme values of

(a) $y = \ln(\cos x)$ on $[-\pi/4, \pi/3]$,

(b) $y = \cos(\ln x)$ on $[1/2, 2]$,

(c) $y = e^x - 2x$ on $[0, 1]$.