

WORKSHEET-7

Course title: MAT101E

Content: Integration Techniques, L'Hopital's Rule and Improper Integrals.

1. Evaluate following limits.

(a) $\lim_{x \rightarrow 0^+} x \ln(x) = ?$

(b) $\lim_{x \rightarrow 0} \left(1 + \sin^2 x\right)^{1/x^2} = ?$

(c) $\lim_{x \rightarrow 0} \left(\cos 2x\right)^{3/x^2} = ?$

(d) $\lim_{x \rightarrow 0} \left(\frac{\sinh^{-1} x}{x}\right)^{1/x^2} = ?$

(e) $\lim_{x \rightarrow \infty} [x - (x+2)e^{1/x}] = ?$

(f) $\lim_{x \rightarrow \infty} (\sinh^{-1} x)^{1/\log_3 x} = ?$

(g) $\lim_{x \rightarrow -2} \frac{x^2 + 4x + 4}{[\sin^{-1}(3x+6)]^2} = ?$

(h) $\lim_{x \rightarrow 0^+} \frac{\int_0^x t \sin^{-1} \sqrt{1-t^2} dt}{\lg_2(x+1) + x \sin \pi x} = ?$

(i) $\lim_{x \rightarrow \infty} \frac{\tan^{-1}(1/x)}{\sin(1/x)} = ?$

(j) $\lim_{x \rightarrow 0} \frac{x}{\tan^{-1}(1+x) - \frac{\pi}{4}} = ?$

(k) $\lim_{x \rightarrow \infty} \frac{e^{1/x^2} - 1}{2 \tan^{-1} x^2 - \pi} = ?$

(l) $\lim_{x \rightarrow 1} \sin^{-1} \frac{x-1}{2} \cot(x-1) = ?$

(m) $\lim_{x \rightarrow 0^+} x^{\frac{1}{4+\ln x}} = ?$

(n) $\lim_{x \rightarrow \infty} x(1 - \tanh x) = ?$

(o) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{2x/3} = ?$

2. Evaluate following limits.

(a) $\lim_{x \rightarrow \infty} (\pi/2 - y) \tan y$

(b) $\lim_{x \rightarrow \infty} (\ln 2x - \ln(x+1))$

(c) $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \cos \sqrt{t} dt}{x^2}$

(d) $\lim_{x \rightarrow 0} \frac{x}{\tan^{-1}(1+x) - \frac{\pi}{4}}$

(e) $\lim_{x \rightarrow 0^+} \left(\cot \frac{x}{2}\right)^{\frac{1}{\ln 3x}}$

(f) $\lim_{x \rightarrow (\pi/2)^-} (\cos x)^{\cos x}$

(g) $\lim_{x \rightarrow 0} \frac{\sinh^{-1} x - \sin^{-1} x}{2^x - e^{\sin x}}$

(h) $\lim_{x \rightarrow \infty} \frac{\tanh x - 1}{e^{1/x} - 1}$

(i) $\lim_{x \rightarrow 0^-} \frac{1}{\sin x} \int_0^x \tan^{-1} \frac{1}{t^2} = ?$

(j) $\lim_{x \rightarrow 0} \frac{1+x^2}{x} \tan^{-1} x = ?$

(k) $\lim_{x \rightarrow 0} (\sin x)^{\sin x} = ?$

(l) $\lim_{x \rightarrow 0} (1 - \tan x)^{\csc x} = ?$

3. Evaluate following integrals.

$$\begin{array}{lll}
 \text{(a)} \int \frac{6-x}{(x-3)(2x+5)} dx & \text{(f)} \int \frac{\cos^2 x dx}{1+\sin 2x} & \text{(k)} \int \sin(\ln x) dx \\
 \text{(b)} \int x^{-1} \ln x dx & \text{(g)} \int \frac{\sqrt[3]{x+1}}{x} dx & \text{(l)} \int x^2 \sin x dx \\
 \text{(c)} \int x \ln(x+3) dx & \text{(h)} \int \frac{\sin 6x}{1-\cos^4 3x} dx & \text{(m)} \int \sin^{-1} x dx \\
 \text{(d)} \int \frac{\sqrt{9-x^2}}{x^2} dx & \text{(i)} \int \frac{(2x^2+2x+1)dx}{(x+1)(x^2+x+1)} & \text{(n)} \int \frac{e^x dx}{(e^x-1)(e^{3x}-1)} \\
 \text{(e)} \int \frac{2y^4}{y^3-y^2+y-1} dy & \text{(j)} \int \cosh x \sin x dx & \text{(o)} \int \sqrt{12+4x-x^2} dx \\
 & & \text{(p)} \int \frac{dx}{(x^2+1)(x^2+x+1)}
 \end{array}$$

