WORKSHEET 9

Course: Mat101E
Content: Polar Coordinates and Graphs

1. Graph the set of points whose polar coordinates satisfy the given equations and inequalities:

   (a) $1 \le r \le 2$ and $0 \le \theta \le \pi/2$,

   (b) $r \le 0$ and $\theta = \pi/4$.

2. Replace the following Cartesian equations by equivalent polar equations:

   (a) $xy = 2$,

   (b) $(x+2)^2 + (y-5)^2 = 16$,

   (c) $x = 7$,

   (d) $x^2 + xy + y^2 = 1$.

3. Replace the following polar equations by equivalent Cartesian equations:

   (a) $r^2 = -4\, r \cos\theta$,

   (b) $r = \theta$,

   (c) $r = (\csc\theta)\, \mathrm{e}^{r\cos\theta}$,

   (d) $r = 2\cot\theta \csc\theta$,

   (e) $r = \dfrac{5}{\sin\theta - 2\cos\theta}$.

4. Graph the following polar curves:

   (a) $r = 2\cos\theta + 1$,

   (b) $r = a\,(1 + \sin\theta)$,

   (c) $r^2 = -\sin 2\theta$,

   (d) $r = 1 + 2\sin\theta$,

   (e) $r = 2\cos 3\theta$.

   (f) $r = \cos 2\theta$

   (g) $r^2 = \cos 2\theta$

   (h) $r = 2/(1 - \cos\theta)$

5. Find the points of intersection of the pairs of curves:

   (a) $r = 1 + \cos\theta,\ r = 1 - \cos\theta$,

   (b) $r = 2\sin\theta,\ r = 2\sin 2\theta$,

   (c) $r = \cos\theta,\ r = 1 - \cos\theta$,

   (d) $r = 1,\ r^2 = 2\ \sin 2\theta$.

6. Find equations for the horizontal and vertical tangent lines to the curves.

   (a) $r = -1 + \sin\theta,\ 0 \le \theta \le 2\pi$,

   (b) $r = 1 + \cos\theta,\ 0 \le \theta \le 2\pi$,

   (c) $r = 2\sin\theta,\ 0 \le \theta \le \pi$,

   (d) $r = 3 - 4\cos\theta,\ 0 \le \theta \le 2\pi$.

7. Find the areas of the following regions:

   (a) Inside the oval limaçon $r = 4 + 2\cos\theta$,

   (b) Inside the cardioid $r = a(1 + \cos\theta)$, $a > 0$,

   (c) Inside the lemniscate $r^2 = 2\,a^2\cos(2\theta)$,

   (d) Inside the six-leaved rose $r^2 = 2\sin(3\theta)$.

8. Find the areas of the following regions:

   (a) Shared by the circles $r = 2\cos\theta$ and $r = 2\sin\theta$,

   (b) Shared by the circle $r = 2$ and the cardioid $r = 2\,(1 - \cos\theta)$,

   (c) Inside the circle $r = -2\,\cos\theta$ and outside the circle $r = 1$,

   (d) Inside the circle $r = 6$, above the line $r = 3\,\csc\theta$,

   (e) Inside the circle $r = 3\,a\cos\theta$ and outside of the cardioid $r = a\,(1 + \cos\theta)$, $a > 0$.

9. Find the lengths of the following curves:

   (a) The spiral $r = \theta^2$, $0 \le \theta \le \sqrt{5}$,

   (b) The curve $r = a\,\sin^2(\theta/2)$, $0 \le \theta \le \pi$, $a > 0$,

   (c) The curve $r = \cos^3(\theta/3)$, $0 \le \theta \le \pi/4$,

   (d) The spiral $r = \mathrm{e}^{\theta}/\sqrt{2}$, $0 \le \theta \le \pi$.