4. Evaluate following integrals.

$$\begin{array}{ll}
 \text{(a)} \int_{e^{\pi/3}}^1 \frac{dx}{x \cos(\ln x)} & \text{(l)} \int 6 \sinh\left(\frac{x}{2} - \ln 3\right) dx \\
 \text{(b)} \int_{\pi/3}^{\pi/4} \frac{dx}{1+\sin x} & \text{(m)} \int \cosh^2(5-x) dx \\
 \text{(c)} \int_{\pi/3}^{\pi/4} \frac{dx}{\sec x + \tan x} & \text{(n)} \int_1^e \frac{dx}{x\sqrt{1+(\ln x)^2}} \\
 \text{(d)} \int_4^9 \frac{dx}{\sqrt{x}(\sqrt{x}-1)} & \text{(o)} \int (\sin 3x \cos 2x - \cos 3x \sin 2x) dx \\
 \text{(e)} \int_1^{1/2} \frac{x-4}{x^2+x} dx & \text{(p)} \int_0^{\pi/4} \frac{1+\sin x}{\cos^2 x} dx \\
 \text{(f)} \int_0^{-1} \frac{x^3}{x^2-2x+1} dx & \text{(q)} \int_0^{2\pi} \sqrt{\frac{1-\cos x}{2}} dx \\
 \text{(g)} \int_3^2 \frac{1}{(x^2-1)^2} dx & \text{(r)} \int \frac{7dx}{(x-1)\sqrt{x^2-2x-48}} \\
 \text{(h)} \int_1^0 \frac{1}{(x^2+9)} dx & \text{(s)} \int_{-1}^1 \frac{dx}{\sqrt[3]{\tan^{-1} x} (1+x^2)} \\
 \text{(i)} \int_0^1 \frac{1}{(x^2+3x+1)} dx & \text{(t)} \int_{-1}^1 \frac{dx}{(1+\cos^{-1} x) \sqrt{1-x^2}} \\
 \text{(j)} \int_e^1 \frac{dx}{x\sqrt{1+(\ln x)^2}} & \text{(u)} \int_0^1 \frac{1+\sinh \sqrt{x}}{\sqrt{x}} dx \\
 \text{(k)} \int_{\pi/2}^{\pi/3} \frac{dx}{\sin x - \cos x} &
 \end{array}$$

5. Evaluate following integrals.

(a) $\int \frac{\sqrt{2-3x}}{\sqrt[3]{2-3x+2}} dx$	(j) $\int \tan^3 x dx$	(s) $\int x^2 e^{-x} dx$
(b) $\int \frac{dx}{x^2 \sqrt{x+7}} dx$	(k) $\int \frac{\sinh x dx}{2 \cosh x - \sinh x - 1}$	(t) $\int \sqrt{\frac{1+x}{1-x}} dx$
(c) $\int x \sqrt{3-2x^2} dx$	(l) $\int \frac{6x-5}{4x+7} dx$	(u) $\int \ln x dx$
(d) $\int \frac{dx}{\cosh x}$	(m) $\int \frac{dx}{2 \sin x - \cos x + 5}$	(v) $\int \frac{2\sqrt{x}}{\sqrt{x}} dx$
(e) $\int \sqrt{x^2+1} dx$	(n) $\int \frac{dx}{\sin^2 x - 2 \cos^2 x}$	(w) $\int e^x \cos x dx$
(f) $\int \sqrt{x^2-9} dx$	(o) $\int \frac{dx}{\sqrt{x^2-3}}$	(x) $\int \frac{dx}{1 + \cosh^2 x}$
(g) $\int \frac{dx}{x^2+4}$	(p) $\int \frac{dx}{\sin^2 x}$	(y) $\int \frac{1 + \sinh 3x}{\cosh^2 3x} dx$
(h) $\int \frac{dx}{\cos^3 x}$	(q) $\int \sqrt{4-6x-x^2} dx$	(z) $\int \frac{1 + \sinh 3x}{\cosh^2 3x} dx$
(i) $\int \frac{dx}{\sin^2 x \cos^3 x}$	(r) $\int \frac{dx}{\sqrt{-5+4x+x^2}}$	

6. Evaluate following integrals.

(a) $\int x^2 \ln x dx$	(i) $\int \frac{(x+1)^2 \tan^{-1} 3x + 9x^3 + x}{(9x^2+1)(x+1)^2} dx$
(b) $\int x(\ln x)^2 dx$	(j) $\int \frac{2x^3 + x^2 + 2x - 1}{(x^2+1)(x^2-1)} dx$
(c) $\int \frac{\cos^2 x dx}{1 + \sin 2x}$	(k) $\int \frac{\sqrt{1-x^2} + \sqrt{1+x^2}}{\sqrt{1-x^4}} dx$
(d) $\int \cos(\ln x) dx$	(l) $\int \frac{1}{x\sqrt{x^6-1}} dx$
(e) $\int \sin x \ln(\cot x) dx$	(m) $\int \frac{x+1}{4-x^2} dx$
(f) $\int \sqrt{x} \cosh \sqrt{x} dx$	(n) $\int \frac{e^x}{\sqrt{e^{2x}-1}} dx$
(g) $\int x(\tan^{-1} x)^2 dx$	
(h) $\int \frac{4 \cos 2x}{(2 - \sin^2 2x) \sin 2x} dx$	

7. Evaluate following integrals.

$$\begin{array}{lll}
 \text{(a)} \int (x+2)^3 \ln x dx & \text{(g)} \int \frac{t^4+9}{t^4+9t^2} dt & \text{(m)} \int \sqrt{1+9x^2} dx \\
 \text{(b)} \int x \sec^2 x dx & \text{(h)} \int \frac{x+3}{2x^3-8x} dx & \text{(n)} \int \frac{6dt}{(9t^2+1)^2} \\
 \text{(c)} \int x^5 e^x dx & \text{(i)} \int \frac{x^3 dx}{x^2+2x+1} & \text{(o)} \int \frac{(1-x^2)^{3/2}}{x^6} dx \\
 \text{(d)} \int z(\ln z)^2 dz & \text{(j)} \int \frac{16x^3 dx}{4x^2-4x+1} & \text{(p)} \int_{\ln(3/4)}^{\ln(4/3)} \frac{e^t}{1+e^{2t}} dt \\
 \text{(e)} \int \ln(a^2+x^2) dx & \text{(k)} \int \frac{\cos y dy}{\sin^2 y + \sin y - 6} & \text{(q)} \int \frac{dt}{1+\sin t + \cos t} \\
 \text{(f)} \int \sin(\ln x^2) dx & \text{(l)} \int \frac{x^2+2}{4x^5+4x^3+x} dx &
 \end{array}$$

8. Evaluate following integrals.

$$\begin{array}{lll}
 \text{(a)} \int_1^2 \frac{1}{x\sqrt{x^2-1}} dx & \text{(c)} \int_0^\pi x \cos 3x dx & \text{(e)} \int_{-\pi/2}^{\pi/2} \frac{2 \cos \theta d\theta}{1+(\sin \theta)^2} \\
 \text{(b)} \int_0^1 \frac{x dx}{(x+1)^2(x^2+1)} & \text{(d)} \int_{-5/2}^{1/2} \frac{2x+5}{\sqrt{8-2x-x^2}} dx & \text{(f)} \int_0^1 \frac{\sin^2(\sin^{-1} x) dx}{\sqrt{1-x^2}}
 \end{array}$$

9. Evaluate following integrals and investigate their divergence.

$$\begin{array}{l}
 \text{(a)} \int_0^\infty \frac{dv}{(1+v^2)(1+\tan^{-1} v)} \\
 \text{(b)} \int_0^1 \frac{4r}{\sqrt{1-r^4}} dr \\
 \text{(c)} \int_{-\infty}^\infty e^{-|x|} dx \\
 \text{(d)} \int_0^1 (-\ln x) dx
 \end{array}$$

10. Which of the following integrals are convergent, which are divergent? Explain your answer.

$$\begin{array}{ll}
 \text{(a)} \int_1^\infty \frac{1}{e^x-2^x} dx & \text{(f)} \int_0^1 \frac{2x-\sin x}{x^3} dx \\
 \text{(b)} \int_0^\infty \frac{x^2+1}{x^4+1} dx & \text{(g)} \int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx \\
 \text{(c)} \int_2^\infty \frac{\sqrt{r}}{\sqrt{r^4-1}} dr & \text{(h)} \int_1^2 \frac{1}{(x^3-x)^{1/4}} dx \\
 \text{(d)} \int_0^1 \frac{1}{\sqrt{x} \ln x} dx & \text{(i)} \int_0^{\pi/2} \tan x dx \\
 \text{(e)} \int_{-\infty}^{-1} \frac{1}{x^2+1} dx &
 \end{array}$$

11. Investigate the convergence of $\int_{-1}^1 \frac{dx}{\sqrt{|x|}}$ by using the definition of the improper integral.

12. Investigate the convergence of $\int_2^\infty \frac{x}{(\ln x)^3} dx$.

13. Evaluate $\int_0^{\infty} \frac{\tan^{-1} x}{1+x^2} dx$.

14. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{3e^x + e^{-x}}$.

15. Investigate the convergence of $\int_{-1}^{\infty} \frac{du}{(u+5)\sqrt{u+1}}$ by evaluating the integral.